Midterm

Advanced Economic Theory, ECO326S1H

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There are three questions. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 120 minutes.

(1) (30 points) Consider the following game with payoffs given in the following Table:

Player 1\Player 2	L	C	R
U	$5,3$	$10,4$	$8,10$
M	$2,5$	$11,4$	$5,3$
D	$8,7$	$10,4$	$5,5$

(a) Find all actions that are never best responses.

(b) What can you predict about behavior of player 1 if you know that she is rational? What can you predict about behavior of player 1 if you know that she is rational and that she knows that player 2 is rational?

(c) Find all pure strategy Nash equilibria.

(d) Does the game have a mixed strategy equilibrium?

(2) (30) Jimmy and Iris study together. The payoff of student $i = J, I$ depend on their own effort $e_i \geq 0$, the effort of their partner e_{-i} and it is equal to

$$e_i (1 + e_{-i}) - C(e_i).$$

Here, $C(e_i)$ is the cost of the effort, equal to

$$C(e_i) = \begin{cases} f + 3e_i^2, & \text{if } e_i > 0, \\ 0, & \text{if } e_i = 0, \end{cases}$$

where $f \geq 0$ is the fixed cost of effort.

(a) Suppose that $f = 0$. Find a best response function and an equilibrium of the above game.

(b) Suppose that $f > 0$. Carefully describe the best response function.

(c) For what values of parameter $f \geq 0$ is there an equilibrium, where both partners students put a strictly positive amount of effort? Describe such an equilibrium.

(d) For what values of parameter $f > 0$ is there an equilibrium in which no student puts an effort?

(3) (40) Oceania and Eurasia are facing an imminent nuclear conflict. Each country $i = O, E$ amassed $n_i \geq 0$ nuclear warheads. The Dear Leaders of the two countries simultaneously decide whether to attack or not. If both countries attack, the payoffs are

$$u_i\left(A_i, A_{-i}\right) = \begin{cases} V - \left(n_i + n_{-i}\right) W, & \text{if } n_i \geq n_{-i} + 10, \\ -\left(n_i + n_{-i}\right) W & \text{if } n_i < n_{-i} + 10. \end{cases}$$

Here, $V > 0$ is the victory bonus that you get only if you shoot 10 more nukes than the other side and $W > 0$ is the per-nuke cost of nuclear war caused by detonating nukes. The nuclear war causes the damage to the whole planet, which is proportional to the number of detonated nukes. If only country $i = O, E$ attacks and the other country does not, the payoff of country i is

$$u_i\left(A_i, N_{-i}\right) = \begin{cases} V - n_i W, & \text{if } n_i \geq 10, \\ -n_i W & \text{if } n_i < 10, \end{cases}$$

and the payoff of the other country is

$$u_{-i}\left(N_{-i}, A_i\right) = -n_i W.$$

(a) Show that if $Wn_i > V_i$ it is strictly dominant for country i not to attack.

(b) Suppose that a country has fewer than 10 nukes. Does it have a strictly dominated strategy?

(c) Find all combinations of (n_i, n_{-i}) for which it is strictly dominant for country i to attack.

(d) Find a combination of parameters, for which there exists a mixed strategy equilibrium in which both countries attack. How does the probability of attack changes with the size of own nuke arsenal? How does it change with the size of the opponent's arsenal?

(e) How does the expected damage to the planet in the mixed strategy equilibrium depend on the number of nukes? Is it possible that the planet would be better off with more nukes? Can you explain why?