Midterm

Economics of Information, ECO421

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There are five questions, but you must solve *any four of them*. (There will be no credit for the extra 5th question.) All questions have the same worth.

Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one.

You have 120 minutes.

(1) (Dirty faces) There are three children $i = a, b, c$. Each child has either a mud on her face or not. The child does not see her own face, but she sees the face of the other two children.

(a) You are going to build a knowledge model of such a situation. First, describe the space of the states of the world and information structures for each of the children. States of the world $\Omega = \{0, 1\}^3$. For each player $i = a, b, c,$, the information structure of i is equal to

$$\mathcal{T}_i = \left\{ \{(0, x_{i+1}, x_{i+2}), (1, x_{i+1}, x_{i+2})\}_{x_{i+1}, x_{i+2}} \right\},$$

where we define $i + 1$ and $i + 2$as modulo operations on the sequence a, b, c.

(b) Let $C_i = \{ \ i \text{ has a clean face } \}$. Describe the events (i.e, the sets of states of the world that correspond to these events)

$$C_a \cup C_b,$$
$$K_a C_a,$$
$$K_a C_b,$$
$$K_c (K_b C_a) .$$

We have

$$C_i = \{(x_1, x_2, x_3) : x_i = 1\} ,$$
$$C_a \cup C_b = \Omega \backslash \{(1,1,0) , (1,1,1)\}$$
$$K_a C_a = \emptyset,$$
$$K_a C_b = C_b,$$
$$K_c (K_b C_a) = C_a.$$

(2) (Investigation) Consider the following version of the Investigation. Trump is guilty for sure, but there is uncertainty whether Mueller has evidence and whether Trump knows about it. There are three states of the world:

- n: no evidence,
  - eu: there is evidence, but it is unknown to Trump,
  - ek: there is evidence and it is known to Trump.

  The information structure is

$$\mathcal{T}_{Mueller} = \{\{n\}, \{eu, ek\}\},$$
$$\mathcal{T}_{Trump} = \{\{n, eu\}, \{ek\}\}.$$

(a) Describe the information structure on a well-labeled picture. TBA

(b) Suppose that both players have a prior belief that there is evidence with probability $p = \frac{1}{2}$ and, if there is evidence, Trump will know about it with probability $q = \frac{2}{3}$. The following table describes the beliefs of each type

Trump	n	eu	ek
U	$\frac{3}{4}$	$\frac{1}{4}$	0
K	0	0	1

,

Mueller	n	eu	ek
N	1	0	0
E	0	$\frac{1}{3}$	$\frac{2}{3}$

.

Bayes formula to explain Trump's beliefs. We have

$$P(n|U) = \frac{P(n,U)}{P(U)} = \frac{P(n,U)}{P(n,U) + P(eu,U)} = \frac{(1-p)}{1-p+p(1-q)} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}\frac{1}{3}} = \frac{3}{3+1} = \frac{3}{4}.$$

(c) Mueller needs to decide whether to try to depose Trump or not. Deposing Trump is costly politically, but it may end up with additional evidence or perjury. Trump needs to decide whether to tell the truth or lie. The latter means that Mueller does not get any new evidence if he does not have any, but also risks a perjury. The payoffs depend on whether Mueller has the evidence and they are in the table below:

$\omega = n$:M\T	Truth	Lie
Depose	1,-1	0,0
Not	0,0	0,0

,

$\omega = eu, ek$:M\T	Truth	Lie
Depose	3,-3	5,-5
Not	1,0	1,0

.

For each of the types of the two players, check whether they have a *strictly* dominant action for each of his types.

Mueller's type E has a strictly dominahnt actin to Depose.
Mueller's type N has a weakly dominant action to Depose.
Trump type K knows the second payoff matrix is the correct one. In such a case, Trump has a strictly dominant strategy to tell the Truth.
The other type of Trump has no strictly dominant action.

(d) Find a Bayesian nash equilibrium. If there is evidence, Mueller will depose (it is strictly domiannt action). If Trump knows that there is evidence, Trump will tell the truth. For the actions of two remaining types, we have the following payoff matrix:

$\omega = n$:Mueller(N)\Trump(U)	Truth	Lie
Depose	$1, \frac{3}{4}(-1) + \frac{1}{4}(-3)$	$0, \frac{3}{4}(0) + \frac{1}{4}(-5)$
Not	0,0	0,0

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$\omega = n$:Muelle
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Hence, there are two equilibria:

- Trump (U) lies and Mueller (N)does not depose,
- Trump (U) lies and Mueller (N)deposes.

(e) Given Trump's behavior, what is Mueller's best response? Mueller will depose if there is no evidence and not depose otherwise.

(3) (Screening) The monopolist sells goods (q, p) where q is the quality and p is the price. The profits from selling each unit of such a good are $p - c(q)$, where the $c(q) = \frac{1}{2}q^2$ is the choice of the quality. The monopolist wants to maximize the profits. The consumer's utility from having a good is equal to

$$(\theta + 1)(1 + q) - p,$$

and the consumer's outside option is equal to 0 (the utility from not having a good and keeping all money). Here, $\theta \geq 0$ is the taste for the quality. The consumer buys the good if the utility from ownership is positive.

(a) Find the optimal choice of p and q in the complete information case, i.e., when the monopolist knows θ. Be careful to state the individual rationality condition. The monopolist will maximize

$$p - \frac{1}{2}q^2 \text{ st. } (\theta + 1)(1 + q) \geq p.$$

In other words,

$$\max_q (\theta + 1)(1 + q) - \frac{1}{2}q^2.$$

The FOCs are

$$\theta + 1 - q = 0,$$

or $q^* = \theta + 1$.

(b) Suppose that there are two types of consumers θ_h with probability π and $\theta_l < \theta_h$ with probability $1 - \pi$. The monopolist wants to design the optimal menu of contracts. Describe the monopolist's problem. Be careful to state the individual rationality and incentive compatibility conditions.

$$\max_{q_h, p_h, q_l, p_l} \pi \left(p_h - \frac{1}{2} q_h^2 \right) + (1 - \pi) \left(p_l - \frac{1}{2} q_l^2 \right)$$

subject to

$$IC_h : (\theta_h + 1)(q_h + 1) - p_h \geq (\theta_h + 1)(q_l + 1) - p_l,$$
$$IC_l : (\theta_l + 1)(q_h + 1) - p_h \leq (\theta_l + 1)(q_l + 1) - p_l,$$

and

$$IR_h : (\theta_h + 1)(q_h + 1) - p_h \geq 0,$$
$$IR_l : (\theta_l + 1)(q_l + 1) - p_l \geq 0.$$

(c) Show that for any incentive compatible menu (i.e, a menu that satisfies the two IC constraints), $q_h > q_l$.

The two IC constraints imply that

$$IC_h : (\theta_h + 1)(q_h + 1) - (\theta_h + 1)(q_l + 1) \geq p_h - p_l,$$
$$IC_l : (\theta_l + 1)(q_h + 1) - (\theta_l + 1)(q_l + 1) \leq p_h - p_l.$$

Putting the two inequalities together, we get

$$(\theta_h + 1)(q_h - q_l) \geq (\theta_l + 1)(q_h - q_l).$$

Because $\theta_h > \theta_l$, it must be that $q_h - q_l > 0$.

(4) (Cheap talk). The CEO observes the quality of the project $\omega = \left\{0, \frac{1}{2}, 1\right\}$, and uses messages $m = \left\{0, \frac{1}{2}, 1\right\}$ to communicate the information to the Investor. The Investor chooses how much to invest $a \in [0,1]$ into the project. The payoffs of the CEO and the Investor are

$$U_{CEO}(a,\omega) = -(a - (\omega + t))^2,$$
$$U_{Investor}(a,\omega) = -(a-\omega)^2.$$

(a) Suppose that the Investor belief is given by $p(0), p\left(\frac{1}{2}\right), p(1) \geq 0$ such that $p(0) + p\left(\frac{1}{2}\right) + p(1) = 1$. Explain that the best response investment level is equal to $a(p) = \sum_{\omega \in \left\{0,\frac{1}{2},1\right\}} \omega p(\omega)$. If the Investor knows that the true state is ω, what is his best response investment? The expected payoff is $E_p U(a,.) = -\sum_{\omega}(a-\omega)^2 p(\omega)$. Investor chooses a to maximize the expected payoff

$$\max_a E_p U(a,.).$$

FOC:

$$\begin{aligned}
0 =& \frac{d}{da}\left(-\sum_{\omega}(a-\omega)^2 p(\omega)\right)\\
=& -\sum_{\omega}\left(\frac{d}{da}(a-\omega)^2\right)p(\omega)\\
=& -\sum_{\omega} 2(a-\omega)p(\omega)\\
=& -2a\sum_{\omega}p(\omega) + 2\sum_{\omega}\omega p(\omega)\\
=& 2(E_p\omega - a).
\end{aligned}$$

Hence, $a_{opt} = E_p\omega = \sum_{\omega}\omega p(\omega)$.

(b) Suppose that the CEO uses truthful communication strategy $m(\omega) = \omega$. Write down the condition that ensures that the CEO does not want to change his message when the true state is equal to $\omega = 0$. $U_{CEO}(a(0), 0) = -(0 - (0 + t))^2 \geq U\left(a\left(\frac{1}{2}\right), 0\right) = -\left(\frac{1}{2} - (0 + t)\right)^2$.In other words,

$$t^2 \leq \left(\frac{1}{2} - t\right)^2,$$

or $t \leq \frac{1}{4}$.

(c) For what values of t the truthful communication is an equilibrium strategy? We need to check all the other incentive conditions. Given the CEO payoffs, it is enough to check that the CEO does not want to deviate upwards:

$$U_{CEO}\left(a\left(\frac{1}{2}\right),\frac{1}{2}\right)=-\left(\frac{1}{2}-\left(\frac{1}{2}+t\right)\right)^{2}\geq U_{CEO}\left(a\left(1\right),\frac{1}{2}\right)=-\left(1-\left(\frac{1}{2}+t\right)\right)^{2},$$

or

$$t^{2}\leq\left(\frac{1}{2}-t\right)^{2}.$$

Hence, the same condition applies, $t<\frac{1}{4}$.]

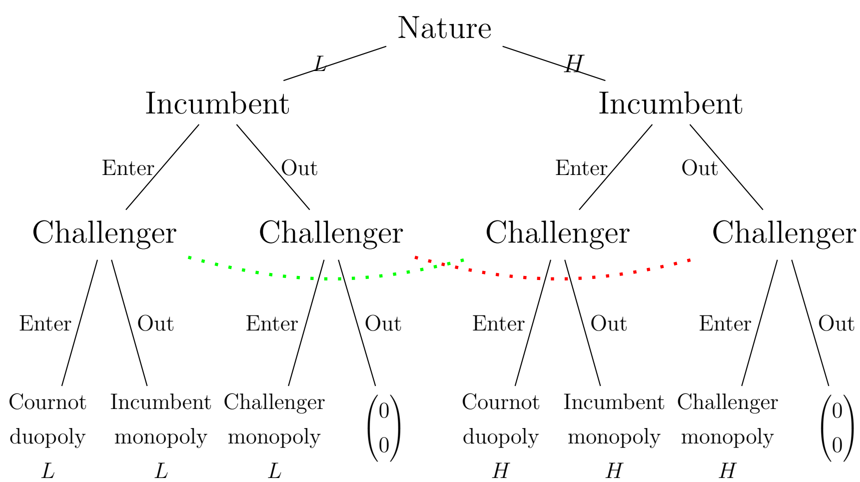
(5) Consider the entry game from the class. Prior to the entry decision, the incumbent observes the entry cost $f \in \{f_L, f_H\}$, where

$$0 < f_L < \frac{1}{9}(a-c)^2 < f_H < \frac{1}{4}(a-c)^2$$

The challenger does not know the entry cost prior to its decision, and it has believes of the high cost is equal to π. We assume that

$$\frac{1}{9}(a-c)^2 < Ef = \pi f_h + (1-\pi) f_l < \frac{1}{4}(a-c)^2.$$

The game is described in the tree below. The payoffs from the Cournot duopoly are $\frac{1}{9}(a-c)^2 - f$ for each of the firm , and the payoff from being monopolist is equal to $\frac{1}{4}(a-c)^2 - f$. The firm that stays out receives payoff of 0.



(a) Suppose that the Incumbent's strategy is "Always Enter$_I$" $a(h) = a(l) = E_I$. What is the Challenger's best response? How does the best response depend on the beliefs? Challenger's best response is

- to stay out after E_I: his beliefs are unupdated, and because the expected cost is higher than the Cournot payoffs, there is not enough room for two firms,
- after O_I, the best response depends on the beliefs. If the beliefs assign sufficiently high probability on the low cost, the Challenger will enter. Otherwise, it will stay out.

Thus, the Challenger's best response is to either Always Out, or Out after Entry, depending on the off-path beliefs.

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(b) Explain that there is an equilibrium (wPBE), in which the the Incumbent's strategy is "Always Enter_I" . Above, we show that Always Out can be a best response to Always Enter, given some off-path beliefs. Because the high cost is smaller than the monopoly's payoffs, Always Enter is a best response to Always Out.

(c) Is there a separating equilibrium (i.e., an equilibrium, where the two types of the Incumbent play different actions)? If the Incumbent plays Enter when Low, the Challenger will best respond with Always Enter_C. If the Challenger always Enters, the Incumbent wants to best respond with Enter when Low.