Midterm

Advanced Economic Theory, ECO326F1H

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There are three questions. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 120 minutes.

(1) (25 points) Consider the following game:

	L	C	R
U	$3, 5$	$4, 3$	$5, 1$
M	$3, 1$	$3, 5$	$3, 2$
D	$4, 2$	$2, 3$	$2, 4$

(a) Find all strictly dominated strategies.

(b) Find all actions that can be eliminated by iterated elimination of strictly dominated strategies.

(c) Find all pure and mixed strategy Nash equilibria.

(2) (40) Two firms cooperate on producing a final product (think a frankfurter and a bun). Each firm produces a part that is later combined with the other part and sold on the market. Each firm $i = 1, 2$, chooses individually (and separately from the other firm) the quality $q_i \geq 0$ of its production. The profits from final product depend on the qualities of both firms and they are equal to

$$\pi(q_1, q_2) = a(q_1 + q_2 - q_1 q_2),$$

where $a \in [0, 2]$. The two firms split the profits equally and each pays its own production cost. The final payoffs are equal to

$$\frac{1}{2}\pi(q_1, q_2) - c(q_i).$$

where $c(q)$ are the costs of producing with quality q.

(a) For questions (a)-(b), suppose that $c(q) = q^2$. Find the best responses and all Nash equilibria.

(b) Do players have any (strictly) dominated strategies? (For partial credit, find actions that are never best responses to pure strategies.) Find all dominated strategies and show that they are dominated.

(c) From now on, suppose that $c(q) = \begin{cases} f + q^2, & \text{if } q > 0, \\ 0, & \text{if } q = 0, \end{cases}$ where $f > 0$ is a fixed cost of producing non-zero quality. Find conditions on parameters such that there is an Nash equilibrium in which both firms choose zero qualities.

(d) Find condition on parameters, such that there exists an equilibrium with strictly positive qualities for both firms.

(e) Do there exist parameters for which there exists an equilibrium in which exactly one firm has a positive quality?

(3) (35 points) A professor wrote two questions for the exam. There is a "boring" question and an "interesting" question. The professor is tempted to choose the "interesting" question but she is afraid that the question might be too difficult (or, it might be not, who knows?).

Instead of choosing herself, the professor decides to let each student choose individually. There are N students. Each student chooses between one of two questions (different students may make different choices). The choices are described in the table

	probability of correct answer	points for correct answer
"boring" question	$\frac{1}{2}$	$A + c\frac{n}{N}$,
"interesting" question	q	B,

Here, $q \in [0, 1]$ is a probability with which a student answers the "interesting" question, B is the number of points awarded for the correct answer to the "interesting" question, $c > 0$ is a constant, and $A + c\frac{n}{N}$ is the number of points awarded for the correct solution to the "boring" question. The latter depends on (is increasing with)

$n =$ the number of students who choose the "interesting" question.

We assume that $A < 2B$. You will find the explanation for such a grading scheme below.

Assume that each student wants to maximize the expected number of points (i.e., (probability of correct answer) * (points for correct answer)).

(a) Find q_* such that a student finds the "boring" question as a strictly dominant action if and only if $q < q_*$ (or, in other words, if the "interesting" question is too difficult).

(b) Find q^* such that a student finds the "interesting" question as a strictly dominant action if and only if $q > q^*$ (or, in other words, if the "interesting" question is too easy).

(c) Given that there are n' other students choosing the interesting question, find the best response function of a student with difficulty parameter q.

(d) Assume that all N students have the same difficulty parameter $q \in [0, 1]$. Find the conditions that characterize the number of students n^* who choose the interesting question in equilibrium.

(e) Is the exam grading system fair to students? Use (d) to explain that if the number of students is large, than, in equilibrium, the expected number of points awarded to each student is (approximately) the same, regardless whether they choose the "boring" or "interesting" question.