Midterm

Advanced Economic Theory, ECO326S1H

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There are three questions. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 120 minutes.

(1) (30 points) Consider the following game:

	L	C	R
U	$3, 1$	$3, 5$	$3, 2$
M	$4, 2$	$2, 3$	$2, 4$
D	$3, 5$	$4, 3$	$5, 1$

(a) Find all strictly dominated strategies.

(b) Find all actions that can be eliminated by iterated elimination of strictly dominated strategies.

(c) Find all pure and mixed strategy Nash equilibria.

(2) (40) There are two firms competing in a version of Bertrand duopoly. Each firm produces the same good at constant unit cost $c > 0$ (the same cost for each firm). Each firm chooses price p_i. The prices can take an arbitrary real value. The profits of firm i are equal to

$$\pi_i\left(p_i, p_{-i}\right) = \left(p_i - c\right) D_i\left(p_i, p_{-i}\right)$$

where $D_i\left(p_i, p_{-i}\right)$ is the demand function, and $a > c$.

(a) For the first question, assume that the demand function has form

$$D_i^{(0)}\left(p_i, p_{-i}\right) = \begin{cases} \left(a - p_i\right) & \text{if } p_i < p_{-i} \\ \frac{1}{2}\left(a - p_i\right) & \text{if } p_i = p_{-i} \\ 0 & \text{if } p_i > p_{-i} \end{cases}$$

Show that the game has unique Nash equilibrium where $p_1^* = p_2^* = c$. Carefully explain why no other action profile is an equilibrium.

(b) The demand function from the previous question is somehow extreme in that the consumers always go to buy the cheaper good, no matter what is the price difference. For all the remaining questions, we assume that the consumers cannot distinguish between small price differences. Precisely, we assume that there exists $\delta > 0$ such that the demand function is given by

$$D_i^{(\delta)}\left(p_i, p_{-i}\right) = \begin{cases} \left(a - p_i\right) & \text{if } p_i < p_{-i} - \delta \\ \frac{1}{2}\left(a - \frac{1}{2}\left(p_i + p_{-i}\right)\right) & \text{if } \left|p_i - p_{-i}\right| \leq \delta \\ 0 & \text{if } p_i > p_{-i} + \delta \end{cases}$$

One can interpret $\delta > 0$ as a measure of attention that the consumers pay to the price difference. Notice that, in the case of similar prices, the demand depends on the average price on the market.

Carefully explain that in each Nash equilibrium, it must be that $\left|p_i - p_{-i}\right| \leq \delta$.

(c) Find a Nash equilibrium. Is the equilibrium unique?

(d) Does the game have strictly dominated actions?

(e) Suppose that it is commonly known that no player can choose price below the cost. Does your answer to the previous question change - do players have strictly dominated actions?

(3) (30) There are two firms on the market. Each firm chooses quantity $q_i \geq 0$. If $q \leq q^*$, the cost of production are equal to cq_i, where $c > 0$ is a constant marginal cost. If the firm wants to expand beyond the threshold and choose $q_i > q^*$, the firm needs to pay a fixed cost $f > 0$. Each firm produces the same good and sells on the same market with the price equal to $(a - (q_1 + q_2))$, where $a > c$. The profits are

$$\pi_i (q_i, q_{-i}) = q_i (q - c - (q_1 + q_2)) - f \mathbf{1}_{q_i > q^*}.$$

(a) Find the best response function of each firm.

(b) For which values of f and q^* is there an equilibrium, where both firms expand their production above q^*.

(c) Find a combination of parameters, for which there exists an equilibrium in which only one firm expands above the threshold. Describe the equilibrium.