Midterm

Advanced Economic Theory, ECO326F1H

October, 2013

There are three questions. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 100 minutes.

(1) (30 points) Consider the following game:

Player 1 \ Player 2	L	C	R
U	$3, 2$	$0, 0$	$0, 1$
M	$0, 0$	$8, 2$	$0, 1$
D	$2, 0$	$2, 0$	$-1, 0$

(a) Find all strictly dominated strategies.

Notice that D is strictly dominated by a mixed strategy $U^{0.7}M^{0.3}$.

(b) Find all pure strategy equilibria.

(L, U) and (C, M)

(c) Find all mixed strategy equilibria.

Notice that D cannot be played in equilibrium. In order to make player 1 indifferent between the two remaining strategies, player 2 has to play $\left(L^{\beta}C^{\frac{3}{8}\beta}R^{\alpha}\right)$ where $\alpha, \beta \geq 0$ and such that $\beta + \frac{3}{8}\beta + \alpha = 1$. In order to make player 2 randomize indifferent between L and C, player 1 must randomize $U^{1/2}M^{1/2}$. In particular, this covers an equilibrium, where player 2 plays only R.

(2) (30 points) Xenia and Zander are involved in a joint project. The value of the project for each them depends on their efforts. Let $e_x \geq 0$ and $e_z \geq 0$ denote the effort of both partners. The utility of player $i = x, y$ is equal to

$$u_i(e_i, e_{-i}) = \begin{cases} e_i(1 + e_{-i}) - e_i^2 - c & , \text{ if } e_i > 0, \\ 0, & \text{if } e_i = 0. \end{cases}$$

Here, $c \geq 0$ is the cost of a strictly positive amount of effort.

(a) Assume first that $c = 0$. Find the best response strategies of each player. Find the Nash equilibrium.

The best responses are

$$e_i^*(e_{-i}) = \frac{1}{2}(1 + e_{-i}).$$

In equilibrium,

$$e_x = e_y = e^* = 1.$$

(b) Assume that $c > 0$. Find the best response strategies of both players.

The best responses are

$$e_i^*(e_{-i}) = \begin{cases} \frac{1}{2}(1 + e_{-i}), & \text{if } \frac{1}{4}(1 + e_{-i})^2 \geq c \\ 0, & \text{otherwise.} \end{cases}$$

(c) For which c does there exist a Nash equilibrium in which no player provides an effort?

If the other player does not provide an effort, $e_{-i} = 0$, then the best response of player i is equal to

$$e_i^*(0) = \begin{cases} \frac{1}{2}, & \text{if } \frac{1}{4} \geq c \\ 0, & \text{otherwise.} \end{cases}$$

Thus, for $e_i(0) = 0$, it must be that $c \geq \frac{1}{4}$.

(d) Find a Nash equilibrium for each c. Be careful of considering separately all the relevant cases. For which c does there exists an equilibrium in which some players provide an effort?

There are cases to consider:

- Case 1: Both players choose 0 effort. By the previous point, this case results in an equilibrium if $c \geq \frac{1}{4}$.
- Case 2: Player i chooses positive effort and player $-i$ chooses 0 effort. In such a case, it must be that $e_i^* = \frac{1}{2}$ (see the solution to the above point) and $e_{-i}^* = 0$. We check whether these strtegies are best responses. For $\frac{1}{2}$ to be player i's best response, it must be that
$$\frac{1}{4} \geq c.$$
(Again, see the solution to the previous point.) On the other hand, the best response of player $-i$ is equal to
$$e_{-i}^*\left(\frac{1}{2}\right) = \begin{cases} \frac{1}{2}\left(1+\frac{1}{2}\right), & \text{if } \frac{1}{4}\left(1+\frac{1}{2}\right)^2 \geq c \\ 0, & \text{otherwise.} \end{cases}.$$
Thus, if the best response is equal to 0, it must be that
$$\frac{1}{4}\left(1+\frac{1}{2}\right)^2 \leq c.$$
The two conditions on c cannot be satisfied simultaneously. Thus, such a case never results in an equilibrium.
- Case 3: Both players choose positive effort. Using first order conditions, we can check that $e_i^* = 1$. Verifying as above the best response conditions, we can check that such strategies are equilibrium whenever $c \leq 1$.

(3) (40 points) A course in behavioral economics is taken by N students. The final grade depends solely on the score $q \in \{0, 1, ..., 100\}$ from an in-class midterm that students take in early October. After the midterm, the professor hangs the list with the midterm scores on her door, so that everybody can observe everybody's grade.[1] After learning the midterm scores, each student has one week to decide between dropping (D) or staying (S) in the class for the rest of the semester. Any student who drops the class receives utility 0. A student with midterm grade q who stays in the class receives utility equal to

$$2\frac{q}{100} - 1 - \frac{1}{2}$$

if the student has the lowest grade *among all the students who decide to stay in the class*, and it is equal to

$$2\frac{q}{100} - 1$$

if there is another student who stayed in the class and who has a strictly lower grade than q. The difference, $\frac{1}{2}$, is the disutility of being at the bottom rank in the class.

In your answers below, you can assume that there are $N = 101$ students and each of them received a different score on the midterm (so that there is one student with score 0, one student with score 1, ..., and one student with score 100). This assumption is not necessary in any way, but it may help you to think through the question.

(a) Consider a student with grade $q = 0$. Does he or she have dominated strategies?

Yes. Strategy S (the largest possible payoff is -1) is dominated by D (with payoff 0).

(b) Consider a student with grade $q = 100$. Does he or she have dominated strategies?

[1] This story does not take place in the University of Toronto, where revealing publicly the grades would be completely illegal.

Yes. Strategy D (the largest possible payoff is 0) is dominated by S (with the lowest possible payoff of $\frac{1}{2}$).

(c) Consider a student with grade $q = 60$. Does he or she have dominated strategies?

No. If everybody stays, then the student prefers to stay as well. If all the other students drop, then the payoff from D is 0 vs payoff from S equal to 1.2-1-0.5=-0.3<0.

(d) Describe all the strictly dominated strategies for student with a score q. How does your answer depend on q?

Strategy S is dominated for all students $q < 50$. The strategy D is dominated for all students q such that $2\frac{q}{100} - 1 - \frac{1}{2} > 0$.

(e) Consider a student with grade q who knows that all the students in the class are rational (but she cannot assume anything else). What can you say about her strategies? How does your answer depend on q? q?

Strategy S is dominated for all students $q \leq 50$. To see why, notice that a student with grade $q$expects all the students with grades lower than 50 to drop (as they are rational). But then, the only students who stay have a grade 50 or more. Thus, a student with grade $q = 50$ expects to have the lowest grade in the class, which leads to the payoff from staying equal to $-\frac{1}{2}$.
The strategy D is dominated for all students q such that $2\frac{q}{100} - 1 - \frac{1}{2} > 0$, or for $q > 75$.

(f) Which strategies survive the iterated elimination of strictly dominated strategies for student with grade q?

Let $q^* = 75$ be chosen so that $2\frac{q^*}{100} - 1 - \frac{1}{2} = 0$. Then, only strategy D survives for students $q < q^*$ and only strategy S survives for $q > q^*$. The fact that S is the only surviving strategy for students $q > q^*$ comes from one of the above points. For the former part, let q_S be the student with the lowest grade for whom S survives the iterated elimination. The payoff from S is equal to at most $2\frac{q_S}{100} - 1 - \frac{1}{2}$ (because all the students with lower grades will drop the class). The payoff from D is 0. Because S is not strictly dominated for student q_S, it must be that $2\frac{q_S}{100} - 1 - \frac{1}{2} \geq 0$, which implies that $q_S \geq q^*$.

(g) Describe an equilibrium of this game.

All students $q \geq q^*$ play S and all students $q < q^*$ play D.