1. (Screening) The monopolist sells goods (q, p) where q is the quality and p is the price. The profits from selling each unit of such a good are $p - c(q)$, where the $c(q) = \frac{1}{2}q^2$ is the choice of the quality. The monopolist wants to maximize the profits. The consumer's utility from having a good is equal to

$$\theta (1 + q) - p.$$

Here, $\theta \geq 0$ is the taste for the quality. The consumer buys the good if the utility from ownership is positive.

(a) Find the optimal choice of p and q in the complete information case, i.e., when the monopolist knows θ. Be careful to state the individual rationality condition.

Solutions:The monopolist will maximize

$$p - \frac{1}{2}q^2 \text{ st. } \theta (1 + q) \geq p.$$

In other words,

$$\max_q \theta (1 + q) - \frac{1}{2}q^2.$$

The FOCs are

$$\theta - q = 0,$$

or $q^* = \theta$.

(b) Suppose that there are two types of consumers θ_h with probability π and $\theta_l < \theta_h$ with probability $1 - q$. The monopolist wants to design the optimal menu of contracts. Describe the monopolist's problem. Be careful to state the individual rationality and incentive compatibility conditions.

Solutions:

$$\max_{q_h, p_h, q_l, p_l} \pi \left(p_h - \frac{1}{2}q_h^2\right) + (1 - \pi) \left(p_l - \frac{1}{2}q_l^2\right)$$

subject to

$$IC_h : \theta_h (q_h + 1) - p_h \geq \theta_h (q_l + 1) - p_l,$$
$$IC_l : \theta_l (q_h + 1) - p_h \leq \theta_l (q_l + 1) - p_l,$$

and

$$IR_h : \theta_h (q_h + 1) - p_h \geq 0,$$
$$IR_l : \theta_l (q_l + 1) - p_l \geq 0.$$

(c) Show that for any incentive compatible menu (i.e, a menu that satisfies the two IC constraints), $q_h > q_l$.

Solutions: The two IC constraints imply that

$$IC_h : \theta_h (q_h + 1) - \theta_h (q_l + 1) \geq p_h - p_l,$$
$$IC_l : \theta_l (q_h + 1) - \theta_l (q_l + 1) \leq p_h - p_l.$$

Putting the two inequalities together, we get

$$\theta_h (q_h - q_l) \geq \theta_l (q_h - q_l).$$

Because $\theta_h > \theta_l$, it must be that $q_h - q_l > 0$.

(d) Show that the IR_h constraint is implied by the other constraints.

Solutions: Because $\theta_h \geq \theta_l$, if IC_h and IR_l are satisifed, we have

$$\theta_h (q_h + 1) - p_h \geq \theta_h (q_l + 1) - p_l \geq \theta_l (q_l + 1) - p_l \geq 0.$$

(e) Show that the IC_h constraint is binding (i.e., it is satisfied with equality).

Solutions: Given that we do not need to worry about IR_h any more, we can always increase p_h to make IC_h bind. It does not afect other constraints, and it increases profits.

(f) Show that IC_l constraint is implied by the other constraints and the previous observations.

Solutions: If IC_h is binding, then we have

$$p_h = \theta_h (q_h - q_l) + p_l.$$

Hence,

$$\begin{aligned} \theta_l (q_h + 1) - p_h &= \theta_l (q_h + 1) - \theta_h (q_h - q_l) + p_l \\ &\leq \theta_l (q_h + 1) - \theta_l (q_h - q_l) + p_l \\ &= \theta_l (q_l + 1) - p_l. \end{aligned}$$

The inequality comes from the fact that $\theta_h > \theta_l$ and $q_h > q_l$.

(g) Show that the IR_l constraint is binding.

Solutions: If not, and given that we do not need to worry about IC_l, we can always increase p_l, raising profits, and not affecting other constraints.

(h) Use the above discussion to describe the simplified problem of the monopolist.

Solutions: Thus,

$$p_l = \theta_l (q_l + 1)$$

and

$$p_h = \theta_h (q_h - q_l) + p_l = \theta_h (q_h - q_l) + \theta_l (q_l + 1).$$

We subsitute the two prices into the monopolist problem and solve the unconstrained problem.

(i) Find the optimal menu.

Solutions: See the notes.

2. (Health insurance) Suppose that you live in one of the countries where you need to buy healthinsurance on a private market. Suppose that insurance charges fee p and pays out the medical costs $C = 1000$ in case of need. An insured receives extra utility equal to $\Delta = 50$ from having insurance. An individual with risk $\pi = 10\%$ buys the insurance if

$$-p + \pi C + \Delta > 0,$$

where Δ is the extra value of peace of mind for an insured individual.

(a) Derive a condition that guarantees that the insurance company makes non-negative profits. Show that there exists an interval for prices p such that the indivduals are happy to buy insurance and the inusrance companies are happy to sell.

Solutions: $p \geq \pi C$.

$$p \in [\pi C, \pi C + \Delta].$$

(b) Suppose that there are two types of individuals, high risk, with probabioity of damage $\pi_h > \pi$ and low risk, with probabioity $\pi_l < \pi$. We assume that the fraction of high risk individuals is such that

$$\pi = \rho \pi_h + (1 - \rho) \pi_l.$$

For what values of π_l, it is possible to have a market with insurance companies making non-zero profites and both types of the indiduals buying insurance?

Solutions: INsurance companies are happy if

$$p \geq \pi C.$$

The low risk types are happy to buy insurance if

$$p \leq \pi_l C + \Delta.$$

Hence, it must be that

$$\pi C \leq \pi_l C + \Delta,$$

or

$$\pi_l \geq \pi - \frac{\Delta}{C}.$$