Midterm

Advanced Economic Theory, ECO326F1H

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February, 2014

There are three questions. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 120 minutes.

(1) (30 points) Consider the following game.

Player 1 \ Player 2	L	C	R
U	$3,-2$	$0,-1$	$-1,1$
M	$-3,2$	$5,-1$	$1,-1$

(a) Find all strictly dominated strategies. Strategy C is strictly dominated by $L^{1/2}R^{1/2}$.

(b) Does the game have a pure strategy Nash equilibrium? No.

(c) Find a Nash equilibrium in mixed strategies. Strategy C is strictly dominated so it is not going to be used in the Nash equilibrium. Two indifference conditions:

$$-2\alpha + 2(1-\alpha) = 1\alpha - 1(1-\alpha),$$
$$3\beta - 1(1-\beta) = -3\beta + 1(1-\beta),$$

imply that $\alpha = \frac{1}{2}, \beta = \frac{1}{2}$ and $\left(U^{1/2}M^{1/2}, L^{1/2}R^{1/2}\right)$ is a mixed strategy Nash equilibrium of the above game.

(d) Suppose that player 2's payoff from playing action C is increased from -1 to 1 (against both actions of player 1 simultaneously). None of the other payoffs are affected. Does the mixed strategy profile that you found in part (c) remains an equilibrium? If not, find a new equilibrium. If the payoffs from C are increased by 1, then the game looks like this:

Player 1 \ Player 2	L	C	R
U	$3,-2$	$0,1$	$-1,1$
M	$-3,2$	$5,1$	$1,-1$

Strategy C is not strictly dominated anymore. In fact, the above profile is not equilibrium, because the strategy C leads to payoff 1 vs payoff 0 from L and R in the equilibrium.

We compute a new equilibrium, in which player 2 randomizes between L and C. The indifference conditions:

$$-2\alpha + 2(1-\alpha) = 1\alpha - 1(1-\alpha),$$
$$3\beta - 1(1-\beta) = -3\beta + 1(1-\beta),$$

so it is not going to be used in the Nash equilibrium. Two indifference conditions:

$$-2\alpha + 2(1-\alpha) = 1,$$
$$3\beta + 0(1-\beta) = -3\beta + 5(1-\beta),$$

imply that $\alpha = \frac{1}{4}, \beta = \frac{5}{11}$. If player 1 is randomizing $U^{1/4}M^{3/4}$, player 2 is indifferent between playing L and C and she strictly prefers both of the actions to R (the payoff from R is $-\frac{1}{2}$). Thus, the profile $\left(U^{1/4}M^{3/4}, L^{5/11}C^{6/11},\right)$ is a mixed strategy Nash equilibrium of the above game.

(2) (40 points) In 2014, there will be N students graduating from the University of Summerville. The students will enter the Summerville job market and apply to one of the N open positions. The employers in each of the positions want to hire the students according to their grades. The University of Summerville ranks all the students and we are going to assume that all the students are arranged from s_1 (the best), $s_2, ...,$ to s_N (the worst). The students want to get the best possible job. We assume that the jobs are also arranged according to their quality: j_1 is the best job, j_2 is the second best, ..., and j_N is the worst job. Assume that all ranikings (of jobs and students) are strict (so, there are no ties).

The application process has two rounds. The first round is decentralized and takes the form of the following game. Each student chooses a job to which she or he sends her or his application. Each job which receives more than one application hires a student with the best grades among all applicants. The hired student and the job leave the market and do not participate in the second round. In the second round, all students who were not hired and all the jobs that were not filled in the first round are matched with each other in a centralized fashion: the best of the remaining students goes to the best of the remaining jobs, the second best remaining student goes to the second best remaining job, etc.

For example, suppose that in the first round, all the students apply to job j_2 (i.e., the second best job). Then, job j_2 hires s_1 (because this is the best student among all applicants). All the remaining students will be allocated in the second round according to their grades and the quality of the job. So, student s_2 will go to job j_1, s_3 will go to j_3, s_4 will go to j_4, etc.

You are supposed to analyze the behavior of the students in the game from the round 1.

(a) Show that student s_1 has a strictly dominant strategy. Applying to j_1

is strictly dominant for s_1. If s_1 applies to j_1, she or he will get hired by j_1 (which is the best possible outcome for s_1) in the first round. If

s_1 applies to any other job, she will get hired to this job, which is worse than j_1. So, she strictly prefers to apply to j_1.

(b) Does student s_2 (or s_n for $n \geq 2$) have any dominant strategy? No. It

is enough to show that student s_2 has at lest two strict best responses, depending on what others are doing. If all students apply to job j_n for $n > 1$, then applying to job j_1 is the strict best response. If s_1 applies to job j_1 and all other students apply to job j_2, then j_2 is the strict best response. A similar argument shows that no other student has a dominant strategy.

(c) Suppose that everybody knows that everybody is rational (i.e., we perform one round of elimination of dominated strategies). Does student s_2 have a dominant or dominated strategy? Knowing that s_1 applies to

job j_1, and knowing that s_1 will be hired by j_1, it is weakly dominant for s_2 to apply to j_2. The reason is that student s_2 gets the best possible outcome (apart from unachievable j_1), and that sometimes applying to j_2 is a strict best response (for example, when all other students apply to j_2). The reason why j_2 is not strictly dominant is that if all the other students apply to j_3, then student s_2 is indifferent between applyting to j_1 or j_2 (if he applies to j_1, he will go to the second round and be allocated to $j_2$in the second round.)

(d) Which strategies survive the iterated elimination of (weakly) dominated strategies? Apply to job j_n for all students s_n and $n < N$.

(e) Describe a Nash equilibrium of the application game. Apply to job j_n

for all students s_n and n. Notice that none of the students can improve his or her payoff by changing the strategy. If the students swicthes up, her payoff is not affected. If she swicthes down, her payoff goes down.

(3) (30 points) There are two firms $i = 1, 2$ selling similar, but slightly different products. Each firm i has a constant marginal cost of production $c_i \geq 0$. Each firm i chooses price $p_i \geq 0$. The demand for products of firm i depends on its own price, as well as the price of its competitor, and it is equal to

$$D_i(p_i, p_{-i}) = \max(0, 10 + 2(p_{-i} - p_i)).$$

Each firm wants to maximize its profits equal to

$$(p_i - c_i)(D_i(p_i, p_{-i})).$$

(a) First, suppose that both firms have the same marginal cost, $c_1 = c_2 = c$. Find the Nash equilibrium of the pricing game. Notice that in the

equilibrium, no firm will have 0 demand (otherwise, at least one of the firms would have an incentive to change its action). We will solve for the equilibrium given the assumption that both firms have strictly positive demand. The best response:

$$\max_{p_i} (p_i - c)(10 - 2p_i + 2p_{-i}).$$

The FOC:

$$10 - 2p_i + 2p_{-i} - 2p_i + 2c = 0,$$

or

$$p_i = \frac{1}{4}(10 + 2p_{-i} + 2c) = \frac{1}{2}(5 + p_{-i} + c).$$

In equilibrium, both prices must be best responses to each other. This implies that

$$p_1 = p_2 = \frac{10 + 2c}{2} = 5 + c.$$

Notice that if both firms choose the same price, then the demand is strictly positive and our initial assumption is correct. We conclude that p_1 and p_2 are the best response strategies.

(b) Now, suppose that $c_1 > c_2 \geq 0$. Carefully describe the best response correspondence of firm i. Suppose that the opposing firm's price is equal to p_{-i}. If the demand at the optimal action is positive, then the FOC imply that the best response for firm i is equal to

$$\frac{1}{2}(5+p_{-i}+c_i).$$

This happens when

$$D\left(\frac{1}{4}(10+2p_{-i}+2c_i), p_{-i}\right)=\max(0,5+p_{-i}-c_i)\geq 0,$$

or when $5+p_{-i}\geq c_i$. Otherwise, if $5+p_{-i}<c_i$, then any price p_i such that the demand is zero,

$$10-2p_i+2p_{-i}\leq 0,$$

is a best response. Thus, the best response correspondence of firm i is equal to

$$b(p_{-i})=\begin{cases}\frac{1}{4}(10+2p_{-i}+2c_i), & \text{if } \frac{1}{4}(10+2p_{-i}+2c_i)\geq c_i\\ [5+p_{-i},\infty], & \text{otherwise.}\end{cases}$$

(c) Find the equilibrium. For what values of parameters firm 1 has positive demand?

Let's consider first the positive demand equilibrium. In such a case,

$$\begin{aligned}p_1&=\frac{1}{2}(5+p_2+c_1)=\frac{1}{2}\left(5+\frac{1}{2}(5+p_1+c_2)+c_1\right)\\&=\frac{15}{4}+\frac{1}{4}p_1+\frac{1}{2}c_1+\frac{1}{4}c_2.\end{aligned}$$

This implies that

$$p_1=5+\frac{2}{3}c_1+\frac{1}{3}c_2,$$

and, similarly,

$$p_2=5+\frac{2}{3}c_2+\frac{1}{3}c_1.$$

This is an equilibrium only if $5+p_{-i}\geq c_i$ for each i, or

$$10+\frac{2}{3}c_2+\frac{1}{3}c_1\geq c_1,\ \text{and}\ 10+\frac{2}{3}c_1+\frac{1}{3}c_2\geq c_2,$$

or

$$15 + c_2 \geq c_1 \text{ and } 15 + c_1 \geq c_2.$$

If $15 + c_2 < c_1$, then the only equilibrium is when