Final exam

Economics of Information, ECO421

Marcin Peski

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There are five questions, but you must solve *any four of them*. (There will be no credit for the extra 5th question.) All questions have the same worth.

Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one.

Please write your name on top of this exam.

You have 120 minutes.

(1) (Lark's singing). A skylark notices a falcon hovering in the air. The falcon wonders whether the skylark is healthy (type h, with probability $\frac{2}{3}$) or sick (type s). The skylark decides whether to run away immediately or sing first. The falcon observes the skylark's behavior and chooses whether to attack or not. If the falcon does not attack, it receives payoff 0. If it attacks, it receives payoff 3 if the skylark is sick and payoff -1 if the skylark is healthy. The skylark's payoffs are described in the table.

Payoffs	not attacked	attacked, type h	attacked, type s
singing	1	$-d_f$	$-d_s$
rnning away	1	0	0

where $d_f < d_s$ is the decrease in the survival chance caused by not running away immediately.

(a) Does the game have pooling equilibria? Carefully describe all off-path beliefs.

Solutions: If nobody sings, the falcon attacks. It is an equilibrium with appropriately chosen off-path beliefs. There is no pooling equilibrium with both types singing.

(b) Does the game have fully separating equilibria?

Solutions: Suppose that both types of skylark are choosing different behavior. The falcon is not going to attack the skylark with the behavior chosen by the healthy type. But then, the sick skylark will try to mimic the healthy one. Not an equilibrium.

(c) Show that there exists an equilibrium, in which the healthy skylark always sings and the falcon always attacks if the skylark runs away immediately. Carefully describe the strategies and beliefs.

Solutions: Yes. In such an equilibrium, the healthy skylark always sings, and the sick one sings with probability $\alpha \in (0, 1)$. The falcon always attacks the non-singing skylark, and attacks the singer with probability β. The falcon's beliefs that the singing skylark is healthy are

$$p = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{1}{3}\alpha} = \frac{2}{2 + \alpha}.$$

The falcon is indifferent between attacking the singing skylark if

$$p\,(-1) + (1 - p)\,3 = 0.$$

This implies that $p = \frac{2}{2+\alpha} = \frac{3}{4}$, which implies $\alpha = \frac{2}{3}$. The falcon always attacks the non-sgning skylark, because such skylark is clearly sick.
The sick skylark is indifferent between singing and running away if

$$0 = \beta\,(-d_s) + (1 - \beta)\,,$$

or $\beta = \frac{1}{1+d_s}$. The healthy skylark wants to sing because the payoff from singing is equal to

$$\beta\,(-d_h) + (1 - \beta) = 1 - \beta\,(1 + d_f) = 1 - \frac{1 + d_f}{1 + d_s} > 0,$$

where the latter is a payoff from non-singing.

(2) (Multitasking) Consider a version of the multi-tasking model. A teacher chooses two types of effort $e_1, e_2 \geq 0$. The teacher's and the school district payoffs given effort choices and wages are equal to

$$\pi_{teacher}(e_1, e_2; w) = w(e_1, e_2; \alpha) - c(e_1, e_2),$$
$$\pi_{district}(e_1, e_2, w) = -w(e_1, e_2; \alpha) + e_2.$$

Notice that the school district cares only about the second type of effort e_2. The cost function is equal to

$$c(e_1, e_2) = (e_1 + e_2)^2 + 2(e_1 - e_2)^2.$$

In particular, the teacher does not like effort, but also does not like to vary effort across activities. The school district observes the first type of effort e_1 and pays the teachers with linear contracts $w(e_1, e_2; \alpha) = w_0 + \alpha e_1$, where $\alpha \geq 0$.

(a) Solve the teacher's problem.

Solutions: The teacher maximizes its utility :

$$\max_{e_1, e_2} w_0 + \alpha e_1 - 3e_1^2 - 3e_2^2 + 2e_1 e_2$$

FOCs are

$$\alpha - 6e_1 + 2e_2 = 0,$$
$$-6e_2 + 2e_1 = 0,$$

This implies that

$$e_1 = \frac{3\alpha}{16},$$
$$e_2 = \frac{\alpha}{16}.$$

(b) Compute the worker's utility from the contract. Explain that the IR constraints are binding.

Solutions: The IR constraints are binding because otherwise the principal can decrease w_0 and increase its own profits.

$$\begin{aligned} U_0 &= w_0 + \alpha\left(\frac{3\alpha}{16}\right) - 3\left(\frac{3\alpha}{16}\right)^2 - 3\left(\frac{\alpha}{16}\right)^2 + 2\left(\frac{3\alpha}{16}\right)\left(\frac{\alpha}{16}\right) \\ &= w_0 + \alpha^2\left[\frac{48}{256} - \frac{27}{256} - \frac{3}{256} + \frac{6}{256}\right] = w_0 + \alpha^2\frac{3}{32}. \end{aligned}$$

(c) State the principal's problem. Use the above observation to reduce the principal's problem to the unconstrained version (you do not need to solve it). Will the district choose a flat-wage contract?

Solutions:

$$\max_{\alpha} \left(U_0 - w_0 - \alpha^2 \frac{3}{32} \right) + \frac{\alpha}{16}.$$

The FOCs are

$$\frac{3}{16}\alpha = \frac{1}{16},$$

or $\alpha^* = \frac{1}{3}$.

(3) (Moral hazard with partially observable effort) An IT security professional (agent) is hired to work for an online retailer (principal). The employer offers a contract: If there is no data breach, the agent receives wage w_0, and if there is a data breach, the agent receives w_1. The employer chooses the contract to maximize the profits. The contract must satisfy the *minimum wage constraint*: namely, the agent's salary must be higher than $w_1, w_0 \geq w_{\min}$, where $w_{\min} \geq 0$ is a constant. The probability of data breach,$q(e) = 1 - e$, depends on the level of effort $e \geq 0$ chosen by the agent. The cost of effort is $c(e) = \frac{1}{2}e^2$. The cost of data breach for the employer is $d > 0$.

(As you recall, the standard principal-agent's problem assumes an IR constraint that ensures that the agent receives at least utility equal to her outside option. Here, we consider the alternative: minimum wage constraint. The goal of this question is to check whether the properties of the standard solution are preserved under a different type of constraint.)

(a) Given w_0, w_1, find the optimal choice of effort by the agent.

Solutions: Let $\Delta = w_0 - w_1$ be the "bonus" for no data breach. The agent solves

$$\max_e w_1 + (1 - q(e)) \Delta - c(e).$$

The FOCs are

$$\Delta - e = 0,$$

or

$$e(\Delta) = \max(\Delta, 0).$$

This solution is valid

(b) State the principal's problem. (Remember to use the minimum wage constraint instead of the IR constraint.)

Solutions:

$$\max_{e^*,\pi_0,\pi_1} -dq(e^*) - w_0(1-q(e^*)) - q(e^*)w_1 \,(= -d(1-e^*) - w_1 - \Delta e^*)$$

$$\text{st.} e^* \in \arg\max_e w_1 + (1-q(e))\Delta - c(e), (IC)$$

$$w_1, w_0 \geq w_{\min}, \text{(minimum wage constraint)}$$

(c) Explain that under the optimal contract, $w_0 \geq w_1$.

Solutions: If $w_0 < w_1$then the principal would be better off by taking ${w_1}' = w_0$. Such contract would lead to the same level of effort, and strictly lower expenditure on salary. Hence, $w_0 < w_1$ cannot be optimal.

(d) Solve the employer's problem. (Hint: Using the fact that under the optimal contract $w_0 \geq w_1$, explain that the minimum wage constraint is binding.) What is the effort level chosen under the principal-optimal contract?

Solutions: We have $e(\Delta) = \Delta$. The minimum wage constraint is binding, and

$$w_1 = w_{\min}$$

Substituting the IC and the IR constraints into the principal's problem, we obtain:

$$\begin{aligned}
&\max_{\Delta} - d(1 - e(\Delta)) - w_{\min} - \Delta e(\Delta) \\
&= -d - w_{\min} + d\Delta - \Delta^2.
\end{aligned}$$

The FOC's imply

$$d = 2\Delta,$$

whcih implies

$$\Delta^* = \frac{1}{2}d.$$

(e) Find the first-best (i.e., socially optimal) level of effort. Is the effort chosen under the optimal contract the same as the first-best?Can the optimal contract be interpreted as selling the firm? Why or why not?

Solutions: Find a solution to

$$\max_{e} -dq\,(e) - c\,(e)\,.$$

The solution is $e = d$, which is strictly larger than $e\,(\Delta^*) = \Delta^* = \frac{1}{2}d$. No. Notice that the "bonus" Δ is *not* equal to the benefit from not having the breach.

(4) (Reputation) A consultant faces a sequence of $T < \infty$ clients. The clients approach the consultant one after another. Each client decides whether to hire the consultant or not; if the consultant is hired, she decides whether to put an effort. Each subsequent clients observe the decisions made in the previous periods. The payoffs are

Consultant, Client	Hire	No hire
Effort	$a, 1$	0,0
No effort	$b, -1$	0,0

,

where we assume $0 < a < b$. Each next client observes the outcome of the previous game (and, in particular, whether the consultant was hired and whether she put an effort) before he makes his decision.

(a) Find the sub game perfect equilibrium of the T repeated game. Is the equilibrium unique?

Solutions: The unique Nash equilibrium of the static game is to Not hire, and no effort. The finitely repeated game, repeats this is as an outcome.

(b) From now on, suppose that, with probability $\varepsilon > 0$, the consultant is an "honest" type, who always puts an effort. With probability $1 - \varepsilon$, the consultant is a "normal" type who has payoffs as described above. Show that if $T = 1$, and ε is very small, then in the unique SPE, the client does not hire the consultant. Find threshold ε^* such that if $\varepsilon \geq \varepsilon^*$, the consultant is hired in one shot game.

Solutions: The normal type puts no effort. If the client hirses the consultant, she expects payoff

$$\varepsilon 1 + (1 - \varepsilon)(-1) = 2\varepsilon - 1.$$

It is a best response to hire if $\varepsilon > \varepsilon^* = \frac{1}{2}$.

(c) Suppose now that $T > 1$. Show that if $a(T-1) > b$ and $\varepsilon > 0$, then, in each pure strategy equilibrium, in the first period, the client hires the consultant and the two types of the consultant put an effort.

Solutions: Consider subgame after the first client hires the consultant. Suppose that the equilibrium strategy is for the consultant not to put an effort. Then, after the clients observe No Effort, the consulatnt is revealed to be normal type, and gets the payoff 0 in all subsequent period. Thus, the payoff from No Effort decision is $b + (T-1)\,0 = b$.
OTOH, if the consultant puts an effort, it is recongized as an "honest" type in the continuation game, hence always hired, and can earn at least $a(T-1)$ in the continuation game. Thus, if

$$a(T-1) > b.$$

Thus, it cannot be an equilibrium.
If there is a pure strategy equilibrium, the two types of consultant must put an effort, and, as a best response, the client must hire the consultant.

(5) (Social learning.) Consider a small generalization of the social learning model from the lecture. There are two states of the world $\omega \in \{0, 1\}$ with a prior probability of state 1 equal to $\pi = \frac{1}{2}$. A sequence of agents observes signals $s \in \{0, 1\}$, with the probability $q > \frac{1}{2}$ that signal s is equal to the state ω (and remaining probability $1 - q$ that s is equal to $1 - \omega$). A sequence of agents , one after another, choose actions $a \in \{0, 1\}$. Each agent observes the actions (but no payoffs) of the previous agents. Each agent receives payoff $u_\omega$if her action is equal to the state and 0 otherwise. We assume that $u_1 > 1 = u_0$, i.e., the payoff of choosing the correct action is greater in state $\omega = 1$ than in state $\omega = 0$.

(a) Suppose that p are some agent's beliefs that state is equal to $\omega = 1$. Explain that there is a threshold p^* such that the agent chooses $a = 1$ if $p > p^*$, and $a = 0$ if $p < p^*$. Show that $p^* < \frac{1}{2}$.

Solutions: Payoff from $a = 1$ is equal to

$$p(h)\, u_1.$$

Payoff from $a = 0$ is equal to

$$(1 - p(h)).$$

$a = 1$ is a best response if the former is larger than the latter, or, if $p(h) > p^* = \frac{1}{1+u_1} < \frac{1}{2}$.

(b) Derive condition on q than ensures that the first agent acts sincerely (i.e., chooses his action to be equal to his signal).

Solutions: The beliefs after signal $s = 1$ are equal to

$$p(s_1 = 1) = \frac{\pi q}{\pi q + (1-\pi)(1-q)} = q.$$

The beliefs after signal $s = 0$ are equal to

$$p(s_1 = 0) = \frac{\pi(1-q)}{\pi(1-q) + (1-\pi)q} = 1 - q$$

For the the first agent to act sincerely, we need

$$1 - q < p^* < q,$$

or

$$p^* < q < 1 - p^*.$$

(c) Does the second agent act sincerely? If so, explain. If not, what is the optimal decision of the second agent?

Solutions:Suppose first that the first agent acts sincerly. Then, if the second agent has the same signal as the first agent, she chooses the same action as the first agent. Otherwise, the two signals cancel, and the second agent chooses 1, according to the prior $\pi > p^*$.

If the first agent is not acting sincerely, no information is released, and the second agent is in the same situation. Hence, the second agent does the same thing as the first agent.