UNIVERSITY OF TORONTO

Faculty of Arts and Science

December 2017 Final exam

Advanced Economic Theory, ECO326H1F

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Duration - 120 minutes

No Aids Allowed

There are three questions with total worth of 100 points. Read the questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You do not need to compute the exact values of algebraic formulas (for example, it is OK to say that $x = 2 * \frac{1}{5} * \sqrt{100}$ instead of $x = 4$.) Questions denoted with (*) are for extra credit; they are more difficult, and you should attempt them only if you have extra time.

You have 120 minutes.

Good luck!

Total pages (including the title page): 4

Total marks: 100

(1) There are two firms $i = 1, 2$ facing a decision whether to enter a new market. The cost of entry is the same for both firms, but it is not fully known. With equal probability $\frac{1}{3}$, the cost can take one of three values $0 < f_L < f_M < f_H$. After doing some research, firms can learn some information about the cost. Precisely, firm 1 is able to learn whether the cost is high (f_H, with probability $\frac{1}{3}$) or not (f_L or f_M, with probability $\frac{2}{3}$). Firm 1 is not able to distinguish between the medium and the low levels of cost. Firm 2 observes whether the cost is low (f_L) , with probability $\frac{1}{3}$ or not (f_M or f_H). or not, with probability $\frac{2}{3}$; but, if it learns that the cost is not low, firm 2 does not know whether the cost is medium or high.

(a) Describe the types and beliefs of each firm.

(b) If firm i enters, it observes and pays the entry cost f, and then chooses the quantity q_i. The payoffs are

$$q_i (\alpha - c - (q_1 + q_2)) - f,$$

where we assume that the quantity produced by the firm that did not enter is equal to 0. The payoffs of the firm that did not enter is equal to 0.

Explain that the Subgame Perfect Equilibrium (SPE) continuation payoffs of each firm are either $\frac{1}{9}(\alpha - c)^2 - f$, $\frac{1}{4}(\alpha - c)^2 - f$, or 0, depending on their decision and the decision of the other firm.

(c) Assume that $\alpha = 2$,$c = 1$, and that

$$f_L < \frac{1}{9} < f_M < \frac{1}{4} < f_H.$$

Show that each firm has a type that has an unique SPE action in the first period.

(d) Find all pure strategy Bayesian Nash equilibria.

(e) Does a game have a BNE in which some of the types randomize? If so, find it; if not, explain.

(2) (40 points) There are three people bargaining over the division of a pie of size 1 unit. The bargaining proceeds through N stages. In each period, one of the players is randomly chosen to be a proposer, independently across periods. Each player can become a proposer with equal probability $\frac{1}{3}$, and in each period, a different person can be chosen. The proposer offers a division of the pie. The remaining players accept or reject. If the players accepts, the game ends and the pie is divided according to the proposal, with each player's payoff equal to its share of the pie. If the players reject, the game proceeds to the next stage, with an exception of the last stage, when the game ends with payoffs 0 for each player. The players discount future payoffs with factor $\delta < 1$.

(a) Suppose that $N = 1$ and that player i is chosen to be a proposer. Describe the SPE strategies and payoffs.

(b) Suppose that $N = 1$. If each player can be chosen a proposer with the same probability, what is the expected payoff of each player before the proposer is chosen?

(c) Suppose that $N = 2$ and player i is chosen to be a proposer. Describe the SPE strategies and payoffs. Suppose that $N = 2$. What is the expected payoff of each player before the proposer is chosen?

(d) Suppose that $N \geq 3$. What is the expected SPE payoff of each player (before the proposer is chosen)?

(e) For the next two questions, assume that the proposal needs to be approved only by the majority (i.e., 2 people) in order to be implemented. Thus, if the proposer and one more player agrees, the proposal is accepted. Suppose that $N = 1$ and that player i is chosen to be a proposer. Describe the SPE strategies and payoffs. If each player can be chosen a proposer with the same probability, what is the expected payoff of player i before the proposer is chosen?

(f) Suppose that $N = 2$ and player i is chosen to be a proposer. Describe the SPE strategies and payoffs. Is the SPE unique? What is the (highest) expected payoff of each player before the proposer is chosen?

(3) (30 points) Ann and Bob are involved in a partnership game over T periods. In each period, player $i = A, B$ chooses effort $e_i \geq 0$ and receives payoffs

$$e_i + e_{-i} - e_i^2.$$

Here, $e_i + e_{-i}$ is the benefit from the partnership and e_i^2 is the cost of one's own effort. The players discount future payoffs with factor $\delta < 1$.

(a) Suppose that $T = 1$. Find the Nash equilibrium (e_A^*, e_B^*) of the game. Is the equilibrium unique? Compute the equilibrium payoffs.

(b) Find a profile of strategies $(e_A^{\max}, e_B^{\max})$ that maximizes the joint payoffs and explain that the equilibrium effort e_i^* is too small compared to the effort that maximizes joint payoffs $e_i^{\max}$.

(c) Suppose that $T > 1$ but $T < \infty$. Find the Subgame Perfect Equilibrium (SPE) of the finitely repeated game. Do players choose optimal level of efforts?

(d) Suppose that $T = \infty$. Can you construct an equilibrium in which players choose efforts $e_i^{\max}$? How does your answer depend on the discount factor?

(e) Suppose that $T = 2$ and that, additionally to the partnership game, the players in each period play the following investment game:

A\B	Invest	Don't
Invest	a, a	$-1, 0$
Don't	$0, -1$	$0, 0$

where $a > 0$ is a parameter. The total payoffs of the player in each period are equal to the sum of payoffs from the partnership and the investment games. Show that there exist a SPE in which the first period efforts are higher than e_i^*. What is the highest first-period effort that is possible in an SPE?

Total pages: 4

Total marks: 100