ECO421: Social learning

Marcin Pęski

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Plan

Wisdom of the crows

- *Wisdom of the crowds*, J. Surowiecki.
- In 1906, F. Galton (statistician, a cousin of C. Darwin) observed an ox-weighing competition.
- 800 farmers estimated the weight of an ox and submitted it to a competition.
- F. Galton checked that the estimates had normal distribution with a significant standard deviation.

Wisdom of the crows

- The average guess was 1,197lb.

Wisdom of the crows

- The average guess was 1,197lb.
- The ox was later weighed to be 1,198lb.

Wisdom of the crows

- The average guess was 1,197lb, which is 1lb less than the true weight of the ox.
- The ox was later weighed to be 1,198lb.
- The average guess was 1lb less than the true weight of the ox.
- There are many examples, where individuals have imperfect information, but when this information becomes aggregated, the wisdom of the crowds
- Economists are interested in when information aggregation works well, and when it fails.

Wisdom of the crows

- Models of information aggregation
  - stock market,
  - prediction markets,
  - voting,
- Today - social learning.

Example

A bag holds n_r of red beans and n_b of brown beans.

- There are two equally likely possible states of the world:
  - R: $n_r = 2n_b$, and
  - $B : 2n_r = n_b$.
- Sequentially, each student
  - privately samples the bag and observes the color of one bean,m
  - poublicly, votes for one of the two states of the world.
- At the end, the state of the world is revealed and students who voted correctly, win.

Social learning

- In the Beans example, each individual has a provide signal about the state of the world.
- Additionally, individuals later in the sequence have an opportunity to observe actions of the earlier individuals.
  - if the earlier indivduals vote depending on their
  - more information.
- An example of social learning.

Social learning

- Social learning, i.e., learning from other actions, is very common.
- We go to popular restaurants, or movies
- Technology adoption.
  - farmers adopting new type of crop,
  - PCs, smartphones, etc.
- Fashion fads, etc.

Plan

Social learning

Simple model

- There are two states of the world, $\omega \in \{x, y\}$
  - common prior probaility of state x is $P(x) = \frac{1}{2}$,
  - both states are equally probable.
- Symmetry is not very important, but helpful.

Social learning
Simple model

- Players $i = 1, 2, 3, \ldots$
- Each player privately observes a signal $s = \{x, y\}$.
  - probability of signal s given state ω is

$$\rho(s|\omega) = \begin{cases} q, & \text{if } s = \omega, \\ 1 - q & \text{otherwise.} \end{cases}$$

  - assume that $q > \frac{1}{2}$.

Social learning

Simple model

- Each player sequentially chooses an action $a \in \{x, y\}$, with payoffs

	$\omega = x$	$\omega = y$
$a = x$	1	0
$a = y$	0	1

- Player i observes her signal, and actions of all players $j < i$ (but not their signals) before she makes her decision.
- Assumption: If a player is indifferent, she randomizes equally.
  - tie-breaking assumption.
- Very simple model, but it will serve to introduce main ideas.
  - we will complicate later.

Social learning
Goals

- Find an equilibrium of the social learning model,
    - beliefs and best responses,
    - beliefs of player 1,
    - beliefs of player 2, etc.
- Does information aggregation work here? Will players learn the true state of the world?

Social learning

Simple model

- Let π be the player's belief that $\omega = x$. Then,
- $a = x$ is a best response if

$$\pi \cdot 1 + (1 - \pi) \cdot 0 > \pi \cdot 0 + (1 - \pi) \cdot 1,$$

  or

$$\pi > \pi^* = \frac{1}{2}.$$

- If $\pi < \pi^*$, then $a = y$ is a best response.
- If $\pi = \pi^*$, player is indifferent and randomizes.

Social learning
Behavior: player 1

- Player 1 will chose action depending on her signal:

$$P(\omega = x | s_1 = x) = \frac{P(\omega = x, s_1 = x)}{P(\omega = x, s_1 = x) + P(\omega = y, s_1 = x)}$$
$$= \frac{\frac{1}{2}q}{\frac{1}{2}q + \frac{1}{2}(1-q)} = q.$$

  - because $q > \frac{1}{2}$m the best response of player 1 with signal $s_1 = x$ is to play $a_1 = x$.
- Similarly, the best response with a signal $s_1 = y$ is to play $a_1 = y$.
- We say that player 1 acts sincerely.

Social learning

Behavior: player 2

- Player 2 has more information: additionally to her signal, she knows that $a_1 = s_1$.
- If $a_1 = s_2$, then player 2 knows that there are two the same signals.
  - she will choose $a_2 = s_1 = s_2 = a_1$.
- If $a_1 \neq s_2$, then pl. 2 knows that there are two conflicting signals.
  - the two signals cancel each other and her posterior is equal to prior $\frac{1}{2}$,
  - she is indifferent and she randomizes (by ass.)
- Player 2 is more likely to choose a_1 if her signal is the same, than if it is different.

Social learning

Behavior:player 3

- Player 3 has even more information.
- **Claim**: If player 3 observes $a_1 = a_2$, then he will ignore his information and always chose the same action as the two previous players.
  - We can compute conditional probabilities, but we have also less formal (but precise) argument.
  - If player 3 observed $a_1 = s_3$, then all the evidence indicates that $a_1 = s_3$ is the right action.
  - Suppose, instead that $a_1 \neq s_3$. Player 3 has conflicting information. Hence, he would be indifferent.
  - But, if additionally he also knows that player 2 chose $a_1 = a_2$, this info tilts the balance towards a_1.
    - remember that player 2 is more likely to choose a_1 if her signal is equal.
- If $a_1 = a_2$, then player 3 ignores her own signal and always chooses the same action as the previous players.
- But it means that player 3 action is not informative any more!

Social learning
Behavior

- Suppose that player 4 observes $a_1 = a_2$,
  - player 4 can ignore a_3 because that is not informative (it's always equal to $a_1 = a_2$ regardless of pl. 3 signal)
  - but then player 4 is in the same situation as player 3,
  - hence, player 4 should choose the same action as the two first players.
- The same argument applies to any other player $i \geq 4$.

Lemma
If $s_1 = s_2$, or $s_1 = a_2$, then all players $i \geq 3$ choose the same action as the first two players, regardless of their private signals.

- This behavior as a *informational cascade.*
  - see references below,
  - more generally, if player i ignores her own signal and chooses a, then it is optimal for player $j = i + 1$ and any subsequent player to ignore their information

Social learning

Behavior

- If the cascade starts, all subsequent players choose the same action as the first two players of the cascade.
- There is no guarantee that this is the correct action.
- "Correct" and "bad" cascades are possible.
- In fact - the first two players can decide that the rest of the society chooses the wrong action.
  - their actions stop the rest from using their own information,
  - negative, "herd" externality,

Social learning

Behavior

- What if $a_1 \neq a_2$?
- Then, player 3 infers that the two first players have different signals that cancel each other.
- Player 3 is in the same situation as player 1.
- The whole process restarts.
- In particular, there will be a cascade if $a_3 = a_4$.

Social learning

Probability of a cascade

- Bad cascade are bad.
- Society in aggregate has all the information to correctly figure out $\omega = x$ (Galton!), but they don't.
- How likely a bad cascade is?

Social learning

Probability of a cascade

- How likely a bad cascade is?
- We can easily compute the probability of a cascade.
- Suppose that the state is $\omega = x$.
- There are 4 possibilities for the two first actions:

$$\begin{aligned} P_{xx}^{x} =& P\left(a_1 = a_2 = x | \omega = x\right) \\ &= q\left(q + (1-q)\frac{1}{2}\right) = \frac{1+q}{2}q, \end{aligned}$$

Social learning

Probability of a cascade

- We can easily compute the probability of a cascade.
- Suppose that the state is $\omega = x$.
- There are 4 possibilities for the two first actions:

$$P_{xx}^{x} = \frac{1+q}{2} q,$$

$$P_{xy}^{x} = q\left((1-q)\,\frac{1}{2} \right) = \frac{1}{2} q\,(1-q)\,,$$

Social learning

Probability of a cascade

- We can easily compute the probability of a cascade.
- Suppose that the state is $\omega = x$.
- There are 4 possibilities for the two first actions:

$$P^x_{xx} = \frac{1+q}{2} q,$$
$$P^x_{xy} = \frac{1}{2} q \left(1-q\right),$$
$$P^x_{yx} = \frac{1}{2} q \left(1-q\right),$$

- We can easily compute the probability of a cascade.
- Suppose that the state is $\omega = x$.
- There are 4 possibilities for the two first actions:

$$P^x_{xx} = \frac{1+q}{2}q,$$
$$P^x_{xy} = \frac{1}{2}q\left(1-q\right),$$
$$P^x_{yx} = \frac{1}{2}q\left(1-q\right),$$
$$P^x_{yy} = (1-q)\left(1-q+q\frac{1}{2}\right) = (1-q)\left(1-\frac{q}{2}\right)$$

- Probability of a good cascaede in period 3:

$$P_{good} = \frac{1+q}{2}q,$$

- Probability of a “bad” cascade in period 3:

$$P_{bad} = (1-q)\left(1-\frac{q}{2}\right).$$

- Probability that no cascade in period 3:

$$P_n = q\,(1-q)\,.$$

Social learning

Probability of a cascade

- Probability of a good cascade eventually

$$P_{good} + P_n P_{good} + P_n^2 P_{good} + \ldots$$

Social learning

Probability of a cascade

- Probability of a good cascade eventually

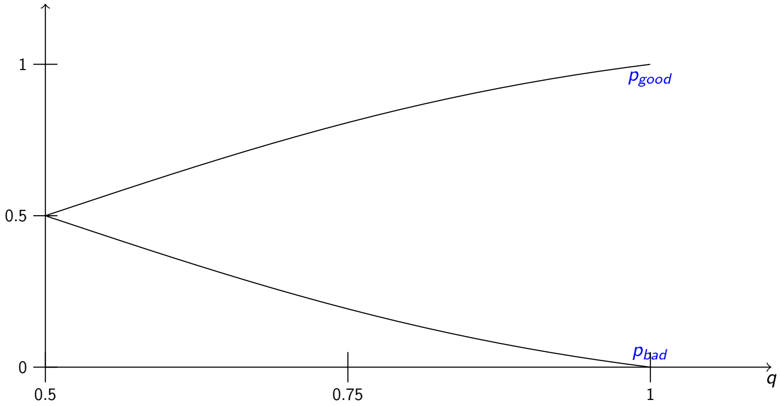
$$
\begin{aligned}
&P_{good} + P_n P_{good} + P_n^2 P_{good} + \ldots \\
=&P_{good} \left(1 + P_n + P_n^2 + \ldots\right)
\end{aligned}
$$

- Probability of a good cascade eventually

$$
\begin{aligned}
& P_{good} + P_n P_{good} + P_n^2 P_{good} + \dots \\
=& P_{good} \left(1 + P_n + P_n^2 + \dots\right) \\
=& P_{good} \frac{1}{1 - P_n} \\
=& \frac{\frac{1}{2}\left(1+q\right) q}{1 - q\left(1-q\right)}.
\end{aligned}
$$

- The remaining probability is of the bad cascade eventually,
  - the cascade will start eventually with prob. 1.

Social learning
Probability of a cascade



Social learning

Probability of a cascade

- The probability of a bad cascade is substantial.
- This is a failure of information aggregation.
- If players could observe each other signals, they would eventually learn the state of the world
  - by the LLN, the fraction of signals s would converge to q iff $\omega = s$ and to $1 - q$ otherwise,
  - thus, by tracking the fraction of signals s, we would eventually learn the state of the world.

Social learning

Results

- Herds and cascades are possible.
- Bad cascades are possible.
- Early deciders are crucial.

Social learning

Results

- In the same time, cascades are fragile.
- Later players base their decision only on few initial players, possibly on the first two.
- Arrival of new public information can upset the cascade.

Social learning

Applications

- Politics: theory of momentum in voting.
- Fads.
- Technology adoption (farmers, hybrid corn seed)
  - initial adoption is very slow,
  - a new information, or exposure can cause a dramatic change to adoption pattern.

Plan

Social learning

Uncertain preferences

- The basic model is very special in many ways.
- After cascade happens, nobody ever does anything different.
- Cascade may arise very fast, after 2 people choose the same action.
- There are other unintuitive consequences. One of them is that for player 2, if he has opposite signal to the player 2 action, the two signals cancel each other out.

$$P(\omega = x | a_1 = x, s_2 = y) = P(\omega = x | a_1 = y, s_2 = x) = \frac{1}{2}.$$

  - in other words, player 2 values the information of player 1 the same as his own.

Social learning
Uncertain preferences

- Two assumptions are responsible:
  - Player 1 and 2 signals are generated using the same technology.
  - But, also, player 1's decision is perfectly transparent to player 2,
- The latter assumption is problematic.
  Often, player 2 is not csure how to interpret player 1's action a: Is it because
  - player 1 got a signal a, or
  - player 1 likes action a very much and chose it regardless of her signal.
- Intuitively, we do not think that other people make decisions in the same way as we do

Social learning

Uncertain preferences

- Model of preferences.

	$\omega = x$	$\omega = y$
$a = x$	$1 - u$	0
$a = y$	0	u

- For each player i.i.d., u is drawn from cdf $F(.)$ on $[0, 1]$.
  - the higher u, player is more inclined to play action y,
  - we assume that cdf F is symmetric around $\frac{1}{2}$: $F(q) + F(1 - q) = 1$.
- Each player observes his own preferences and actions of other players,
  - but not their preferences.
- Other assumptions the same: $P(x) = \pi(x) = \frac{1}{2}$, $\text{Prob}(s = \omega | \omega) = q$, etc.

Social learning
Goal

- We will show that, with uncertain preferences,

$$P(\omega = x | a_1 = x, s_2 = y) < P(\omega = x | a_1 = y, s_2 = x), \quad (1)$$

  or player 2 values his information more than the information of player 1.
- Two parts of the argument:
- First, we use Bayes formula to show that inequality (1) is equivalent to

$$P(a_1 = x | \omega = x) < P(s_2 = x | \omega = x),$$

- Second, we analyze the best response behavior to show that the above inequality holds.

Social learning

Bayes formula

- Notice that

$$P(\omega = x | a_1 = x, s_2 = y) = \frac{P(\omega = x, a_1 = x, s_2 = y)}{P(a_1 = x, s_2 = y)}$$
$$P(\omega = x | a_1 = y, s_2 = x) = \frac{P(\omega = x | a_1 = y, s_2 = x)}{P(a_1 = x, s_2 = y)}.$$

- The symmetry implies that $P(a_1 = x, s_2 = y) = P(a_1 = x, s_2 = y)$.
- Hence, our inequality (1) is equivalent to

$$P(\omega = x, a_1 = x, s_2 = y) < P(\omega = x, a_1 = x, s_2 = y).$$

Social learning

Bayes formula

- Notice that

$$P\left(\omega = x, a_1 = x, s_2 = y\right) = P\left(\omega = x\right) P\left(a_1 = x|\omega = x\right) P\left(s_2 = y|\omega = x\right),$$
$$P\left(\omega = x, a_1 = y, s_2 = x\right) = P\left(\omega = x\right) P\left(a_1 = y|\omega = x\right) P\left(s_2 = x|\omega = x\right).$$

- Hence, our inequality (1) is equivalent to

$$P\left(a_1 = x|\omega = x\right) P\left(s_2 = y|\omega = x\right) < P\left(a_1 = y|\omega = x\right) P\left(s_2 = x|\omega = x\right).$$

Social learning

Bayes formula

- Further notice that

$$P(a_1 = x|\omega = x) = 1 - P(a_1 = y|\omega = x) = q_a,$$
$$P(s_2 = x|\omega = x) = 1 - P(s_2 = y|\omega = x) = q.$$

- The inequality looks like

$$q_a(1-q) < (1-q_a)\,q.$$

After adding qq_a on both sides, we obtain the inequality (1) is equivalent to

$$P(a_1 = x|\omega = x) = q_a < q = P(s_2 = x|\omega = x).$$

- To show the inequality, we need to compute $P(a_1 = x|\omega = x)$.

Social learning

Best responses

- Suppose player with preferences u has belief π that the state of the world is x.
- Best response behavior: choose $a = x$ if

$$\pi (1 - u) > (1 - \pi) u,$$

or, after adding πu to both sides, if

$$\pi > u.$$

- Denote the belief that the state is equal to x after a signal s_1 as $\pi(s_1)$. Then,

$$\pi(x) = q,$$
$$\pi(y) = 1 - q.$$

- Player 1 makes her decision based on signal s_1 she receives, and her preferences. Hence

$$P(a_1 = x | s_1 = x) = \text{Prob}(u < \pi(x)) = F(q),$$
$$P(a_1 = x | s_1 = y) = \text{Prob}(u < \pi(y)) = F(1 - q) = 1 - F(q),$$

where the last equality comes from the symmetry property of the distribution.

- Finally, we compute

$$
\begin{aligned}
P\left(a_1 = x | \omega = x\right) = & P\left(a_1 = x | s_1 = x\right) P\left(s_1 = x | \omega = x\right) \\
& + P\left(a_1 = x | s_1 = y\right) P\left(s_1 = y | \omega = x\right) \\
= & F\left(q\right) q + \left(1 - F\left(q\right)\right)\left(1 - q\right) \\
= & 1 - q + F\left(q\right)\left(2q - 1\right).
\end{aligned}
$$

  But because $F\left(q\right) < 1$, the above is smaller than

$$
1 - q + F\left(q\right)\left(2q - 1\right) < 1 - q + 2q - 1 < q.
$$

- This is what we wanted to show.

Plan

Conclusions

What did we learn - skills

- how to find
- the size of herd necessary for a cascade
- probability of a bad cascade
- stable point of social learning

Conclusions

Further reading

- Herds
  - A. Banerjee, "A simple model of herd behavior" (1992) Quarterly Journal of Economics,
  - Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch. "A theory of fads, fashion, custom, and cultural change as informational cascades." Journal of political Economy 100.5 (1992): 992-1026.
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