Midterm

Advanced Economic Theory, ECO326S1H

Marcin Peski

12th February, 2018

There are three questions. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 120 minutes.

(1) (30 points) Consider the following game:

	L	C	R
U	$3,1$	$3,5$	$3,2$
M	$4,2$	$2,3$	$2,4$
D	$3,5$	$4,3$	$5,1$

(a) Find all strictly dominated strategies. Action U is strictly dominated by $D^{1/2+\varepsilon}M^{1/2-\varepsilon}$ for some very small $\varepsilon > 0$. Nothing else is (Every other action is a best response against some pure belief).

(b) Find all actions that can be eliminated by iterated elimination of strictly dominated strategies. After the first round of elimination, we get

	L	C	R
D	$3,5$	$4,3$	$5,1$
M	$4,2$	$2,3$	$2,4$

In this game, none of the action is strictly dominated (action C is equivalent is not dominated by a mixed strategy $L^{\alpha}R^{1-\alpha}$, and all other actions are best responses against pure belief.)

(c) Find all pure and mixed strategy Nash equilibria. After the first round of elimination, we get

	L^{β}	C^{γ}	$R^{1-\beta-\gamma}$
D^{α}	$3,5$	$4,3$	$5,1$
$M^{1-\alpha}$	$4,2$	$2,3$	$2,4$

There are no pure strategy equilibria. For similar reasons, there are no equilibria, where player 1 is playing only D or only M, nor player 2 playing only L or only C, or only R. Consider a strategy $D^{\alpha}M^{1-\alpha}$. In order for player 2 to be indifferent between any pair of her actions, it must be that $\alpha = \frac{1}{3}$. In fact, if player 1 randomizes $D^{1/3}M^{2/3}$, then player 2 is indifferent among all actions.

In order for player 1 to be indifferent, it must be that

$$3\beta + 4\gamma + 5(1 - \beta - \gamma) = 4\beta + 2\gamma + 2(1 - \beta - \gamma).$$

(2) (40) There are two firms competing in a version of Bertrand duopoly. Each firm produces the same good at constant unit cost $c > 0$ (the same cost for each firm). Each firm chooses price p_i. The prices can take an arbitrary real value. The profits of firm i are equal to

$$\pi_i (p_i, p_{-i}) = (p_i - c) D_i (p_i, p_{-i})$$

where $D_i (p_i, p_{-i})$ is the demand function, and $a > c$.

(a) For the first question, assume that the demand function has form

$$D_i^{(0)} (p_i, p_{-i}) = \begin{cases} (a - p_i) & \text{if } p_i < p_{-i} \\ \frac{1}{2} (a - p_i) & \text{if } p_i = p_{-i} \\ 0 & \text{if } p_i > p_{-i} \end{cases}$$

Show that the game has unique Nash equilibrium where $p_1^* = p_2^* = c$. Carefully explain why no other action profile is an equilibrium. In

equilibrium, it must be that $\min (p_1, p_2) \geq c$. Otherwise, the lower price firm is making strictly negative profits and can deviate to $p = c$ with 0 profits.

Second, $c \leq p_i < p_{-i}$ cannot be an equilibrium, because firm i has a profitable deviation to a higher price (and strictly lower then p_{-i}).

Finally, $c < p_i = p_{-i}$ cannot be an equilibrium because each firm can undercut the rival by a tiny bit, increasing its profits.

(b) The demand function from the previous question is somehow extreme in that the consumers always go to buy the cheaper good, no matter what is the price difference. For all the remaining questions, we assume that the consumers cannot distinguish between small price differences. Precisely, we assume that there exists $\delta > 0$ such that the demand function is given by

$$D_i^{(\delta)} (p_i, p_{-i}) = \begin{cases} (a - p_i) & \text{if } p_i < p_{-i} - \delta \\ \frac{1}{2} \left(a - \frac{1}{2} (p_i + p_{-i})\right) & \text{if } |p_i - p_{-i}| \leq \delta \\ 0 & \text{if } p_i > p_{-i} + \delta \end{cases}$$

One can interpret $\delta > 0$ as a measure of attention that the consumers pay to the price difference. Notice that, in the case of similar prices, the demand depends on the average price on the market.

Carefully explain that in each Nash equilibrium, it must be that $|p_i - p_{-i}| \leq \delta$. First, notice that $\min(p_1, p_2) \geq c$ for the same reason as in the above question.

If both prices are not smaller than c, then if $p_{-i} > p_i + \delta$, then player $-i$ gets 0 profits, whereas she could get positive profits by choosing $p_{-i}{}' = p_i + \delta$.

(c) Find a Nash equilibrium. Is the equilibrium unique? By the previous point, we can restrict ourselves to looking among strategy profiles such that $|p_i - p_{-i}| \leq \delta$. Let's find the best responses. Player i maximizes profits

$$\max_{p_i} (p_i - c) \frac{1}{2} \left(a - \frac{1}{2} (p_i + p_{-i}) \right).$$

The FOCs are

$$\left(a - \frac{1}{2} (p_i + p_{-i}) \right) - \frac{1}{2} (p_i - c) = 0,$$

or

$$p_i = a - \frac{1}{2} p_{-i} + \frac{1}{2} c.$$

Solving the system of equations for both players, we obtain

$$p_i = p_{-i} = \frac{2}{3} (a + c).$$

The equilibrium is unique.

(d) Does the game have strictly dominated actions?

There are no strictly dominated actions. This is because player i is indifferent between any two prices p_i and $p_i{}'$ if $p_{-i} < \min(p_i.p_i') - \delta$.

(e) Suppose that it is commonly known that no player can choose price below the cost. Does your answer to the previous question change - do players have strictly dominated actions? Action $p_i = c$ is strictly dominated by $p_i = c + \delta$.The former always leads to 0 profits, the latter always leads to strictly positive profits (given that $p_{-i} \geq c$).

(3) (30) There are two firms on the market. Each firm chooses quantity $q_i \geq 0$. If $q \leq q^*$, the cost of production are equal to cq_i, where $c > 0$ is a constant marginal cost. If the firm wants to expand beyond the threshold and choose $q_i > q^*$, the firm needs to pay a fixed cost $f > 0$. Each firm produces the same good and sells on the same market with the price equal to $(a - (q_1 + q_2))$, where $a > c$. The profits are

$$\pi_i (q_i, q_{-i}) = q_i (q - c - (q_1 + q_2)) - f \mathbf{1}_{q_i > q^*}.$$

(a) Find the best response function of each firm.

Suppose that player $-i$ chooses q_{-i}. We consider two cases:

- the best possible payoff when $q_{-i} \leq q^*$. By solving the maximization problem, we obtain

$$q_{-i} = \min \left(\max \left(0, \frac{1}{2} (a - c - q_{-i}) \right), q^* \right).$$

The payoff is equal to

$$q_i (a - c - q_i - q_{-i}) = \begin{cases} 0, & \text{if } a - c - q_{-i} < 0 \\ \frac{1}{4} (a - c - q_{-i})^2 & \text{if } \frac{1}{2} (a - c - q_{-i}) < q^* \\ \frac{1}{2} q^* (a - c - q^* - q_{-i}) & \text{otherwise} \end{cases}$$

- The best possible payoff when $q_{-i} \geq q^*$ (and $\frac{1}{2} (a - c - q_{-i}) \geq q^*$) is

$$\frac{1}{4} (a - c - q_{-i})^2 - f.$$

Hence, the best response function is

$$b(q) = \begin{cases} 0, & \text{if } a - c - q_i < 0 \\ q^* & \text{if } \frac{1}{2} (a - c - q_{-i}) > q^* \text{ and } \frac{1}{4} (a - c - q_{-i})^2 - \frac{1}{2} q^* (a - c - q^* - q_{-i}) < f \\ \frac{1}{2} (a - c - q_i) & \text{otherwise.} \end{cases}$$

(b) For which values of f and q^* is there an equilibrium, where both firms expand their production above q^*.

It must be that

$$q_i = \frac{1}{2}\left(a - c - q_{-i}\right) > q^*,$$

for both firms, and

$$\frac{1}{4}\left(a - c - q_{-i}\right)^2 - \frac{1}{2}q^*\left(a - c - q^* - q_{-i}\right) > f.$$

We can check that

$$q_i^* = \frac{1}{3}\left(a - c\right),$$

hence, it must be that

$$\frac{1}{3}\left(a - c\right) > q^*$$

and

$$\frac{1}{9}\left(a - c\right)^2 - \frac{1}{2}q^*\left(\frac{2}{3}\left(a - c\right) - q^*\right) > f.$$

(c) Find a combination of parameters, for which there exists an equilibrium in which only one firm expands above the threshold. Describe the equilibrium. We are looking for equilibrium, where $q_i \leq q^*$ and $q_{-i} > q^*$ We

consider two cases.

Case 1. $q_i < q^*$, which means $\frac{1}{2}\left(a - c - q_{-i}\right) < q^*$. In such a case,

$$q_i = \frac{1}{2}\left(a - c - q_{-i}\right),$$
$$q_{-i} = \frac{1}{2}\left(a - c - q_{-i}\right).$$

for both players. But the only solution is that the two quantities are equal. Contradiction.

Case 2. $q_i = q^*$.Then,

$$q_{-i} = \frac{1}{2}\left(a - c - q^*\right).$$

$$\frac{1}{4}\left(a - c - \frac{1}{2}\left(a - c - q^*\right)\right)^2 - \frac{1}{2}q^*\left(a - c - q^* - \frac{1}{2}\left(a - c - q^*\right)\right) < f$$

to make sure that player i does not want to expand, and

$$\text{and } \frac{1}{4}\left(a - c - q^*\right)^2 - \frac{1}{2}q^*\left(a - c - 2q^*\right) > f$$

to make sure that player $-i$ does not want to lower production to q^*. Together, the two conditions mean

$$\begin{aligned}0 >& \frac{1}{16}\left((a-c)+q^*\right)^2 - \frac{1}{4}q^*\left(a-c-q^*\right) - f \\ =& \frac{1}{16}\left((a-c)^2 - 2\left(a-c\right)q^* + q^{*2}\right) + \frac{1}{4}q^{*2} - f\end{aligned}$$

and

$$0 < \frac{1}{4}\left((a-c)^2 - 2\left(a-c\right) + q^{*2}\right) + q^{*2} - f.$$