UNIVERSITY OF TORONTO

October 2014 Midterm

Advanced Economic Theory, ECO326S1H

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Duration - 120 minutes

WRITE YOUR NAME AND ID ON THE BLUE BOOK. There are no aids allowed.

There are three questions with a total number of 100 points. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 110 minutes.

(1) (25 points) Consider the following game:

Player 1 (She) \ Player 2 (He)	L	C	R
U	$2,3$	$4,1$	$1,4$
M	$5,0$	$2,3$	$3,3$
D	$1,1$	$1,3$	$6,2$

(a) What can you conclude about the behavior of the players from the following statements:

(i) Player 1 is rational.

(ii) Player 1 is rational and she knows that player 2 is rational.

(iii) Player 1 is rational and she knows that player 2 is rational and that player 2 knows that player 1 is rational.

(b) Find all Nash equilibria of the above game. (For a complete answer, find all pure strategy and mixed strategy equilibria and explain why there are no more.)

(2) (35 points) There are $N \geq 2$ players playing partnership game. Each player i chooses a level of effort $q_i > 0$. The payoff of each player depends on their own efforts and the average effort in the group and it is equal to

$$u_i(q_i, q_{-i}) = q_i \left(a + \frac{1}{N}(q_1 + ... + q_N) \right) - cq_i^2.$$

Here, $a > 0$ and $c > 0$ are positive constants. We assume that $c > 1$. (For a partial answer, you can assume that $N = 2$ when solving this problem.)

(a) Find the best response function for each player.

(b) Find the Nash equilibrium strategies. (For a complete answer, you are not allowed to assume upfront that the equilibrium is symmetric.)

(c) Show that all actions

$$q_i < q^1 = \frac{1}{2} \frac{1}{c - \frac{1}{N}} a.$$

are strictly dominated for player i. Are there any other strictly dominated actions?

(d) What strategies are eliminated at the second stage of IESDS?

(3) (40 points) Consider the following version of a Cournot duopoly in which firms can upgrade their technology to produce at lower cost level. There are two firms. Each firm simultaneously chooses the cost level $c \in \{c_L, c_H\}$ and quantity $q \geq 0$. The payoffs of firm i depend on is quantity, the quantity of the competitor q_{-i}, and its own cost and they are equal to

$$\pi_i\left(\left(c_i, q_i\right), \left(c_{-i}, q_{-i}\right)\right) = \begin{cases} q_i\left(\alpha - c_i - \left(q_i + q_{-i}\right)\right) - f, & \text{if } c_i = c_L, \\ q_i\left(\alpha - c_i - \left(q_i + q_{-i}\right)\right) & \text{if } c_i = c_H. \end{cases}$$

Here, $f > 0$ is the cost of investment in the technology of producing at lower cost. We assume that $c_L < c_H < \alpha$.

(a) Compute best response quantity of firm i against (c_{-i}, q_{-i}) assuming that $i's$ cost is equal to $c_i = c_H$. Find the associated profit.

(b) Compute best response quantity of firm i against (c_{-i}, q_{-i}) assuming that $i's$ cost is equal to $c_i = c_L$. Find the associated profit.

(c) Find conditions on the parameters for which there are equilibrium in which both players choose low cost.

(d) Find conditions on the parameters for which there are equilibrium in which each player chooses a different cost.