Midterm

Advanced Economic Theory, ECO326S1H

Marcin Pęski

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There are three questions. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 120 minutes.

(1) (30 points) Consider the following game:

	L	C	R
U	$2,5$	$10,4$	$6,5$
M	$7,2$	$0,3$	$3,6$
D	$4,8$	$0,7$	$5,6$

(a) Find all strictly dominated strategies. Action C is strictly dominated

by $L^{1/2+\varepsilon}R^{1/2-\varepsilon}$ for some very small $\varepsilon > 0$. Nothing else is. (Every other action is a best response against some pure belief).

(b) Find all actions that can be eliminated by iterated elimination of strictly dominated strategies. After the first round of elimination, we get

	L		R
U	$2,5$		$6,5$
M	$7,2$		$3,6$
D	$4,8$		$5,6$
$U^\alpha M^{1-\alpha}$	$7-5\alpha,$		$3\alpha+3$

Actions L, R, U, M are not strictly dominated because they are best responses against some pure belief. To check whether D is strictly dominated, consider strategy $U^\alpha M^{1-\alpha}$. If D is strictly dominated, there exists $\alpha \in [0,1]$ such that

$$7-5\alpha > 4 \text{ and } 3\alpha+3 > 5.$$

But such α does not exists.

(c) Find all pure strategy Nash equilibria.

(d) Find all mixed strategy Nash equilibria. After the first round of elimination, we get

	L		R
U	$2,5$		$6,5$
M	$7,2$		$3,6$
D	$4,8$		$5,6$

There is a pure strategy Nash equilibrium (U, R). There is no other equilibrium in which the column player or the row player plays pure strategy.

We check whether there are mixed strategy equilibria. In order to make the column player indifferent between his actions, the row player must play $U^{\alpha}M^{\beta}D^{\gamma}$ so that

$$5\alpha + 2\beta + 8\gamma = 5\alpha + 6\beta + 6\gamma.$$

This implies that $\gamma = 2\beta$. There are three possibilities:

(a) $\alpha = 1$: This will be an equilibrium if the row player prefers to play U rather than M and D. the row player chooses $L^{\delta}R^{1-\delta}$ where δ ensures that the support condition holds:

$$2\delta + 6(1-\delta) \geq 7\delta + 3(1-\delta),$$
$$2\delta + 6(1-\delta) \geq 4\delta + 5(1-\delta).$$

Any $\delta \leq \frac{1}{3}$ is good.

(b) the row player is indifferent between all his actions. It is easy to check that this option is not possible.

(c) the row player mixes between M and D. For the latter, the row player must play $M^{1/3}D^{2/3}$. In order to make her indifferent, the row player chooses $L^{\delta}R^{1-\delta}$ where δ solves the indifference condition:

$$3 + 4\delta = 5 - \delta,$$

or $\delta = \frac{2}{5}$. Thus, the only mixed strategy equilibrium is $\left(M^{1/3}D^{2/3}, L^{2/5}R^{3/5}\right)$.

(2) (35) Three politicians, Donald, Ted and Marco compete for the Republican nomination in a series of primary elections. The next primary is going to be in Nevada. The politicians must decide how many resources (money, candidate time, etc.) to allocate to compete in the Nevada before the primary. Let $r_i \geq 0$ be the amount of resources allocated by politician $i = D, T, M$. The payoff of politician i is equal to

$$\alpha r_i - \beta \left(r_D + r_T + r_M\right)^2,$$

where $\alpha, \beta > 0$ are parameters. The payoff includes both the benefits of winning more delegates than the competitors as well as the cost of resources that are spent in Nevada rather than allocated to the other states. Answer the following questions.

(a) Find the Donald's best response given the choices of the other politicians.

The FOC:

$$\alpha - 2\beta \left(r_D + r_T + r_M\right) = 0.$$

Because the payoff function is concave, the best response is either solution to the FOCS, or if the latter is negative, it is equal to $r_D = 0$. Thus, the best response is

$$r_D^{BR} = \max\left(0, \frac{\alpha}{2\beta} - r_T - r_M\right).$$

(b) Find the Nash equilibrium decisions. Is the equilibrium unique? What is the equilibrium sum of resources $r_D + r_T + r_M$? Any profile of choices $r_D, r_T, r_M \geq 0$ such that $r_D + r_T + r_M = \frac{\alpha}{2\beta}$ is a Nash equilibrium. So, the equilibrium is not unique.

(c) Which Nash equilibrium maximizes the Donald's payoff? The Donald's payoff in equilibrium is equal to

$$\alpha r_D - \beta \left(\frac{\alpha}{2\beta}\right)^2 = \alpha r_D - \frac{\alpha^2}{4\beta}.$$

It is maximized by $r_D = \frac{\alpha}{2\beta}$.

(d) Suppose that each politician has a fixed cost $f > 0$ of competing. The fixed cost is paid only if $r_i > 0$, otherwise, there is no cost. That is, the payoffs are equal to

$$\begin{cases} \alpha r_i - \beta\left(r_D + r_T + r_M\right)^2 - f, & \text{if } r_i > 0, \\ -\beta\left(\sum_{j \neq i} r_j\right)^2 & \text{if } r_i = 0. \end{cases}$$

Find conditions on parameters under which there is an equilibrium such that only one politician competes. (For a partial credit, you can assume that there are only two players, say Donald and Ted, rather than three players.) Suppose that $r_i = \frac{\alpha}{2\beta}$ and $r_j = 0$ for $j \neq i$ (that is the equilibrium in the game without fixed costs that maximizes i's payoffs). Such a profile is an equilibrium in the game with fixed costs if i's payoffs from competing are larger than fixed cost,

$$\frac{\alpha}{2\beta} - \frac{\alpha^2}{4\beta} \geq f.$$

(e) (Extra credit) Find conditions on parameters under which there is an equilibrium such that all politicians compete. (For a partial credit, you can assume that there are only two players, say Donald and Ted, rather than three players.) Suppose that $r_i = \frac{1}{3}\frac{\alpha}{2\beta}$ for all i (that is the equilibrium in the game without fixed costs that maximizes the worst off player's payoffs). Such a profile is an equilibrium in the game with fixed costs if

$$\alpha\frac{1}{3}\frac{\alpha}{2\beta} - \frac{\alpha^2}{4\beta} = \frac{1}{3}\frac{\alpha}{2\beta} - \frac{\alpha^2}{4\beta} - f \geq -\beta\left(\frac{2}{3}\frac{\alpha}{2\beta}\right)^2.$$

(3) (35 points) There are N investors, each holding a packet of shares of a company *Next New Shiny Thing.* Each investor i decides when to sell his or her shares. Let $t_i = 0, 1, 2, ..., T$ to be the decision of player i (you can assume that $T = 10$). Let

$$t^*_{-i} = \max_{j \neq i} t_j - 1$$

be the last period in which there is still another player who wants to hold on to his shares. (For example, if all other players decide to sell in period 10, $t_j = 10$ for $j \neq i$, then the last period in which there are still players other than i willing to hold on to their shares is $t^*_{-i} = 9$.) The payoffs are equal to

$$\begin{cases} 5t_i, & \text{if } t^*_{-i} \geq t_i \\ 5t_i - 10\left(t_i - t^*_{-i}\right), & \text{otherwise.} \end{cases}$$

Here, $A_t = 5t > 0$ is a function that describes the benefit of holding on to the shares till period t (that includes the dividend, exogenous growth, etc.) The idea is that as long as the investors willing to hold an asset, we say that the market for the asset is sufficiently *thick*, and the price is high. If nobody else wants to hold the asset, the market is *thin* and the price falls by $\beta = 10$ in each period in which the market is thin.

(a) Show that waiting till the last possible moment $t = 100$ is strictly dominated. Are there any other strictly dominated actions? The payoff from

waiting till $T = 100$ is equal to $A_T - \beta\left(T + 1 - \max_{j \neq i} t_j\right) = -5T - \beta + \beta \max_{j \neq i} t_j$. The payoff from waiting till $T - 1$ is equal to (a) A_{T-1} if $\max_{j \neq i} t_j = T$, and (b) equal to $A_{T-1} - \beta\left(T - 1 + 1 - \max_{j \neq i} t_j\right) = -5\left(T - 1\right) - \beta + \beta \max_{j \neq i} t_j$ otherwise. In each case, (a) or (b) the payoff from waiting till $T - 1$ is higher than the payoff from waiting till T. than $5T - \beta + \beta \max_{j \neq i} t_j$.

No other action is strictly dominated. In fact, if everybody else waits till the last period, it is a best response to wait till the period last before last.

(b) Which actions are consistent with players being rational and knowing that everybody else is rational? All actions $t \leq T - 2$. The above argument shows that rational players won't play T. The same argument as above applies to show that rational players in a game with actions $t = 0, 1, ..., T - 1$ won't play $T - 1$.

(c) Find all the actions that survive the iterated elimination of strictly dominated actions. The only action that survives the iterated elimination is $t = 0$.

(d) Find all Nash equilibria of the investment game. Compute the payoffs. There is only one Nash equilibrium, in which all investors sell immediately. The payoff of each player is equal to $-\beta$

(e) In order to improve the functioning of the markets, the central bank decides to impose a penalty $R = 10$ on all short-term investors. The penalty is paid by each investor who sells before (but not including) period $t^* = 50$. Is the outcome that you described in (d) an equilibrium of the game with penalty? Does the game have new equilibria? Find an equilibrium that maximizes the sum of the payoffs.

The previous outcome remains the equilibrium (notice that in order to avoid the penalty, the investor would have to wait for 50 periods, which is much worse than paying the penalty).
There are new equilibria. For example, $t_i = t^*$ for all players i is a new equilibrium. Such equilibrium maximizes the sum of all player's payoffs.