

Midterm

Advanced Economic Theory, ECO326S1H

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There are three questions. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 110 minutes.

(1) (30 points) Consider the following game:

	L	C	R
U	3, 5	3, 1	3, 3
M	4, 2	2, 4	4, 3
D	3, 5	5, 1	2, 3

- Find all never best responses. Be careful!
- Find all actions that can be eliminated by iterated elimination of never best responses.
- Find all pure and mixed strategy Nash equilibria.

- (2) (30) Many courses at the University are offered in different versions, one of them is typically more advanced, or more accelerated than the other. Students have often a choice in which version to enroll. Here, we analyze this choice. There are $N \geq 50$ students. Each student has ability $g_i \in [0, 1]$, and we assume that $g_1 < \dots < g_N$. Each student chooses whether to take course version $a_i = 1$ or $a_i = 2$. (Here, a_i denotes i 's action..) Student i 's payoff from class $k = 1, 2$ is equal to

$$u_i(k, a_{-i}) = x_k - (g_{\max}^k - g_i)$$

depends on how much she will learn x_k , and it decreases with the distance between her ability and the highest ability in the course, $g_{\max}^k - g_i$, where

$$g_{\max}^k = \max \{g_j : a_j = k\}.$$

is the highest ability among all students who enroll in course k . (This is because there is always a bit of possibility that the professor may curve the grade.) We assume that students learn more in course 2: $x_1 < x_2$.

- (a) What is the payoff of the highest ability student from choosing course k ? Explain that she or he has a strictly dominant action.
- (b) What is the payoff of the lowest ability student from choosing course k ? From now on, assume that $x_2 - x_1 < g_N - g_1$. Show that the lowest ability student does not have a strictly dominant action.
- (c) Show that there exists $1 \leq i^* < N$ such that all students $i \geq i^* + 1$ have a strictly dominant action, and all students $i \leq i^*$ do not.
- (d) Show that there is a unique equilibrium. Can the equilibrium be found by the iterated elimination of strictly dominated actions? How many stages of elimination are required?

- (3) There are N citizens in the country. Each of the citizens chooses the level of environmentally negative activity $a_i \geq 0$. The activity yields direct utility $ua_i - ca_i^2$, where $u > 0$ is the utility and $c > 0$ is the cost, but it also has a negative environmental effect on the entire country (pollution, climate change, etc.). The payoff of each citizen is equal to

$$ua_i - \frac{1}{2}ca_i^2 - \frac{1}{2}(a_1 + \dots + a_N)^2.$$

- (a) Find a best response function.
 (b) Find the Nash equilibrium. Compute the equilibrium utility of each citizen u_N^{eq} as a function of the size of the population N .
 (c) Find the levels of activity that maximize the total welfare. Compare with the Nash equilibrium: in equilibrium, is there too much or too little of the activity? Compute the utility of each citizen u_N^W .
 (d) The government is considering a tax on the activity ta_i paid by everybody (i.e., carbon tax). The proceeds from the tax are immediately returned to citizens, in equal amounts per citizen. Thus, the payoff function changes to :

$$ua_i - \frac{1}{2}ca_i^2 - \frac{1}{2}(a_1 + \dots + a_N)^2 - ta_i + t\frac{1}{N}(a_1 + \dots + a_N).$$

Find the new equilibrium and equilibrium utility $u_N^{eq}(t)$ as a function of t .

- (e) Find the welfare-maximizing level of tax. (To simplify the calculations, you can assume that the population is very large, i.e., $N \rightarrow \infty$, in which case,

$$u_N^{eq}(t) \rightarrow -\frac{1}{2}(u - t)^2.)$$

Compare with the answer that you found in question (c).

- (f) Would your answer to the previous question changed if the government did not return the carbon tax proceeds to the citizens but used it to finance other programs?