Midterm

Advanced Economic Theory, ECO326S1H

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There are three questions. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 120 minutes.

(1) (30 points) Consider the following game with payoffs given in the following Table:

Player 1\Player 2	L	C	R
U	$5, 3$	$10, 4$	$8, 10$
M	$2, 5$	$11, 4$	$5, 3$
D	$8, 7$	$10, 4$	$5, 5$

(a) Find all actions that are never best responses. Action C is strictly dominated by $L^{2/3}R^{1/3}$. Hence, C is a never best response. Nothing else is. (Every other action is a best response against some pure belief).

(b) What can you predict about behavior of player 1 if you know that she is rational? What can you predict about behavior of player 1 if you know that she is rational and that she knows that player 2 is rational? Player 1 does not have any strictly dominated actions in the original game. After the first round of elimination, we get

Player 1\Player 2	L		R
U	$5, 3$		$8, 10$
M	$2, 5$		$5, 3$
D	$8, 7$		$5, 5$

Action M becomes strictly dominated by U. Hence, you can predict that 1 will play U or D.

(c) Find all pure strategy Nash equilibria.

Player 1\Player 2	L		R
U	$5, 3$		$8, 10$
D	$8, 7$		$5, 5$

There are two pure strategy equilibria

(d) Does the game have a mixed strategy equilibrium? Yes, $\left(U^{\alpha}D^{1-\alpha}, L^{\beta}R^{1-\beta}\right)$, where α and β solve equations:

$$\alpha 3 + (1-\alpha)\,7 = \alpha 10 + (1-\alpha)\,5,$$
$$\beta 5 + (1-\beta)\,8 = \beta 8 + (1-\beta)\,5.$$

(2) (30) Jimmy and Iris study together. The payoff of student $i = J, I$ depend on their own effort $e_i \geq 0$, the effort of their partner e_{-i} and it is equal to

$$e_i \left(1 + e_{-i}\right) - C\left(e_i\right).$$

Here, $C\left(e_i\right)$ is the cost of the effort, equal to

$$C\left(e_i\right) = \begin{cases} f + 3e_i^2, & \text{if } e_i > 0, \\ 0, & \text{if } e_i = 0, \end{cases}$$

where $f \geq 0$ is the fixed cost of effort.

(a) Suppose that $f = 0$. Find a best response function and an equilibrium of the above game. FOC: For each i,

$$1 + e_{-i} - 6e_i = 0.$$

Because $e_{-i} \geq 0$, there is a unique equilibrium with $e_i = e_{-i} = \frac{1}{5}$.

(b) Suppose that $f > 0$. Carefully describe the best response function. The

best response positive effort can be derived form the first order conditions and it is equal to

$$e_i^+\left(e_{-i}\right) = \frac{1}{6}\left(1 + e_{-i}\right).$$

The best payoff from the positive effort is equal to

$$\frac{1}{6}\left(1 + e_{-i}\right)^2 - f - 3\left(\frac{1}{6}\left(1 + e_{-i}\right)\right)^2 = \frac{1}{12}\left(1 + e_{-i}\right)^2 - f.$$

Thus, the best response is

$$e_i^{BR}\left(e_{-i}\right) = \begin{cases} \frac{1}{6}\left(1 + e_{-i}\right), & \text{if } \frac{1}{12}\left(1 + e_{-i}\right)^2 \geq f, \\ 0, & \text{if } \frac{1}{12}\left(1 + e_{-i}\right)^2 \leq f, \end{cases}$$

(c) For what values of parameter $f \geq 0$ is there an equilibrium, where both partners students put a strictly positive amount of effort? Describe such an equilibrium.

In such an equilibrium, both players put $e_1 = e_2 = \frac{1}{5}$ effort. This is an equilibrium if

$$f \leq \frac{1}{12}\left(1+\frac{1}{5}\right)^2 = \frac{1}{12}\frac{36}{25} = \frac{3}{25}.$$

(d) For what values of parameter $f > 0$ is there an equilibrium in which no student puts an effort? If not student puts an effort, then it must be that $\frac{1}{12}(1+0)^2 = \frac{1}{12} \leq f$.

(3) (40) Oceania and Eurasia are facing an imminent nuclear conflict. Each country $i = O, E$ amassed $n_i \geq 0$ nuclear warheads. The Dear Leaders of the two countries simultaneously decide whether to attack or not. If both countries attack, the payoffs are

$$u_i(A_i, A_{-i}) = \begin{cases} V - (n_i + n_{-i})W, & \text{if } n_i \geq n_{-i} + 10, \\ -(n_i + n_{-i})W & \text{if } n_i < n_{-i} + 10. \end{cases}$$

Here, $V > 0$ is the victory bonus that you get only if you shoot 10 more nukes than the other side and $W > 0$ is the per-nuke cost of nuclear war caused by detonating nukes. The nuclear war causes the damage to the whole planet, which is proportional to the number of detonated nukes. If only country $i = O, E$ attacks and the other country does not, the payoff of country i is

$$u_i(A_i, N_{-i}) = \begin{cases} V - n_i W, & \text{if } n_i \geq 10, \\ -n_i W & \text{if } n_i < 10, \end{cases}$$

and the payoff of the other country is

$$u_{-i}(N_{-i}, A_i) = -n_i W.$$

(a) Show that if $Wn_i > V_i$ it is strictly dominant for country i not to attack.

(b) Suppose that a country has fewer than 10 nukes. Does it have a strictly dominated strategy? Suppose i has less than 10 nukes. The payoff from attacking is equal to $-W\left(n_i + \mathbf{1}_{a_{-i}=A} n_{-i}\right)$. The payoff from not attacking is $-W\left(\mathbf{1}_{a_{-i}=A} n_{-i}\right)$. It is strictly dominant not to attack if $n_i > 0$; otherwise, player i is indifferent.

(c) Find all combinations of (n_i, n_{-i}) for which it is strictly dominant for country i to attack.

We consider two cases:

(a) $n_i \geq n_{-i} + 10$,

(b) $n_i < n_{-i} + 10$.

In case (a), i attacks, she receives the victory bonus and total payoff of

$$V - \left(n_i + \mathbf{1}_{a_{-i}=A} n_{-i}\right) W.$$

If she does not attack, she gets payoff

$$-\mathbf{1}_{a_{-i}=A} n_{-i} W.$$

Hence, attacking is strictly dominant if $V > n_i W$, and $n_i \geq n_{-i} + 10$.
In case (b), if player $-i$ attacks, and $n_i > 0$, then it is the best response for player i not to attack. If $n_i = 0$, player i is indifferent. Hence, attack is not strictly dominant in such a case.

(d) Find a combination of parameters, for which there exists a mixed strategy equilibrium in which both countries attack. How does the probability of attack changes with the size of own nuke arsenal? How does it change with the size of the opponent's arsenal? By the first question above, it

must be $W n_i \leq V$ for each i. By the answer to question (c), it must be d $n_i < n_{-i} + 10$ and $n_i \geq 10$.
Let α_i be the probability of attacking for player i. The expected payoff of player i from attacking is equal to

$$(1 - \alpha_{-i}) V - (n_i + \alpha_{-i} n_{-i}) W.$$

(This is because the player will win if and only if the other guy does not attack. The payoff from not attacking is equal to

$$-(\alpha_{-i} n_{-i}) W.$$

Hence, in equilibrium, it must be that

$$(1 - \alpha_{-i}) V = n_i,$$

or $\alpha_{-i} = 1 - \frac{n_i}{V}$. The probability of attack decreases with the opponent arsenal.

(e) How does the expected damage to the planet in the mixed strategy equilibrium depend on the number of nukes? Is it possible that the planet would be better off with more nukes? Can you explain why? The expected damage to the planet is equal to

$$\alpha_i n_i W + \alpha_{-i} n_{-i} W = n_i \left(1 - \frac{n_{-i}}{V}\right) W + n_{-i} \left(1 - \frac{n_i}{V}\right) W = \left(n_1 + n_2 - \frac{2n_1 n_2}{V}\right) W.$$

In particular, if $2n_{-i} > V$, the damage to the planet is decreasing with n_i. The reason is that large nuke arsenal has two effects: larger direct damage to the planet, and smaller equilibrium probability that the other country attacks. if n_{-i} is sufficiently high, the other effect dominates.