Midterm

Advanced Economic Theory, ECO326F1H

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There are three questions. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 120 minutes.

(1) (30 points) Consider the following game.

Player 1 \ Player 2	L	C	R
U	$3, -2$	$0, -1$	$-1, 1$
M	$-3, 2$	$5, -1$	$1, -1$

(a) Find all strictly dominated strategies.

(b) Does the game have a pure strategy Nash equilibrium?

(c) Find a Nash equilibrium in mixed strategies.

(d) Suppose that player 2's payoff from playing action C is increased from -1 to 1 (against both actions of player 1 simultaneously). None of the other payoffs are affected. Does the mixed strategy profile that you found in part (c) remains an equilibrium? If not, find a new equilibrium.

(2) (40 points) In 2014, there will be N students graduating from the University of Summerville. The students will enter the Summerville job market and apply to one of the N open positions. The employers in each of the positions want to hire the students according to their grades. The University of Summerville ranks all the students and we are going to assume that all the students are arranged from s_1 (the best), $s_2, ...,$ to s_N (the worst). The students want to get the best possible job. We assume that the jobs are also arranged according to their quality: j_1 is the best job, j_2 is the second best, ..., and j_N is the worst job. Assume that all ranikings (of jobs and students) are strict (so, there are no ties).

The application process has two rounds. The first round is decentralized and takes the form of the following game. Each student chooses a job to which she or he sends her or his application. Each job which receives more than one application hires a student with the best grades among all applicants. The hired student and the job leave the market and do not participate in the second round. In the second round, all students who were not hired and all the jobs that were not filled in the first round are matched with each other in a centralized fashion: the best of the remaining students goes to the best of the remaining jobs, the second best remaining student goes to the second best remaining job, etc.

For example, suppose that in the first round, all the students apply to job j_2 (i.e., the second best job). Then, job j_2 hires s_1 (because this is the best student among all applicants). All the remaining students will be allocated in the second round according to their grades and the quality of the job. So, student s_2 will go to job j_1, s_3 will go to j_3, s_4 will go to j_4, etc.

You are supposed to analyze the behavior of the students in the game from the round 1.

(a) Show that student s_1 has a strictly dominant strategy.

(b) Does student s_2 (or s_n for $n \geq 2$) have any dominant strategy?

(c) Suppose that everybody knows that everybody is rational (i.e., we perform one round of elimination of dominated strategies). Does student s_2 have a dominant or dominated strategy?

(d) Which strategies survive the iterated elimination of (weakly) dominated strategies?

(e) Describe a Nash equilibrium of the application game.

(3) (30 points) There are two firms $i = 1, 2$ selling similar, but slightly different products. Each firm i has a constant marginal cost of production $c_i \geq 0$. Each firm i chooses price $p_i \geq 0$. The demand for products of firm i depends on its own price, as well as the price of its competitor, and it is equal to

$$D_i\left(p_i, p_{-i}\right) = \max\left(0, 10 + 2\left(p_{-i} - p_i\right)\right).$$

Each firm wants to maximize its profits equal to

$$\left(p_i - c_i\right)\left(D_i\left(p_i, p_{-i}\right)\right).$$

(a) First, suppose that both firms have the same marginal cost, $c_1 = c_2 = c$. Find the Nash equilibrium of the pricing game.

(b) Now, suppose that $c_1 > c_2 \geq 0$. Carefully describe the best response correspondence of firm i.

(c) Find the equilibrium. For what values of parameters firm 1 has positive demand?