Midterm

Advanced Economic Theory, ECO326F1H

October, 2013

There are three questions. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 100 minutes.

(1) (30 points) Consider the following game:

Player 1 \ Player 2	L	C	R
U	$3,2$	$0,0$	$0,1$
M	$0,0$	$8,2$	$0,1$
D	$2,0$	$2,0$	$-1,0$

(a) Find all strictly dominated strategies.

(b) Find all pure strategy equilibria.

(c) Find all mixed strategy equilibria.

(2) (30 points) Xenia and Zander are involved in a joint project. The value of the project for each them depends on their efforts. Let $e_x \geq 0$ and $e_z \geq 0$ denote the effort of both partners. The utility of player $i = x, y$ is equal to

$$u_i\left(e_i, e_{-i}\right) = \begin{cases} e_i\left(1 + e_{-i}\right) - e_i^2 - c & \text{, if } e_i > 0, \\ 0, & \text{if } e_i = 0. \end{cases}$$

Here, $c \geq 0$ is the cost of a strictly positive amount of effort.

(a) Assume first that $c = 0$. Find the best response strategies of each player. Find the Nash equilibrium.

(b) Assume that $c > 0$. Find the best response strategies of both players.

(c) For which c does there exist a Nash equilibrium in which no player provides positive effort?

(d) Find a Nash equilibrium for each c. Be careful of considering separately all the relevant cases. For which c does there exist an equilibrium in which at least one player provides positive effort?

(3) (40 points) A course in behavioral economics is taken by N students. The final grade depends solely on the score $q \in \{0, 1, ..., 100\}$ from an in-class midterm that students take in early October. After the midterm, the professor hangs the list with the midterm scores on her door, so that everybody can observe everybody's grade.[1] After learning the midterm scores, each student has one week to decide between dropping (D) or staying (S) in the class for the rest of the semester. Any student who drops the class receives utility 0. A student with midterm grade q who stays in the class receives utility (a) equal to

$$2\frac{q}{100} - 1 - \frac{1}{2}$$

if the student has the lowest grade *among all the students who decide to stay in the class*, and (b) equal to

$$2\frac{q}{100} - 1$$

if there is another student who stayed in the class and who has a strictly lower grade than q. The difference, $\frac{1}{2}$, is the disutility of being at the bottom rank in the class.

In your answers below, you can assume that there are $N = 101$ students and each of them received a different score on the midterm (so that there is one student with score 0, one student with score 1, ..., and one student with score 100). This assumption is not necessary in any way, but it may help you to think through the question.

(a) Consider a student with grade $q = 0$. Does he or she have dominated strategies?

(b) Consider a student with grade $q = 100$. Does he or she have dominated strategies?

(c) Consider a student with grade $q = 60$. Does he or she have dominated strategies?

(d) Describe all the strictly dominated strategies for student with a score q. How does your answer depend on q?

[1] This story does not take place in the University of Toronto, where revealing publicly the grades would be completely illegal.

(e) Consider a student with grade q who knows that all the students in the class are rational (but she cannot assume anything else). What can you say about her strategies? How does your answer depend on q?

(f) Which strategies survive the iterated elimination of strictly dominated strategies for student with grade q?

(g) Describe an equilibrium of this game.