ECO421: Signaling

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Plan

Introduction

Signaling games

- We talked about communication
- cheap talk
  - verifiable talk
- "*Talk is cheap. Actions speak.*"

Introduction

Signaling games

- Ann and Bob
- Ann observes the state of the world θ and chooses action a.
- Bob observes a (but not θ) and chooses action b.
- Players receive payoffs that depend on a, b, and θ.
  - Special case: *cheap talk (communication) games*, where payoffs do not depend on a.
- Today: general case.

- Ann's choice of *a* may reflect Ann's preferences.
- But, Ann may also have an interest in communicating her type to Bob
  - Bob's action depend on his beliefs,
  - Ann cares about Bob's action.
- Ann may want to use her action to signal her type to Bob.

Signaling models in this class:

- Why do we go to the university?
- Why do consultants wear suits?
- Signaling everywhere.

Plan

Suits

- Why do we wear suits to the job interview?
- Or why do we wear suits at all?
- They are uncomfortable.

Suits

Example

A consulting firm is interviewing job market candidates.

- There are two types $\theta = l, h$ and the prior probability of the high type is $\pi \in (0, 1)$.
- The type of the candidate is unobservable, but the firm can see whether the candidate wears a suit.
- The cost of the suit is the same for both types $c > 0$.
- The firm decides whether to employ the candidate. If the candidate θ is hired, the payoffs are u_θ for the candidate and v_θ for the firm.
- We assume that

$$\begin{aligned} v_h &> 0 > v_l, \\ \pi v_h + (1 - \pi) v_l &< 0, \\ u_h &> u_l. \end{aligned}$$

Suits

Assumptions

- $v_h > 0 > v_l$,
  - the firm would like to hire the high type, but not the low type,
- $\pi v_h + (1 - \pi) v_l < 0$
  - nobody would be hired without the interview.
- $u_h > u_l$,
  - the high type has a higher payoff from being hired,
  - perhaps because he expects better career than the low type.
  - in the Spence model, the employment payoff was the same for both types.
- cost of the suit is the same for everybody:
  - model works if the cost is lower for the high type,
  - in the Spence model, the cost was lower for the high type.

Suits

Types of equilibria

- pooling equilibrium
  - nobody wears suits: always exists,
  - everybody wears suits: never exists,
- separating equilibrium
  - high types only wear suit,
    - exists if $u_h > c > u_l$,
  - low types only wear suit: do not exist.
- partially separating equilibrium
  - mixed strategies.

Suits

Pooling equilibrium

- There is an equilibrium in which nobody wears a suit.
- Indeed, take beliefs

$$p_0 = \pi,$$
$$p_1 = \pi.$$

  The suit is treated as uninformative.
- Because average candidate is not worthy, he is not hired.
- Because wearing suit does not increase chances of being hired, nobody wears the suit.
- Pooling equilibrium.
- If $c > u_h$, this is the only equilibrium.

Suits

Equilibrium with suits - full separation

- If $u_h > c > u_l$, then there is a fully separating equilibrium.
- If $c < u_l, u_h$, then, no fully separating equilibrium.

Suits

Strategies and beliefs

- From now on, assume $c < u_h, u_l$.
- Notation: *mixed* actions
  - α_θ - probability that candidate type θ wears the suit.
  - β_i - probability that candidate is hired if he wears the suit ($i = 1$) or no ($i = 0$),
- beliefs
  - p_i - probability that suited ($i = 1$) or unsuited ($i = 0$) candidate is type h.

Suits

Strategies and beliefs

- We are looking for equilibrium, where

$$1 > \pi\alpha(h) + (1-\pi)\alpha(l) > 0,$$

  or suits are worn, but not always.
  - all other cases already considered:
    - nobody ever wears suits (equilibrium)
    - everybody wears suits (no equilibrium)
  - no off-path beliefs.
- On-path beliefs: Bayes formula:

$$p_1 = \frac{\pi\alpha(h)}{\pi\alpha(h) + (1-\pi)\alpha(l)}, \text{ and}$$

$$p_0 = \frac{\pi(1-\alpha(h))}{\pi(1-\alpha(h)) + (1-\pi)(1-\alpha(l))}.$$

Suits

Employer's decision

- Employer with beliefs p_i:
  - payoffs
    - from hiring $p_i v_h + (1 - p_i) v_l$,
    - not hiring: 0,
- Define the threshold belief as

$$p^* = \frac{-v_l}{v_h + (-v_l)},$$

  - $v_l < 0 \Longrightarrow p^* > 0$,
  - $\pi v_h + (1 - \pi) v_l < 0 \Longrightarrow p^* > \pi$,
    - the latter means that the employer needs to have a better information than the average to hire the candidate.

Suits

Employer's decision

- Employer with beliefs p_i: best response
  - hire ($\beta_i = 1$) if $p_i > p^*$,
  - indifferent ($\beta_i \in [0, 1]$) if $p_i > p^*$,
  - hire ($\beta_i = 0$) if $p_i > p^*$.

Suits

Worker's decision

- Worker type θ wants to wear the suit if

$$\beta_1 u_\theta - c > \beta_0 u_\theta.$$

- Define

$$t_\theta = \frac{c}{u_\theta}.$$

  - notice that $t_h < t_l$.
- Worker's best response
  - wear suit ($\alpha_\theta = 1$) if $\beta_1 - \beta_0 > t_\theta$,
  - indifferent ($\alpha_\theta \in [0, 1]$) if $\beta_1 - \beta_0 = t_\theta$,
  - not wear the suit ($\alpha_\theta = 0$) if $\beta_1 - \beta_0 < t_\theta$.

Suits

Worker's decision

- Monotonicity: because $t_h < t_l$, there are 5 possibilities:
  1. $\alpha_h = \alpha_l = 1$
  2. $\alpha_h = 1, \alpha_l \in (0,1)$,
  3. $\alpha_h = 1,\ \alpha_l = 0$,
  4. $\alpha_h \in (0,1), \alpha_l = 0$
  5. $\alpha_h = 0 = \alpha_l$.
- notice that both types of the worker cannot beindifferent because it would require $t_h = \beta_1 - \beta_0 = t_l > t_h$-> contradiction.

Suits

Worker's decision

- Monotonicity: because $t_h < t_l$, there are 5 possibilities:
- ~~$\alpha_h = \alpha_l = 1$~~
  - "everybody wears suits" never exists
- $\alpha_h = 1$,$\alpha_l \in (0, 1)$,
- ~~$\alpha_h = 1, \alpha_l = 0$~~
  - "full separation" never exists
- $\alpha_h \in (0, 1)\,, \alpha_l = 0$
- ~~$\alpha_h = \alpha_l = 0$,~~
  - "nobody wears suits already considered.

Suits

Equilibrium with suits

- Let us consider $\alpha_h \in (0, 1)\,, \alpha_l = 0$.
- Bayes formula -> $p_1 = 1, p_0 \in (0, 1)$
  - suit is only worn by the high type,
- Because $1 > p^*$, we have $\beta_1 = 1$.
  Because $\alpha_h \in (0, 1)$, we have

$$\beta_0 = \beta_1 - t_h = 1 - \frac{c}{u_h} \in (0, 1)\,.$$

- Becuase $\beta_0 \in (0, 1)$, we have $p_0 = p^*$.

Suits

Equilibrium with suits

- But then

$$
\begin{aligned}
\pi < p^* =& p_0 \\
=& \frac{\pi(1-\alpha_h)}{\pi(1-\alpha_h)+(1-\pi)(1-\alpha_l)} \\
=& \frac{\pi(1-\alpha_h)}{\pi(1-\alpha_h)+(1-\pi)} \\
=& \pi\frac{1-\alpha_h}{1-\pi\alpha_h} < \pi.
\end{aligned}
$$

- Contradiction.
- Hence, no equilibrium so that $\alpha_h \in (0,1), \alpha_l = 0$.

Suits

Equilibrium with suits

- Let us consider $\alpha_h = 1, \alpha_l \in (0, 1)$.
- Bayes formula -> $p_0 = 0, p_1 \in (0, 1)$
  - suit is never worn by the low type,
- Because $0 < p^*$, we have $\beta_0 = 0$.
  Because $\alpha_l \in (0, 1)$, we have

$$\beta_1 = \beta_0 + t_l = \frac{c}{u_l} \in (0, 1) .$$

- Becuase $\beta_1 \in (0, 1)$, we have $p_1 = p^*$.

- Hence,

$$p^* = p_1 = \frac{\pi\alpha_h}{\pi\alpha_h + (1-\pi)\,\alpha_l} = \frac{\pi}{\pi + (1-\pi)\,\alpha_l},$$

which can be used to find the strategy of the low type

$$\alpha_l = \frac{\pi}{1-\pi}\left(\frac{1}{p^*} - 1\right).$$

Suits

Equilibrium with suits

Theorem

If $c < u_l, u_h$, there is an equilibrium in which $\alpha_h = 1, \alpha_l \in (0, 1)$ and the unsuited candidate is never hired.

Signaling everywhere

- Suit is a signal that the worker is of the high type.
- The high type is willing to pay the signal because it increases her chances of getting a job.
- The signal is more valuable for the high type, because she has a higher utility from being hired.
- But some low types also wear the suit.
  - If not, the suited candidates would be always hired, which would make the low type want to wear the suit.
- In equilibrium, the unsuited type is never hired and the suited type is hired with probability that makes the low type indifferent between wearing the suit or not.
  - the high type strictly prefers the suit.

Plan

Education
Model

- Why do we go to the university?
- University is costly (tuition, lost opportunities, etc.)
- Is education worth it?

- Signaling theory of education (M. Spence):
  - education is intrinsically useless,
  - it provides an evidence that a worker is smart and hard-working,
  - employers do not care about the education, but they want to hire “smart and hardworking” workers.

Education
Model

- M. Spence's model of education (1973).
- Two players: worker and employer.
  - worker can be either high θ_h or low quality $\theta_l < \theta_h$,
    - $\Delta\theta = \theta_h - \theta_l$,
  - workers knows her type,
  - employer believes that π is the probability of θ_h.
- Worker chooses education level $e \geq 0$
  - education costs $c(e, \theta) = \frac{1}{\theta} e^2$,
  - the cost is lower for high quality worker.

Education

Model

- Employer observes the education, forms a belief that worker is a high type and offers wage w:

$$w = E_{p(e)}\theta$$
$$= p(e)\,\theta_h + (1 - p(e))\,\theta_l$$

  - beliefs $p(e)$
- In the original Spence's model, there are many employers who Bertrand compete for the worker and offer her the wage equal to their expected productivity.
- Here, we shortcut. We can assume that the employer maximizes:

$$-(w - \theta)^2 .$$

  - Optimal response is $w = E_{p(e)}\theta$.

Education

Model

- Employer observes the education, forms a belief that the worker is a high type and offers wage w:

$$\begin{aligned} w &= E_{p(e)}\theta \\ &= p(e)\,\theta_h + (1 - p(e))\,\theta_l \\ &= \theta_l + p(e)\,\Delta\theta. \end{aligned}$$

Education

Equilibrium

- worker's strategy $e_h, e_l \geq 0$,
- employer beliefs $p(e)$ (prob. that the worker is θ_h) after observing e,
- employer's strategy $w(e)$.

Definition

WPBE $(e(.), w(.), p(.))$:

- worker best responds to $w(.)$,
- employer's strategy: for each e, the employer's strategy

$$w(e) = \theta_l + p(e)\,\Delta\theta,$$

- beliefs after after $e \in \{e(\theta_l), e(\theta_h)\}$,

$$p(e) = \frac{P(e, \theta_h)}{P(e)},$$

Education

Equilibrium

- *Observation*: In any wPBE, we can assume that for each off-path history $e \notin \{e(\theta_h), e(\theta_l)\}$,

$$p(e) = 0,$$
$$w(e) = \theta_l.$$

- Why?
  - take any equilibrium $(e(.), w(.), p(.))$,
  - we can always modify the beliefs and wages so that the off-path beliefs put mass 1 on the low type,
  - this lowers off-path wages, and strenghten incentives to stay on-path.
- The observation simplifies looking for an equilibrium.
- But it is not necessary for an equilibrium:
  - there are plenty of equilibria consistent with other off-path beliefs.

Education

Equilibrium

Two types of pure strategy equilibria:

- pooling equilibria $e_h = e_l$,
- separating equilibria $e_h \neq e_l$.

Education

Pooling equilibrium

- In a pooling equilibrium, both types of the worker choose the same effort e^*.
- Beliefs after e^*:

$$p\left(e^*\right) = \pi \text{ and } w\left(e^*\right) = \theta_l + \pi\Delta\theta = w^*,$$

- Beliefs after $e \neq e^*$,

$$p\left(e\right) = 0 \text{ and } w\left(e\right) = \theta_l.$$

Education

Pooling equilibrium

- Worker type θ's payoff from e^*

$$w^* - \frac{1}{\theta}e^{*2} = \theta_l + \pi\Delta\theta - \frac{1}{\theta}e^{*2}.$$

- Worker's payoff from deviation to $e \neq e^*$

$$w(e) - \frac{1}{\theta}e^2 = \theta_l - \frac{1}{\theta}e^2.$$

  - the best deviation is $e = 0$, with payoff θ_l.
- Thus the worker best responds if

$$\theta_l + \pi\Delta\theta - \frac{1}{\theta}e^{*2} \geq \theta_l,$$

or

$$e^* \leq \sqrt{\theta(\pi\Delta\theta)}.$$

Education

Pooling equilibrium

- Example: $e^* = 0$
  - this equilibrium requires that the employer beliefs that any positive education means that the worker no smarter than the average,
  - not very natural beliefs,
  - but wPBE does not restrict off-path beliefs.
- There are many pooling equilibria.

Education

Separating equilibrium

- In a separating equilibrium, two effort levels $e_l \neq e_h$.
  - beliefs

$$p(e_l) = 0, w(e_l) = \theta_l,$$
$$p(e_h) = 1, w(e_h) = \theta_h,$$

- In equilibrium, the low type cannot get lower wage by choosing $e = 0$.
  0 effort costs 0.
- Hence, it must be that

$$e_l^* = 0 < e_h^*.$$

Education

Separating equilibrium

- In equilibrium, none of the types should want to imitate each other:

$$IC_h : \theta_h - \frac{1}{\theta_h} e_h^2 \geq \theta_l - \frac{1}{\theta_h} e_l^2,$$
$$IC_l : \theta_h - \frac{1}{\theta_l} e_h^2 \leq \theta_l - \frac{1}{\theta_l} e_l^2.$$

  - incentive compatibility!
  - using $e_l^* = 0$, we can rewrite it as

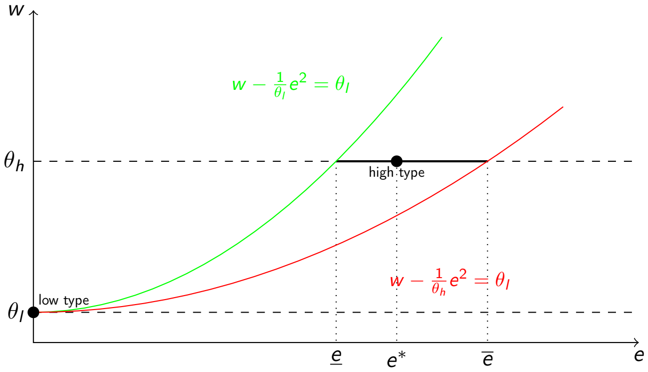
$$\theta_h \Delta\theta \geq e_h^2 \text{ and } e_h^2 \geq \theta_l \Delta\theta, \text{ or}$$

$$\sqrt{\theta_l \Delta\theta} =: \underline{e} \leq e_h^* \leq \overline{e} := \sqrt{\theta_h \Delta\theta}.$$

  - the lower bound is for the low type not to choose the high type effort,
  - the upper bound stops the high type from choosing low type effort.

Education

Separating equilibrium



Education

Separating equilibrium

- Off-path beliefs:
  - if $w(e) = \theta_l$ for $e \neq e_h^*, e_l^*$, we ensure no profitable off-path deviations.
  - all such deviations lead to low wages
- Alternative off-path beliefs:
  - we can also assume that $p(e) = 1$ and $w(e) = \theta_h$ for each $e \geq e_h^*$
  - any beliefs for $e \geq e_h^*$ are good for equilibrium,
  - check the incentives.
- Also, it is ok to assume (some) positive beliefs below e_h^*,
  - as long as the IC conditions are satisfied.

Education

Separating equilibrium

Lemma

There exist a continuum of separating equilibria:

- $e(\theta_l) = 0, e(\theta_h) = e^* \in [\underline{e}, \overline{e}]$,
- $p(e) = 0$ *for each* $e < e^*$ *and* $p(e) = 1$ *for each* $e = e^*$,
- $w(e) = \theta_l + (\Delta\theta)\, p(e)$.

- So,
  - a continuum of pooling equilibrium, and
  - a continuum of separating equilibria.
- Which one is better?

Education

Welfare

- The payoffs in *pooling equilibrium*:
  - the same 0 effort,
  - the same wage $\theta_l + \pi(\Delta\theta)$ for each type,
  - payoffs:$w_{pool}(\theta) = \theta_l + \pi(\Delta\theta)$ for each type θ,

Education

Welfare

- The payoffs in *separating equilibrium* with high effort e^*.
  - Type θ_l: wage θ_l, zero effort,
    - payoff $w_{sep}(\theta_l) = \theta_l < \theta_l + \pi(\Delta\theta) = w_{pool}$,
    - the low type is always worse off.
  - Type θ_h: wage θ_h, effort e^*,
    - payoff $w_{sep}(\theta_h) = \theta_h - \frac{1}{\theta_h}(e^*)^2$,
- The most efficient separating equilibrium is when e^* is the smallest possible, or

$$e^* = \underline{e} = \theta_l(\Delta\theta).$$

- the high type type is better off if

$$\theta_h - \frac{1}{\theta_h}\underline{e}^2 \geq w_{pool} = \theta_l + \pi(\Delta\theta)$$

Education

Welfare

Summary:

- Separation is costly and wasteful.
- But it may raise welfare, because it improves match rate between employers and employees.
- Low types are worse off.

Education

Welfare

Is the signaling model of education correct?

- Alternative model:
  - education creates human capital,
  - more human capital creation for higher ability types.
  - education is costly
- Very similar predictions:
  - higher types get more education,
  - more education means more wages.

Education
Welfare

Is the signaling model of education correct?

- Bryan Caplan (recent book) argues that 30% of the value of education is signaling,\
- but he is rather alone among economists:
  - intelligence is easy to test for, no need to prove it by 4 years of useless education,
  - how about on-the-job signaling (by working for 4 years in different tasks)?
  - most labor economists only work with human capital of higher education.
- Even if incorrect, nice cute model to understand signaling.

Education

Intuitive criterion

- A theoretical point.
- Which of the separating equilibria is most natural?
- Depends on our theory of off-path beliefs.

Education

Intuitive criterion

- Some deviations are very stupid for some types.
- We should not belief that certain deviation was done by a type θ if such a deviation would be very stupid for type θ.
- Seems intuitive.

Education

Intuitive criterion

- The idea is as follows.
- The highest possible wage is θ_h
- For any $e > \tilde{e} = \theta_l(\Delta\theta)$, the upper bound on the payoffs of lower type is

$$w(e) - \frac{1}{\theta_l}e^2 \leq \theta_h - \frac{1}{\theta_l}e^2.$$

Education

Intuitive criterion

- The idea is as follows.
- The highest possible wage is θ_h
- For any $e > \tilde{e} = \theta_l (\Delta\theta)$, the upper bound on the payoffs of lower type is

$$
\begin{aligned}
w(e) - \frac{1}{\theta_l} e^2 &\leq \theta_h - \frac{1}{\theta_l} e^2 \\
&< \theta_h - \frac{1}{\theta_l} \tilde{e}^2 = 0
\end{aligned}
$$

  - the inequality comes from $e > \underline{e}$, and
  - the equality comes the definition of $\underline{e}$ as the effort for which the low type is indifferent between signaling truthfully and imitation the high type.

Education

Intuitive criterion

- We say that action $e > \underline{e}$ is *equilibrium dominated for type* θ.
- The intuitive criterion says that the beliefs after e should not assign positive value to any type θ for which e is equilibrium dominated.
- The intuitive criterion eliminates all equilibria $e^* > \underline{e}$.

Plan

Signaling everywhere

- Features of signaling models
- Ann wants to convince Bob that she is the high type.
- She uses a costly signal:
  - either the cost of signal is lower for high types (Spence), or
  - the benefits from convincing Bob are higher for high types (Suits).

Signaling everywhere

- Conspicuous consumption.
- Handicap principle.
- Contracts.

Signaling everywhere

Conspicuous consumption

- Conspicuous consumption
  - consumption that is not done for its utility value, but for show,
  - T. Veblen (1899)
- Example: luxury watches
- Signaling explanation
  - it is very expensive and wasteful,
  - but I can afford it because my income ability is very high.

Signaling everywhere

Other examples:

- Diamond ring for engagement.
- Large wedding.
- Charity.
- Purdah.

Signaling everywhere
Handicap principle

- Some animal behavior or features seems clearly useless or harmful
    - peacock tail
    - Gazelle stotting
    - larks songs
- Theory of Honest Signals (Amotz Zahavi):
    - a certain debilitating feature can represent a proof of otherwise very fit genes,
    - “if I survived with a long tail, I must be amazing otherwise”
    - mates who select for this feature can have healthier progeny
    - -> evolution.

Signaling everywhere

Handicap principle

- Other examples
  - begging for food (see also Osborne, Ex 335.2)
  - autumn colors,
  - Red frogs

Signaling everywhere

Contract negotiations

- Often contract negotiations are difficult because of the uncertainty of the partner's quality
  - adverse selection (market for lemons)
- Possible solutions: trade terms that are bad on some dimensions to credibly distinguish themselves from "bad" types
  - "lifetime warranty!"
  - "we accept all returns, no questions asked"
  - "we will match the lowest price"

Plan

Conclusions

What did we learn - concepts

- Signaling games
  - features,
  - how to identify signaling in real world.
- Pooling and separating equilibria.
- Intuitive criterion.

Conclusions

What did we learn - skills

- Find equilibria of signaling games,
  - separating and pooling,
  - mixed and pure strategy.
- Find the best payoff equilibria.

Conclusions

Further reading

- Spence's model of education
  - Michael Spence (1973). "Job Market Signaling". Quarterly Journal of Economics. 87 (3): 355–374
  - Andrew Weiss (1995). "Human Capital vs. Signaling Explanations of Wages". The Journal of Economic Perspectives. 9 (4): 133–154
- Handicap principle
  - "Honest Signaling Theory: A Basic Introduction". By Carl T. Bergstrom
- Conspicuous consumption
  - Theory of the Leisure Class by Thorstein Veblen
  - "A signaling explanation for charity" A Glazer, KA Konrad - The American Economic Review, 1996