ECO421: Communication

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Plan

Introduction

- Asymmetric information means some players know more than the others.
- In real life, information can be transmitted
- A simple model of communication.

Introduction

Communication games

- Two players.
- Player 1 (Sender):
  - learns something about the world,
  - attempts to communicate it to the other player,
- Player 2 (Receiver)
  - observes the message sent by the Sender,
  - chooses an action.
- Both players payoffs depend on the action and the state of the world,
  - but not on the message ($\Rightarrow$signaling).

Introduction

Types of communication games

- Cheap talk:
  - messages have no cost, no payoff consequences at all.
  - Example: everyday communication but no punishment for lying.
  - "*Mere prattle, without practice.*" Shakespeare
  - An article heading in the Portsmouth Times published on August 21, 1958 carried the headline: "United Nations: Talk Is Cheap." The story was about another skirmish in the Middle East and reported in part:
    "Those who have criticized the United Nations for doing nothing but talk can be thankful there has been a place to talk, which is cheap and much to the preferred over armed conflict, which is costly."
- Verifiable talk
  - Bob has an full-proof evidence and may choose to show it.

Plan

Cheap talk I

- Cheap talk: messages have no cost, no payoff consequences at all.
- When can it work?
- Three examples:
  - I: common interests,
  - II: conflict of interests,
  - III: partial conflict of interests

Cheap talk

Doctor and patient with aligned interests

Example

Ann is a doctor, Bob is a patient.

- Ann asks Bob "Do you have frequent headaches"?
  - Before he answers, she thinks that Bob has headaches with prob. $p < \frac{1}{2}$.
- Bob answer ($m \in \{Y, N\}$) depends on the factual state of the world $\omega \in \{H, NH\}$.
- Depending on his answer, Ann may decide whether to send Bob for expensive MRI, $a \in \{MRI, NMRI\}$.
- Payoffs:

	$\omega = H$	$\omega = NH$
MRI	1,1	-1,-1
NMRI	-1,-1	1,1

Cheap talk

Doctor and patient with aligned interests

- Bob's strategy: $m : \{H, NH\} \to \{Y, N\}$
  - TT: truth-telling: $m(H) = Y$, $m(NH) = N$
  - AY: always yes (babbling): $m(H) = m(NH) = Y$,
  - AN: always no (babbling): $m(H) = m(NH) = N$,
  - L: lying: $m(H) = N$, $m(NH) = Y$.

Cheap talk

Doctor and patient with aligned interests

- Ann's strategy: $a : \{Y, N\} \to \{MRI, NMRI\}$
  - "straightforward": $a(Y) = MRI$, $a(N) = NMRI$
  - "always MRI": $a(Y) = a(N) = MRI$,
  - "never MRI": $a(Y) = a(N) = NMRI$,
  - "contrarian": $a(Y) = NMRI$, $a(N) = MRI$.
- Can any of these strategies be used in equilibrium?

Cheap talk

Idea of an equilibrium in Cheap Talk games

- Both Ann and Bob must best respond given the strategy of the opponent, and beliefs.
- Bob's best responses:
  - "truth-telling" is a best response to "straightforward",
  - "lying" is a best response to "contrarian",
  - any strategy is a best response to "always MRI" or "always NMRI"
- Ann is more complicated:
  - for Ann to best respond, she needs to form beliefs about the state of the world.

Cheap talk

Idea of an equilibrium in Cheap Talk games: beliefs

- Ann's beliefs about the state of the world (i.e., headaches) after Bob's message.
  - depend on Bob's strategy (and her prior belief p)
- Suppose that Bob is "telling the truth".
- Ann's posterior beliefs that Bob has headaches after message m:

$$\begin{aligned} p_{TT}(Y) &= \text{Prob}(\omega = H | m = Y) \\ &= \frac{\text{Prob}(m = Y, \omega = H)}{\text{Prob}(m = Y, \omega = H) + \text{Prob}(m = Y, \omega = NH)} \\ &= \frac{p \cdot 1}{p \cdot 1 + (1-p) \cdot 0} = \frac{p}{p} = 1, \end{aligned}$$

Cheap talk

Idea of an equilibrium in Cheap Talk games: beliefs

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  - depend on Bob's strategy (and her prior belief p)
- Suppose that Bob is "telling the truth".
  - Ann's posterior beliefs that Bob has headaches after message m:

$$
\begin{aligned}
p_{TT}(Y) &= \text{Prob}(\omega = H | m = Y) \\
&= \frac{\text{Prob}(m = Y, \omega = H)}{\text{Prob}(m = Y, \omega = H) + \text{Prob}(m = Y, \omega = NH)} \\
&= \frac{p \cdot 1}{p \cdot 1 + (1-p) \cdot 0} = \frac{p}{p} = 1, \\
p_{TT}(N) &= \text{Prob}(\omega = H | m = N) \\
&= \frac{\text{Prob}(m = N, \omega = H)}{\text{Prob}(m = N, \omega = H) + \text{Prob}(m = N, \omega = NH)} \\
&= \frac{p \cdot 0}{p \cdot 0 + (1-p) \cdot 1} = 0.
\end{aligned}
$$

Cheap talk

Doctor and patient with aligned interests

- Suppose that Bob is "telling the truth".
- Ann's best response:
  - given Y,she knows that $\omega = H$, hence best response MRI,
  - given N,she knows that $\omega = NH$, hence best response $NMRI$,
  - overall best response strategy is "straightforward"

Lemma

There is an equilibrium, where

- *Bob uses the truth-telling strategy,*
- *Ann uses "straightforward" strategy, and*
- *Ann's beliefs are* $p(Y) = 1$, $p(N) = 0$.

Cheap talk

Doctor and patient with aligned interests

- (Perfect Bayesian) Equilibrium:
  - Ann's and Bob's strategies,
  - Describe Ann's posterior beliefs $p(m)$ about $\omega = H$ that are derived from
    - her prior belief p,
    - Bob's strategy,
    - Bayes formula and Bob's strategy as long as it is possible,
  - check that Ann's strategy is a best response given her posterior beliefs,
  - check that Bob's strategy is a best response to Ann's strategy.

Cheap talk

Doctor and patient with aligned interests

Lemma

There is another (perfect Bayesian) equilibrium, where

- *Bob uses the lying strategy,*
- *Ann uses the "contrarian strategy, and*
- *Ann's beliefs are* $p(N) = 1$, $p(Y) = 0$.

- Check!
- In PBE, meaning of the message comes only from equilibrium.
  - In real-life, meaning comes also from semantic context.

Cheap talk

Doctor and patient with aligned interests

Lemma

If $p < p^* = \frac{1}{2}$, there is an equilibrium, where

- *Bob "babbles yes",*
- *Ann never prescribes MRI, and*
- *Ann's beliefs are $p(Y) = p$, $p(N) \leq p^*$.*

- If $p < \frac{1}{2}$, if there is no information, Ann would never prescribe MRI.
- Hence, "never MRI" is a best response to "babbling".
- Bob's babbling is also best response given that Ann ignores his messages.

Cheap talk

Doctor and patient with aligned interests

- In summary, each of Bob's strategy can be used in equilibrium.
- But this was a very special game, where Ann and Bob's payoffs were completely aligned.
- Next, we consider misaligned payoffs.

Plan

Cheap talk II

Doctor and patient with misaligned interests

Example

Ann is a doctor, Bob is a patient. Ann asks Bob "Do you have frequent headaches"?

- Before he answers, she thinks that Bob has headaches with prob. $p < \frac{1}{2}$.
- Bob answer ($m \in \{Y, N\}$) depends on the factual state of the world $\omega \in \{H, NH\}$.
- Depending on his answer, Ann may decide whether to send Bob for expensive MRI, $a =\in \{MRI, NMRI\}$.
- Payoffs: Bob

	$\omega = H$	$\omega = NH$
MRI	1,1	-1,1
$NMRI$	-1,-1	1,-1

- Bob always wants MRI (maybe because he is hypochondriac)

Cheap talk II

Doctor and patient with misaligned interests

Lemma

In all equilibria, Bob babbles.

- Indeed, suppose that Bob is using a truth-telling strategy. Then, Ann's beliefs are $p(Y) = 1$, $p(N) = 0$, and Ann wants to do MRI only if the message is Y.
- But then, Bob's best response is to always say Y.
- Similar argument for "contrarian".
- There is no meaningful communication in the game with misaligned interests.

Plan

Cheap talk III

Meaningful but not truthful communication

Example

Investor asks CEO to tell him how much to invest.

- CEO knows the optimal level of investment $\omega \in [0,1]$. Investor prior belief is uniform across $[0,1]$.
- After listening to the CEO, Investor chooses investment level $a \in [0,1]$.
- Payoffs:

$$U_I(\alpha, \omega) = -(a-\omega)^2,$$
$$U_{CEO}(a, \omega) = -(a-\omega-t)^2.$$

t is a measure of the misallignment of preferences.

Cheap talk III

Meaningful but not truthful communication

- Payoffs:

$$U_I(\alpha, \omega) = -(a - \omega)^2,$$
$$U_{CEO}(a, \omega) = -(a - \omega - t)^2.$$

- Given ω,
  - the "bliss-point" of the Investor is $a = \omega$,
  - the "blisspoint" of the CEO is $a = \omega + t$.

Cheap talk III

Meaningful but not truthful communication

- Given uncertainty about ω, the Investor's best response is always to choose the expected valuue of the state:
  - suppose that the Investor beliefs are given by p.d.f. $p(\omega)$ on $[0,1]$, where $\int p(\omega)\, d\omega = 1$,
  - the expected payoff is $EU = -\int (a-\omega)^2 p(\omega)\, d\omega$.
  - FOC:

$$\begin{aligned}
0 =& \frac{d}{da}\left(-\int (a-\omega)^2 p(\omega)\, d\omega\right) \\
=& -\int \left(\frac{d}{da}(a-\omega)^2\right) p(\omega)\, d\omega \\
=& -\int 2(a-\omega) p(\omega)\, d\omega \\
=& -2a\int p(\omega)\, d\omega + 2\int \omega p(\omega)\, d\omega \\
=& 2(E_p\omega - a).
\end{aligned}$$

  - hence, $a_{opt} = E_p\omega = \int \omega p(\omega)\, d\omega$.

Cheap talk III

Meaningful but not truthful communication

- CEO's strategy
  $m : [0,1]_{\text{states of the world}} \to [0,1]_{\text{levels of investment}}$
  - truthful: $m(\omega) = \omega$,
  - babbling: $m(\omega) = 0$ (or any other constant).

Cheap talk III

Babbling equilibium

Lemma

There is always a babbling equilibrium,

- Investor strategy: always invest $a(m) \equiv E\omega = \frac{1}{2}$.
- CEO is best responding because his message is irrelevant.
- Investor's beliefs $p(m)$ are uniform over $[0, 1]$. So, Investor is best responding.

Cheap talk III

No truthful equilibrium

Lemma

There is no truthful equilibrium

- Suppose that *CEO* says truthfully $m = \omega$.
- Then, the Investor who hears message m thinks that the state of the world is m and chooses investment level $a = m$.
- But then, the investor would like to report $m' = \min(\omega + t, 1)$.

Cheap talk III

Doctor and patient with aligned interests

Lemma

If $t < \frac{1}{4}$, then for $x = \frac{1}{2} - 2t$, there is an equilibrium such that the CEO sends a message

$$m(\omega) = \begin{cases} Low, & \text{if } \omega < x, \\ High, & \text{if } \omega > x, \end{cases}$$

and the Investor invests

$$a(Low) = \frac{1}{2}x,$$

$$a(High) = \frac{1}{2}(1 + x).$$

Cheap talk III

Doctor and patient with aligned interests

- Assume that beliefs

$$p(m) = \begin{cases} \text{uniform on } [0, x], & \text{if } m = \text{Low}, \\ \text{uniform on } [x, 1], & \text{if } m = \text{High} \end{cases}$$

- Investor's best response

$$a(m) = \begin{cases} E_{p(0)}\omega = \frac{1}{2}x, & \text{if } m = \text{Low}, \\ E_{p(1)}\omega = \frac{1+x}{2}, & \text{if } m = \text{High} \end{cases}$$

Cheap talk III

Meaningful but not truthful communication

- Let's check whether the CEO is best responding.
- The payoffs of CEO ω from sending the two messages are

$$U_{CEO}\left(a\left(\text{Low}\right),\omega\right)=-\left(\frac{1}{2}x-\left(\omega+t\right)\right)^{2}$$

$$U_{CEO}\left(a\left(\text{High}\right),\omega\right)=-\left(\frac{1+x}{2}-\left(\omega+t\right)\right)^{2}$$

- If $\omega=x$, then the CEO is indifferent between sending the indifference (notice that $t=\frac{1}{4}-\frac{1}{2}x$):

$$\left(\frac{1}{2}x-\left(x+t\right)\right)^{2}=\left(\frac{1}{2}x-x-\frac{1}{4}+\frac{1}{2}x\right)^{2}$$

$$=\left(-\frac{1}{4}\right)^{2}=\left(\frac{1}{4}\right)^{2}=\left(\frac{1}{2}+\frac{x}{2}-x-\frac{1}{4}+\frac{1}{2}x\right)^{2}$$

$$=\left(\frac{1+x}{2}-\left(\omega+t\right)\right)^{2}.$$

Cheap talk III

Meaningful but not truthful communication

- If $\omega < x$, the CEO prefers to send message Low. (CHECK!).
  If $\omega > x$, the CEO prefers to send message High.
- The equilibrium makes sense. Some communication can be restored by partial information.

Plan

Verifiable talk

- So far, talk was entirely cheap: there was no way to put up anything behind the messages.
- But sometimes, one can prove that what one is saying is true.
- Verifiable talk.

Verifiable talk

Transcript

Example

A recent UofT graduate approaches an employer. The employer asks: "Can you show me your transcript?"

- The graduate, who knows his grades ($\omega = h$ or $\omega = l$), can reply:
  - Y: "Sure, here it is," or
  - N: "I am sorry, I cannot find it, I must have lost it."
- The employer prior is that the two grades are equally likely. She decides whether to hire or not hire the graduate.
- The payoffs are such that employer wants to hire only if the graduate is high ability. The graduate always wants to be hired.

Employer\graduate	$\omega = h$	$\omega = l$
H	1,1	-2,1
NH	0,0	0,0

Verifiable talk

Transcript

- Graduate strategies: $d : \{h, l\} \to \{Y, N\}$.
  - always show: $d(\omega) = Y$ for each ω,
  - show only good grades: $d(h) = Y$, $d(l) = N$,
  - show only bad grades: $d(h) = N$, $d(l) = Y$,
  - never show: $d(\omega) = N$ for each ω.

Verifiable talk

Transcript

- Always show: $d(\omega)^{\cdot} = Y$ for each ω.
  - employer always learns the type of the graduate.
  - employer always hire type h, and rejects type l graduate.
- For this to be an equilibrium, we need to check that the graduate does not want to deviate.
  - the type h cannot be better off, so no profitable deviation,
  - in order to ensure no profitable deviation for type l , we need to ensure that employer does not want to hire a graduate who shows no grades,
  - beliefs after N must low enough that the employer does not want to hire,
    - notice that such beliefs cannot be derived through Bayes formula, because N does not happen in equilibrium (and the denominator is 0).

Verifiable talk

Transcript

- Show only good grades: $d(h) = Y$, $d(l) = N$.
  - when h, the employer learns the type and hires the graduate.
  - when $m = N$, the employer does not learn the type directly, but she has beliefs that employer is type h with prob:

$$p(N) = \frac{\text{Prob}(\omega = h, d(\omega) = N)}{\text{Prob}(\omega = h, d(\omega) = N) + \text{Prob}(\omega = l, d(\omega) = N)}$$
$$= 0!$$

- Because only low type does not show the grades, the employer has correct beliefs that the graduate is type l

Verifiable talk

Transcript

- There is no equilibrium, where graduate never shows the grades.
- On the contrary, suppose that $d(h) = d(l) = N$. Then, the beliefs that graduate has type h are

$$p(N) = \frac{\text{Prob}(\omega = h, d(\omega) = N)}{\text{Prob}(\omega = h, d(\omega) = N) + \text{Prob}(\omega = l, d(\omega) = N)}$$
$$= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{2}.$$

- The payoffs are such that the employer does not want to hire.
- But then, type h prefers to show his grades and be hired!

Verifiable talk

Transcript

- There is no equilibrium, where only type *l* shows his grades.
- Why?

Verifiable talk

Transcript

Theorem

In each equilibrium of the Transcript game, the employer correctly infers the type of the graduate, even if the transcript is not shown.

Verifiable talk

- With verifiable information, in equilibrium, the employer will always know what are the graduate grades, even, if the graduate is not going to show them.
- Remains true for more than two types $\omega = 0, 1, 2, ..., N$ as well.
- Whenever an agent can prove high quality, but refuses to do it, this action is interpreted against him.
  - no news = bad news,
  - Milgrom (82) "Good news, bad news".
- Application:
  - missing documents in application,
  - omission of certain topics in reference letters.

Plan

Communication under commitment

- In the Transcript game, we assumed that the graduate has a choice whether to reveal the grades or not.
- What if not?
- Here, we assume that the choice whether to reveal the grades is made by the school, in the interest of *all* students.
- The school has commitment power and can ensure, by its policy, never to reveal the grades, regardless of whether they are good or bad.
- Would the school want to do it?

Communication under commitment

MBA schools

Fact

Top MBA schools (Harvard, Stanford) do not reveal grades their graduates.

Fact

Lower rank MBA schools do reveal their grades.

- *Rotman: "All grades, including failed (FZ) and all non-grade course reports appear on the official U of T transcript."*

Fact

Top MBA schools are very competitive and the graduate have higher employability.

Communication under commitment

Prep schools

Fact

Top private prep schools consider stopping revealing grades of their students.
See: "Elite High Schools Plot to Undermine College Admissions".

Fact

Ivy League universities use high school grades as an important criterion for the admission.

Fact

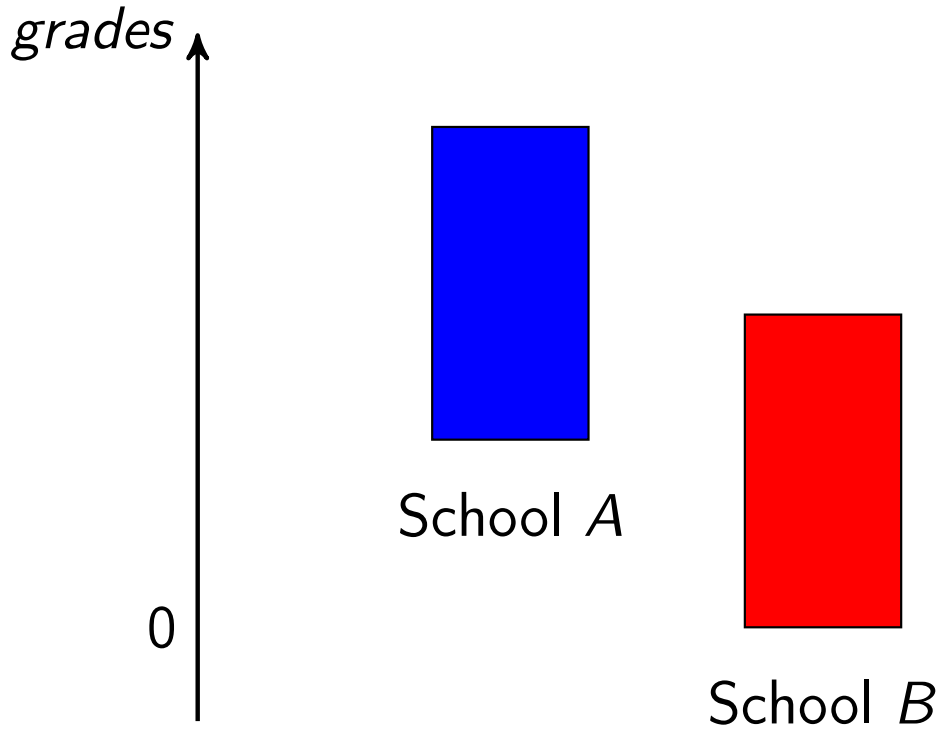
Top private prep schools have very high fraction of their students admitted to the Ivy League.

Communication under commitment

- Why?
- Model: two schools A and B. Both schools have 100 students.
- The grades in school A are distributed uniformly on $[60, 160]$.
- Te grades in school B are distributed uniformly on $[0, 100]$.
- The employers have 100 vacancies. They want to hire the best students.

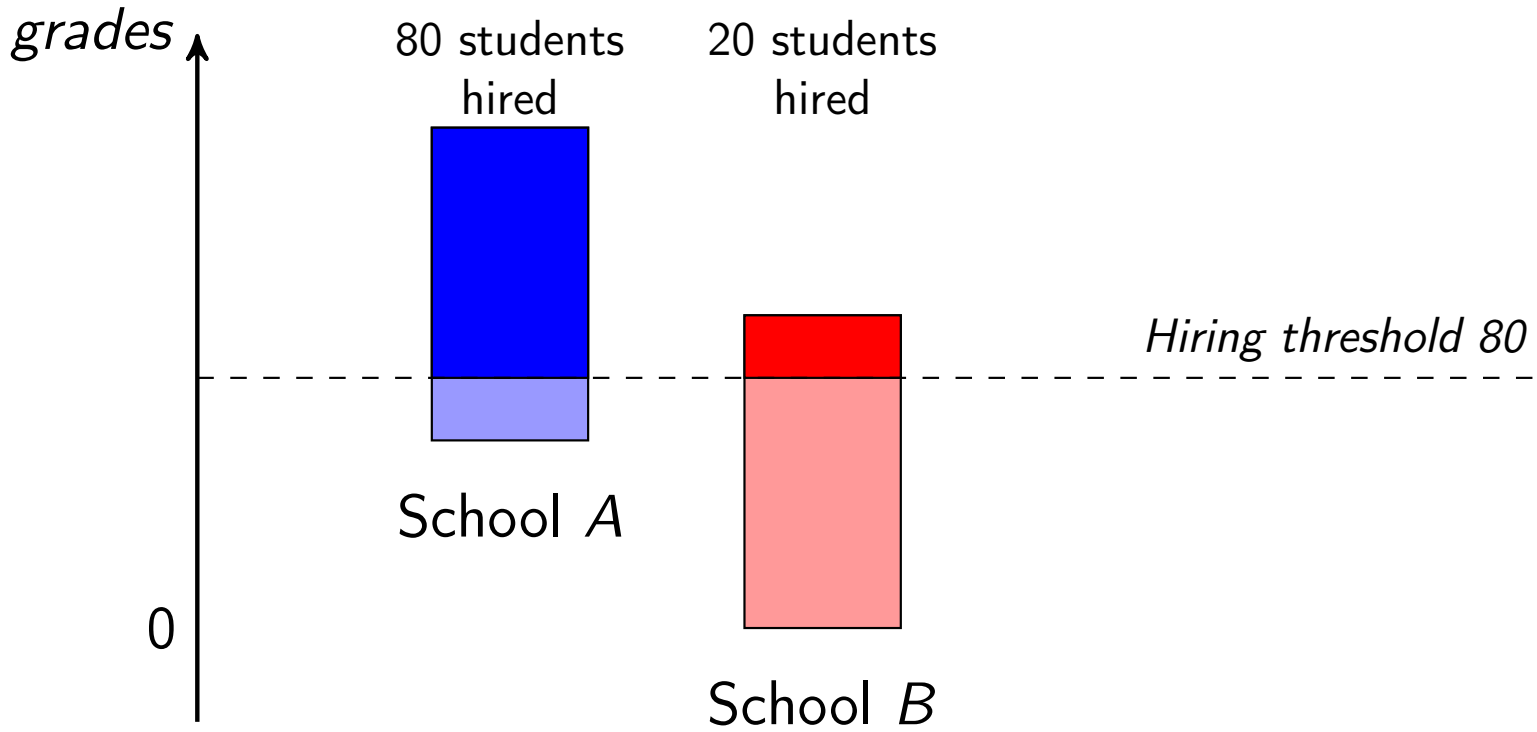
Communication under commitment

- Two schools, A and B



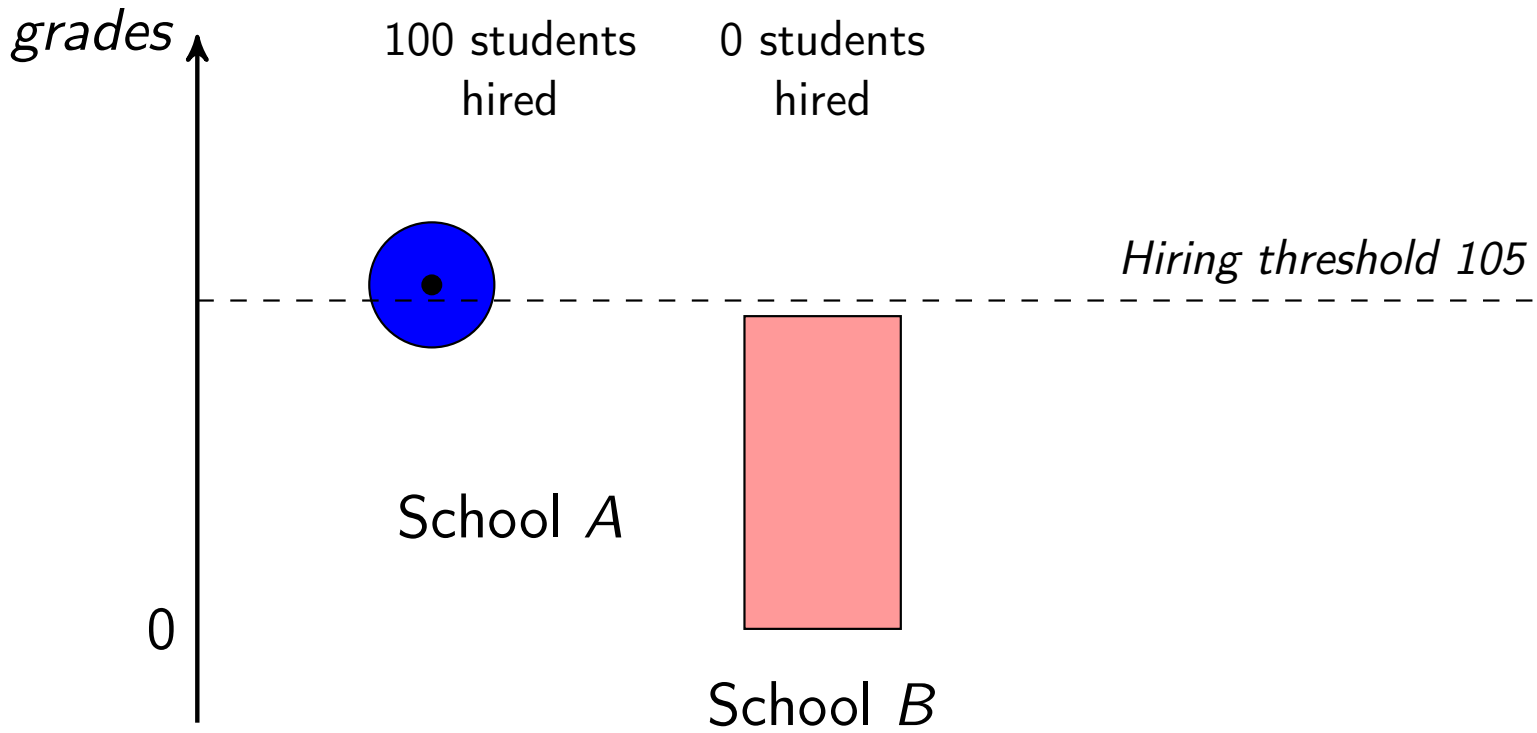
Communication under commitment

- Suppose that both schools reveal the grades.
- Employers hire the best 100 students.



Communication under commitment

- Suppose that School A does not reveal grade.
- The expected quality of students of A is 110> the top quality of student in B.



Communication under commitment

- You can play with the distribution numbers. But, as long as there is a clearly defined better and worse school, and
- both schools want to maximize the number of the hired graduates,

Theorem

The worse school always reveals grades. The better may or may not, depending on the parameters.

Plan

Conclusions

What did we learn - concepts

- Communication game.
- Incentives to communicate in cheap talk games:
  - with possible communication,
  - impossible communication, or with partial communication.
- Communication with verifiable talk.
- Commitment and revelation of information.

Conclusions

What did we learn - skills

- Find equilibria in simple cheap talk games.
- Explain when refusing to reveal information is informative.