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Crosscutting Areas

# Duopoly Competition with Network Effects in Discrete Choice Models

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**Abstract.** We consider two firms selling products to a market of network-connected customers. Each firm is selling one product, and the two products are substitutable. The customers make purchases based on the multinomial logit model, and the firms compete for their purchasing probabilities. We characterize possible Nash equilibria for homogeneous network interactions and identical firms: When the network effects are weak, there is a symmetric equilibrium that the two firms evenly split the market; when the network effects are strong, there exist two asymmetric equilibria additionally, in which one firm dominates the market; interestingly, when the product quality is low and the network effects are neither too weak nor too strong, the resulting market equilibrium is never symmetric, although the firms are ex ante symmetric. We extend these results along multiple directions. First, when the products have heterogeneous qualities, the firm selling inferior product can still retain market dominance in equilibrium due to the strong network effects. Second, when the network effects are heterogeneous, customers with higher social influences or larger price sensitivities are more likely to purchase either product in the symmetric equilibrium. Third, when the network consists of two communities, market segmentation may arise. Fourth, we extend to the dynamic game when the network effects build up over time to explain the first-mover advantage.

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## 1. Introduction

Network effects are widely observed when human beings engage in social interactions. They arise from the "payoff externality," in the sense that an individual's payoff depends not only on her own action, but also on the actions chosen by others. Examples abound, from the classical videotapes, keyboards, operating systems, and telecommunication networks, to recent online games, cloud services, and mobile apps. For instance, Dropbox allows users to conveniently share their files across platforms; users are likely to choose Dropbox over other service providers (e.g., Google Drive and OneDrive) if many of their colleagues have already adopted Dropbox. As another example, an increasing amount of users on one side of a two-sided market (such as Airbnb) attracts more users on the other side and, thus, generates network effects for the users on the same side.

Because network effects largely enhance individuals' willingness to pay through the cascade of externalities, the aforementioned industries are highly profitable, and network effects lift the potential and competition there to the next level. Competition over networks gives rise to various phenomena that could not be observed in conventional industry structures. In particular, it is widely believed that network effects create the winner-take-all phenomenon (Bruner 2014): A product (such as Dropbox) may capture a dominant market share over its seemingly identical alternatives (such as OneDrive). It is also pointed out in several studies (Banerji and Dutta 2009, Bimpikis et al. 2016) that the network structure is related to market segmentation: Product adoption may vary tremendously from segment to segment in the network (e.g., WeChat in China, Facebook's Messenger in North America, and WhatsApp in other regions; see Sevitt 2016). Such market dominance and market segmentation do not arise if the market were monopolized by a single firm.

This paper attempts to gain deep understanding of the firms' competitive strategies when customers' purchasing decisions are influenced by network effects. We pay particular attention to the emergence of market dominance and market segmentation in the form of *asymmetric equilibria*. In pursuit of this goal, we consider two firms that sell their substitutable products to a market of network-connected customers. The customers choose between the two products or leave the market without purchasing. They make their decisions based on *price*, *quality*, and *the anticipated network effect*, in the sense that they are influenced by the choices of their neighbors in the network. We describe the choice process by a multinomial logit (MNL) model (Anderson et al. 1992, McFadden 2001). The MNL model has been very successful in capturing discrete choices, due to its analytical tractability and interpretability. We focus on the Cournot-type competition: The firms' decision variables are customers' choice probabilities, or, equivalently, the *market shares*, rather than the prices. We provide practical and theoretical justifications for this modeling choice in Section EC.3.1 of the online appendix.

Despite the parsimony of our model, the equilibrium analysis turns out to be highly intractable. The complexity can be attributed to three sources: the network interactions, the nonconcave payoff function arising from the MNL model, and asymmetric equilibria. To the best of our knowledge, the first issue has only been addressed under models with special structures (such as Hotelling's model and the linear-quadratic framework; see Section 2 for more details) in the literature, and the second and third issues have not been addressed in this context. To circumvent the difficulty and find the pure-strategy Nash equilibria of the duopoly competition, we adopt a novel approach that focuses on the inverses of the best-response functions (see Section 4). It allows us to obtain analytical results, whereas selecting best responses from the local maxima of a nonconcave function is virtually impossible.

We show that, depending on the products' qualities and the strength of the network effects, the Nash equilibria exhibit highly distinct features. When the products are symmetric and customers are homogeneous—that is, the network that connects customers is a complete graph—a single symmetric Nash equilibrium arises if the network effects are weak. At the other extreme, when the network effects are strong enough, there exist three Nash equilibria: two stable asymmetric Nash equilibria, in which one firm captures almost all the entire market and the other firm is left with little market share, and an unstable symmetric Nash equilibrium. The stable equilibria exhibit some form of *market dominance*, and it emerges because of strengthened competition.

To understand the intuition, note that when the network effects are not strong, customers' purchasing decisions are primarily driven by their intrinsic consumption values. Thus, given that the firms are ex ante symmetric, they equally split the market. When the network effects are very strong, however, the network effects start to take over. This, in fact, homogenizes the products, because now the firms primarily compete on attracting the critical mass of customers. In this vein, equal splitting becomes unstable, and the market structure is geared toward one firm dominating the other. Notably, this asymmetric equilibrium is also more efficient in terms of boosting customers' willingness to pay.

Under the same assumption of homogeneous customers and products, when the product quality is low and the network effects are neither too weak nor too strong, the resulting market equilibrium is *never symmetric*, even if the firms are ex ante symmetric. This result is somewhat paradoxical, but particularly robust, because it implies the nonexistence of even unstable symmetric equilibria. In this sense, our result demonstrates a strong rebuttal to the conventional, and perhaps naïve, intuition that symmetric equilibria shall exist for symmetric firms. We are not aware of any prior work that proves this in the context of network effects with competition.

We next consider products with heterogeneous qualities. When the network effects are strong enough, we establish the existence of two Nash equilibria corresponding to the respective market-dominance positions of each firm, *regardless of their quality difference*. This implies that, even though one product is far inferior to its competitor, it may still retain market dominance due to the strong network effects. This could happen, for example, if the product has a first-mover advantage. Some classical examples include the battle between VHS and Beta in the videotape industry and the victory of the QWERTY keyboard over others.[1] Our second result justifies the investment in quality improvement: If the quality difference is sufficiently large, the superior product can penetrate the network effects and secure its dominating position, which is the only equilibrium outcome of the duopoly competition.

When the network effects are heterogeneous among customers, we show that the firms are more likely to sell to customers with higher social influences and larger price sensitivities in the symmetric equilibrium. We also study networks consisting of two communities—that is, the connectivity inside each community is homogeneous, which is different from the externality between communities. Such a network structure encompasses many special networks that are investigated in the literature, such as star graphs and complete bipartite graphs. We show that *market segmentation* may arise under the network effects: One product is dominating in one community, while the other product is popular in the other community. This market segmentation emerges if the network effects inside communities are relatively strong in comparison with those between communities. In the opposite case, wherein the network effects between communities is stronger than those within communities, the aforementioned segmentation is no longer sustainable.

Finally, the game is extended to a dynamic setting, when customers cannot coordinate perfectly and the network effects build up over time. We show the existence of the pure-strategy, open-loop Nash equilibrium. The steady state of the dynamic game is closely related to the Nash equilibrium of a static game with transformed parameters. We also provide an explanation for the first-mover advantage using the dynamic game.

The rest of this paper is organized as follows. Section 2 reviews some relevant literature. Section 3 introduces our model setup. In Section 4, we consider the case with homogeneous customers—that is, the network is a complete graph. We discuss the possible equilibria with both symmetric and heterogeneous firms. Section 5 discusses heterogeneous network effects. In Section 6, we investigate the dynamic version of the game. Section 7 concludes. All proofs are included in the online appendix.

## 2. Literature Review

The study of the network effect and its influence on economic activities has attracted many researchers in the last few decades. Because of the seminal paper by Katz and Shapiro (1985), monopoly pricing for network goods has been studied extensively in various settings; see Economides (1996) for a review. Recently, *local* network effects have become a central research topic in this area (see, e.g., Ballester et al. 2006). As opposed to *global* network effects, local network effects capture the heterogeneous interaction between any pair of agents in the network and, thus, allow for the analysis of agents' "centrality" measure. For example, Candogan et al. (2012), Bloch and Quérou (2013), and Fainmesser and Galeotti (2015) investigate the pricing problem when customers have local network effects. Hu et al. (2015) incorporate social influence into the newsvendor model and study marketing strategies to leverage the network effect. Abeliuk et al. (2015) and Maldonado et al. (2018) study a monopoly offering multiple products to customers with social influence and position bias.

Our paper is related to a stream of literature that investigates competition and network effects. Network effects create complementarity between customers' choices and may significantly change the market dynamics. For example, Laffont et al. (1998) study duopoly competition for vertically differentiated products in Hotelling's model. They show the existence of two asymmetric customer equilibria, in which the market is cornered by one of the firms, in addition to a symmetric equilibrium. See also Einhorn (1992), Grilo et al. (2001), and Katz and Shapiro (1992). See Farrell and Klemperer (2007) for a comprehensive review of earlier papers.

In a more recent paper, Aoyagi (2018) also demonstrates the multiplicity of Nash equilibria in the customers' buying subgame. His model uses a graph to describe the network effects, and the two firms can price-discriminate. In contrast, Chen et al. (2018) characterize the unique Nash equilibrium and the associated prices and consumptions in a closed form, using quadratic utility functions and linear network effects. Banerji and Dutta (2009) study market segmentation when two firms use uniform pricing for their network goods. In terms of model assumptions, the setups in Calzada and Valletti (2008) and Tan and Zhou (2021) are close to ours. They consider Bertrand-type network competition under the MNL model (or more general choice models). However, they only study symmetric equilibria and cannot obtain the insights provided by our model, such as the nonexistence of symmetric equilibria when the network effects are neither too weak nor too strong. In a setting similar to ours, Feng and Hu (2017) observe that market concentration increases with the network effects. Different from us, their model does not use the MNL model for customer choice.

Our paper differs from the previous literature in the following ways. First, we introduce customers' discrete choice modeling into competition. Customers do not select a product simply because it generates a higher utility than its competitor; instead, a choice probability is assigned to each product, as well as the no-purchase option. Second, in some previous papers, the multiplicity of Nash equilibria arises from customers' buying subgame, as customers have different expectations of the network effects. In contrast, we focus on Cournot-type competition rather than price competition, following papers such as Katz and Shapiro (1985), Economides (1996), and De Palma and Leruth (1996). As a result, there is no purchasing subgame in our model and the source of the multiplicity of Nash equilibria is the strategic interactions between the firms. Third, we identify a scenario (see Proposition 4) in which *no symmetric equilibrium exists* for symmetric firms. This happens because the network effects make the firms' best response discontinuous. To the best of our knowledge, this phenomenon has not been analyzed in the literature.

As aforementioned, we use the multinomial logit model for customers' discrete choices; see Anderson et al. (1992) and McFadden (2001) for comprehensive reviews. In this regard, our paper is related to the following studies. Du et al. (2016) study the profit-maximization problem of a monopoly offering a set of substitutable products when customers have network effects. The customers make their choices according to the MNL model. Like us, they transform the decision variable from pricing to market share. Wang and Wang (2016) show that assortment optimization under the MNL model with network effects is NP-hard, but a simple heuristic can achieve near-optimal performance. Cohen and Zhang (2017) use the MNL model to capture the decision process of riders choosing between platforms. Anderson et al. (1992), Bernstein and Federgruen (2004), Gallego et al. (2006), Aksoy-Pierson et al. (2013), and Gallego and Wang (2014) establish various conditions for the existence and uniqueness of Nash equilibria for the MNL model and its generalizations, such as the mixed multinomial logit model. They do not consider network effects among customers. Brock and Durlauf (2001) investigate the equilibrium properties when individuals make choices in the presence of social interactions, according to the MNL model. They find that there can be one or three choice levels in equilibrium, depending on the parameter values. Although this observation seems similar to our Proposition 2 and Proposition 3, our model and findings differ, in that there is no competition between firms involved in their model, and, as mentioned above, symmetric equilibrium may not exist in our setup.

## 3. The Model

Consider two firms, $X$ and $Y$, selling their substitutable products to a market of $n$ customers. We use superscripts $X$ and $Y$ to denote the features of their respective products. The utilities and choices of customers are subject to the *network effects*—that is, customer $j$ imposes *network externality* $g_{ij} \geq 0$ on customer $i$. The externality $g_{ij}$ quantifies the level of influence customer $j$ has on customer $i$, or, equivalently, customer $i$'s susceptibility to the behavior of customer $j$. The role of $g_{ij}$ will become clear when we define the choice modeling of customers later in the section.

### 3.1. Customer Choices

Let $\alpha^X$ and $\alpha^Y$ be the quality parameters of the products and $\beta_i$ be the price sensitivity of customer $i$.

**3.1.1. Choice Probabilities.** Given prices $p_i^X$ and $p_i^Y$ for $i \in \{1, \ldots, n\}$, the customers anticipate the choice probabilities of others and the resulting network effects. It leads to a rational equilibrium, and the customers form their choice probabilities by the MNL model. More precisely, customer $i$'s choice probability of product $X$ and $Y$, denoted $x_i$ and $y_i$, satisfies[2]

$$x_i = \frac{\exp\left\{\alpha^X - \beta_i p_i^X + \sum_{j=1}^n g_{ij} x_j\right\}}{1 + \exp\left\{\alpha^X - \beta_i p_i^X + \sum_{j=1}^n g_{ij} x_j\right\} + \exp\left\{\alpha^Y - \beta_i p_i^Y + \sum_{j=1}^n g_{ij} y_j\right\}}, \tag{1}$$

$$y_i = \frac{\exp\left\{\alpha^Y - \beta_i p_i^Y + \sum_{j=1}^n g_{ij} y_j\right\}}{1 + \exp\left\{\alpha^X - \beta_i p_i^X + \sum_{j=1}^n g_{ij} x_j\right\} + \exp\left\{\alpha^Y - \beta_i p_i^Y + \sum_{j=1}^n g_{ij} y_j\right\}}. \tag{2}$$

The MNL model has been very popular and successful in modeling customer choices. It can be rationalized by a random utility model with extreme value distributions. More precisely, the utility of product $X$ ($Y$) to customer $i$ is the sum of a deterministic part $\alpha^X - \beta_i p_i^X + \sum_{j=1}^n g_{ij} x_j$ (or $\alpha^Y - \beta_i p_i^Y + \sum_{j=1}^n g_{ij} y_j$) and a random utility following a standard Gumbel distribution. The deterministic part features the quality of the product itself, the price, and the network effects. We refer the readers to Anderson et al. (1992) for more details.

We investigate personalized pricing here—that is, different prices are set for different customers. Given the unique positions of customers in a network, personalized pricing is more natural within the theoretical framework and generates more profits. This is why it is the focus of many papers in this area (Candogan et al. 2012, Bloch and Quérou 2013). Personalized pricing is also made increasingly feasible for digital services and products. For example, Dropbox used to provide free storage for customers making referrals, which can be translated to lower prices. These are the reasons behind our modeling choice. Nevertheless, some of our results apply to uniform pricing; see Section 4 and Section EC.3.4 in the online appendix.

**3.1.2. Network Externality or Network Effects.** From (1) and (2), we observe that customer $i$ is more likely to choose product $X$ than $Y$ if $\alpha^X - \beta_i p_i^X + \sum_{j=1}^n g_{ij} x_j$ is larger than $\alpha^Y - \beta_i p_i^Y + \sum_{j=1}^n g_{ij} y_j$. This can happen if product $X$ has a better quality ($\alpha^X > \alpha^Y$), a lower price ($p_i^X < p_i^Y$), or stronger network externalities ($\sum_{j=1}^n g_{ij} x_j > \sum_{j=1}^n g_{ij} y_j$). The quantity $g_{ij}$ captures the heterogeneous interactions between customers in the network. For example, in the choice of competing social networks, if $i$ and $j$ are close friends, then the choice of $j$ may have a stronger impact on the choice of $i$ than mere acquaintances, and, thus, $g_{ij}$ is large. In the case of collaboration tools such as Dropbox, the choice is mainly driven by the network effects, and we expect $g_{ij}$

to be large if $i$ and $j$ are collaborators. Moreover, we do not assume $g_{ii} = 0$. Notice that the choice probability of customer $i$ depends on the choice probabilities of other customers, not the realization of their discrete choices. This is because the choice decision is made simultaneously, and customers do not communicate or coordinate their decisions.

### 3.2. Nash Equilibrium

The firms' expected revenues are given by $\sum_{i=1}^n x_i p_i^X$ and $\sum_{i=1}^n y_i p_i^Y$, respectively, which are simply the (expected) sales multiplied by the prices. They are the payoff functions of both firms in the game. We use the choice probabilities $\{x_i\}_{i=1}^n$, or, equivalently, the market shares, as the firms' decision variables. That is, we consider Cournot competition. We refer to Section EC.3.1 in the online appendix for the justification for this modeling choice. The key step is the following transformation (see, e.g., Du et al. 2016):

$$p_i^X = \frac{1}{\beta_i}\left(\alpha^X + \sum_{j=1}^n g_{ij}x_j - \log\left(\frac{x_i}{1 - x_i - y_i}\right)\right). \tag{3}$$

The second term in the parentheses represents the total network externality customer $i$ receives; the third term is the logarithm of the relative choice probability of choosing product $X$ to that of the no-purchase option. In general, $p_i^X$ can be negative for given $\{x_i\}_{i=1}^n$ and $y_i$. This cannot happen in equilibrium, however, when customers are homogeneous (see Proposition EC.1 in the online appendix). For heterogeneous customers (Section 5), negative prices may arise and can be interpreted as coupons used to attract high-value customers.[3]

As Cournot-type competition, the *best response* of firm $X$ is therefore to choose a vector $\{x_i\}_{i=1}^n$ to maximize its revenue for given $\{y_i\}_{i=1}^n$:

$$\max_{\{x_i\}_{i=1}^n} \sum_{i=1}^n \frac{x_i}{\beta_i}\left(\alpha^X + \sum_{j=1}^n g_{ij}x_j - \log(x_i) + \log(1 - x_i - y_i)\right)$$
$$\text{subject to } 0 \le x_i \le 1 - y_i. \tag{4}$$

Having defined the game primitives, our next proposition establishes the existence of pure-strategy Nash equilibria. We transform the strategy of firm $X$ to obtain a supermodular game. The existence, thus, follows from Milgrom and Roberts (1990).

**Proposition 1.** *The duopoly game defined above has at least one pure-strategy Nash equilibrium.*

To characterize the equilibrium, we first argue that the constraints $0 \le x_i \le 1 - y_i$ must not be binding in any best response of firm $X$, unless $y_i = 1$. Note that $x_i = 1 - y_i$ generates $-\infty$ revenue and, thus, can never be optimal. On the other hand, $x_i = 0$ garners zero revenue from customer $i$, while setting $x_i = \epsilon < 1 - y_i$ for a sufficiently small $\epsilon$ always guarantees a positive revenue from customer $i$ and does not reduce the revenues from other customers for firm $X$. Therefore, any equilibrium must satisfy $0 < x_i < 1 - y_i$. It implies that the equilibria are fully characterized by the first-order conditions. That is, the best response $\{x_i^*\}_{i=1}^n$ satisfies

$$\alpha^X - 1 + \sum_{j=1}^n \left(g_{ij} + \frac{\beta_i g_{ji}}{\beta_j}\right)x_j^* - \log\frac{x_i^*}{1 - x_i^* - y_i} - \frac{x_i^*}{1 - x_i^* - y_i} = 0, \tag{5}$$

for all $i = 1, \ldots, n$. A similar first-order condition is satisfied by the best response of firm $Y$ given $\{x_i\}_{i=1}^n$:

$$\alpha^Y - 1 + \sum_{j=1}^n \left(g_{ij} + \frac{\beta_i g_{ji}}{\beta_j}\right)y_j^* - \log\frac{y_i^*}{1 - x_i - y_i^*} - \frac{y_i^*}{1 - x_i - y_i^*} = 0. \tag{6}$$

If $\{x_i^*\}_{i=1}^n$ and $\{y_i^*\}_{i=1}^n$ is a Nash equilibrium in the duopoly competition, then the two sets of first-order conditions ((5) and (6)) must be satisfied simultaneously. In the remainder of this paper, we will use this necessary condition to identify pure-strategy Nash equilibria in different cases.

## 4. Homogeneous Customers

In this section, we assume that

**Assumption 1** (Homogeneous Customers). $\beta_i \equiv \beta$, $\forall i = 1, \ldots, n$, *and* $g_{ij} \equiv \gamma/2n$, $\forall i, j = 1, \ldots, n$.

Assumption 1 indicates that customers are equally price-sensitive ($\beta_i \equiv \beta$), and the network effects are homogeneous ($g_{ij} \equiv \gamma/2n$). This is an assumption adopted by many previous papers (e.g., Katz and Shapiro 1985, Cabral 2011).[4] Because customers are homogeneous, we also restrict the set of strategies in the assumption below.

**Assumption 2.** *The strategies are symmetric for all customers*: $x_i \equiv x$ *and* $y_i \equiv y$.

In general, even though customers are homogeneous and identical to the firms, there may exist Nash equilibria in which the choice probabilities of a product differ across customers.[5] We restrict our attention to symmetric strategies because: (1) Our goal is to examine the asymmetric equilibria arising from the *interaction between the firms*, rather than those from customers. Therefore, focusing on symmetric strategies allows us to isolate the firms' competitive behaviors that lead to market dominance and market segmentation. (2) Some prior papers have examined the asymmetry arising from consumers (such as Brock and Durlauf 2001 and Du et al. 2016). We deviate from these papers because asymmetry arising from firms' strategic interactions is orthogonal to their analysis and less explored. (3) In practice, it is often not feasible to discriminate customers with identical features. (4) In Section 5, we relax Assumptions 1 and 2 and show that many insights carry over to the general case based on the analysis in this section.

### 4.1. Preliminaries

Given $y_i \equiv y$, the best response of firm $X$ solves the following univariate optimization problem:

$$x^*(y) \equiv \underset{0 \le x \le 1-y}{\arg\max} \left\{ x\left(\alpha^X + \frac{\gamma x}{2} - \log(x) + \log(1-x-y)\right)\right\}. \tag{7}$$

By Assumption 1 and the first-order condition (5), $x = x^*(y)$ satisfies $\log\frac{x}{1-x-y} + \frac{x}{1-x-y} = \alpha^X - 1 + \gamma x$. The first-order condition for $y^*(x)$ can be obtained by symmetry.

The difficulty in analyzing the Nash equilibrium lies in the fact that (7) may have multiple solutions because of the *nonconcavity* of the objective function. To obtain $x^*(y)$, one has to compare several potential local maxima. This is virtually infeasible because those local maxima do not have tractable expressions.

To deal with this difficulty, we use the following novel approach. In (7), if the best response of $X$ is given as $x = x^*(y)$, then $y$ has a unique solution that can be expressed in closed form. More precisely,

**Definition 1.** Define

$$\bar{y}(x) = 1 - x - \frac{x}{h^{-1}\left(\alpha^X - 1 + \gamma x\right)},$$

where $h^{-1}(\cdot)$ is the well-defined inverse of $h(t) \equiv t + \log(t)$.[6]

Clearly, a given $x$ is the optimal solution to (7) only if $y = \bar{y}(x)$. The function $\bar{y}(x)$ (and symmetrically $\bar{x}(y)$) can be interpreted as follows: If $x$ is the action of firm $X$, then $\bar{y}(x)$ is the *action of firm Y that makes x the best response*. We shall also elaborate on the mathematical connection between $\bar{y}(\cdot)$ and $x^*(\cdot)$. If $\bar{y}(\cdot)$ has a well-defined inverse, for example, when $\bar{y}(\cdot)$ is strictly deceasing, then the inverse $\bar{y}^{-1}(\cdot)$ coincides with the best response $x^*(\cdot)$. If $\bar{y}(\cdot)$ does not have an inverse, then there may be multiple solutions of $x$ to $\bar{y}(x) = y$, and one of them is the best response $x^*(y)$.

In our approach, we focus on $\bar{y}(\cdot)$ instead of $x^*(\cdot)$, thanks to the explicit expression of the former. The main technical challenge remains to be the nonmonotonicity of $\bar{y}(\cdot)$. As we shall see in Figure 3, such nonmonotonicity may lead to discontinuities in the best response, a surprising outcome in symmetric games.

### 4.2. Products with Identical Qualities

In this section, our analysis is based on the following assumption.

**Assumption 3** (Identical Qualities). $\alpha^X = \alpha^Y = \alpha$.

Assumption 3 suggests that the game is entirely symmetric, and in the sequel, we characterize Nash equilibria that may arise.

Under Assumption 3, symmetric equilibria $x^* = y^*$ satisfy the first-order condition

$$\log\frac{x^*}{1-2x^*} + \frac{x^*}{1-2x^*} = \alpha - 1 + \gamma x^*, \tag{8}$$

following (5).

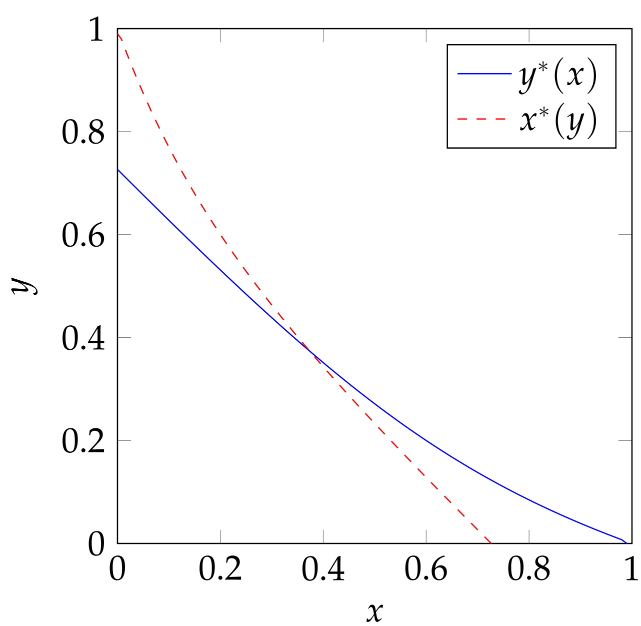
#### 4.2.1. Weak Network Effects.

When the network effects are not strong, our first result features a single symmetric Nash equilibrium. For a given $\alpha$, define $\gamma_1$ to be the unique solution to $\gamma_1 = h^{-1}(\alpha - 1 + \gamma_1) + 1$.

**Proposition 2.** *Suppose Assumptions 1–3 hold. If $\gamma < \gamma_1$, then there exists one and only one equilibrium $(x^*, x^*)$, where $x^*$ is the only solution to (8).*

The characteristics of symmetric equilibria for symmetric firms have been documented in, for example, Calzada and Valletti (2008), Chen et al. (2018), and Laffont et al. (1998), where they use the linear demand model or Hotelling's model to describe the network effects. To understand Proposition 2, we illustrate the best responses in Figure 1. We note that a sufficient condition for the existence of a single symmetric equilibrium is the following: The derivative of the best response, $(x^*)'(y)$, is between −1 and 0 in the domain. We can show that when $\gamma < \gamma_1$, the derivative of $\bar{y}(\cdot)$ is

**Figure 1.** (Color online) One Symmetric Nash Equilibrium Corresponding to Proposition 2




*Note.* In this example, we set $\alpha^X = \alpha^Y = 1$ and $\gamma = 5$.

less than −1. Because $\bar{y}(x)$ is strictly decreasing, it follows that its inverse is exactly $x^*(y)$. By the property of function inverses, the derivative of $x^*(\cdot)$ is between −1 and 0. Symmetrically, the same property can be obtained for $y^*(x)$, and there must be a single symmetric and stable Nash equilibrium.

When the network effects are not strong, both firms take up equal market shares in equilibrium, according to Proposition 2. This result is consistent with our observations of traditional industries: If two products are similar, such as Pepsi and Coca-Cola, then the market is more or less evenly split, and both firms coexist. This may no longer be the case when the network effects are strong.

**4.2.2. Strong Network Effects.** Our next result features three Nash equilibria when the network effects are strong enough.

**Proposition 3.** *Suppose Assumptions 1–3 hold. For any $\delta \in (0, 1/4)$, there is a constant $\Gamma(\alpha, \delta)$ such that when $\gamma > \Gamma$, there exist three Nash equilibria:*

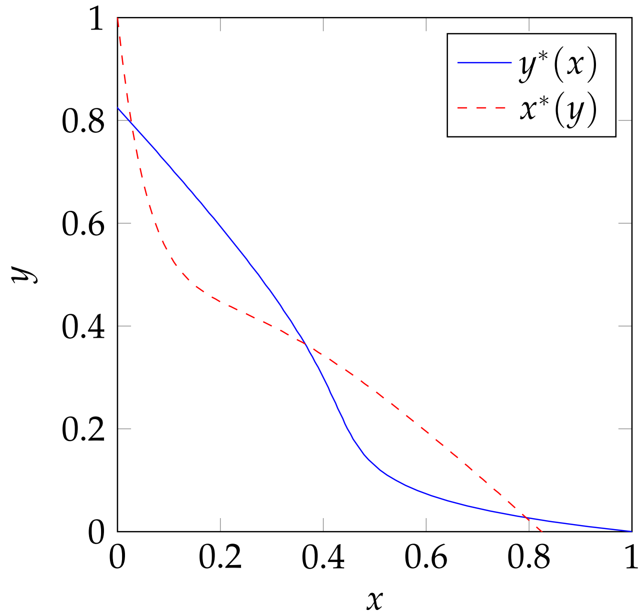
- *Two stable asymmetric Nash equilibria $(x_1^*, y_1^*)$ and $(x_3^*, y_3^*)$, in which $x_1^* = y_3^* \in (1-\delta, 1)$ and $y_1^* = x_3^* \in (0, \delta)$; and*
- *An unstable symmetric Nash equilibrium $(x_2^*, y_2^*)$.*

The rise of asymmetric equilibria is pertinent to discrete-choice modeling. When customers have linear utilities (Chen et al. 2018) or make choices by Hotelling's law (Laffont et al. 1998), only symmetric equilibria exist.

The best responses and the Nash equilibria of Proposition 3 are illustrated in Figure 2. The derivative of the best response $y^*(x)$ changes more drastically compared with Figure 1. In particular, there is an interval of $x$ (when $x$ is small), in which $(y^*)'(x) < -1$, whereas in Proposition 2, we always have $(y^*)'(x) > -1$. This phenomenon suggests that $y^*(x) + x$ may decrease in $x$. It, therefore, implies that when the network effects strengthen, the total market share may *decrease* as the dominated firm (firm $X$ when $x$ is small) tries to promote its product. Because of the dominated firm's aggressive behavior, the leading firm (firm $Y$) holds back its market share substantially in order to maintain a high profit margin. This crowding-out effect is more pronounced than the usual strategic substitution, which only indicates the direction rather than the magnitude of firm $Y$'s adjustment.

An implication of the above difference is that when the network effects are strong enough, there are two asymmetric equilibria that can be arbitrarily close to a *market dominance*. Here, market dominance refers to a Nash equilibrium $(x^*, y^*)$, in which one of the products takes almost all the market share—for example, $x^* < \delta$ and $y^* > 1 - \delta$ for some small $\delta$. Moreover, the two asymmetric Nash equilibria are

**Figure 2.** (Color online) One Symmetric Nash Equilibrium and Two Asymmetric Nash Equilibria Corresponding to Proposition 3




*Note.* In the example, we set $\alpha^X = \alpha^Y = -1$ and $\gamma = 10$.

more likely to emerge than the unstable symmetric equilibria in the following sense: If the two firms start from a tuple of actions $(x_0, y_0)$ and play the best response sequentially $(x_t, y_t) = (x^*(y_{t-1}), y^*(x_t))$ for $t = 1, 2, \ldots$, then the actions converge to one of the asymmetric Nash equilibria as $t \to \infty$. In other words, when the network effects are strong, one of the products will eventually dominate the market, rather than two products equally split the market.

The emergence of market dominance arises from strengthened competition. Specifically, recall from (1) and (2) that customers' choices are influenced by two factors: the appeal of the product itself (measured by $\alpha^X - \beta_i p_i^X$ and $\alpha^Y - \beta_i p_i^Y$) and the network externality (measured by $\sum_{j=1}^n g_{ij} x_j$ and $\sum_{j=1}^n g_{ij} y_j$). In addition, the choice model also embeds a random utility that captures product differentiation (i.e., the two products are differentiated in nature). When the network effects are not strong, customers' purchasing decisions are primarily driven by their intrinsic consumption values. Thus, given that the firms are ex ante symmetric, they naturally equally split the market.

When the network effects are very strong, however, the network externality terms start to take over. This, in fact, homogenizes the products, because now the firms primarily compete on attracting the critical mass of customers. In this vein, equal splitting becomes unstable, and the market structure is geared toward one firm dominating the other. Notably, this asymmetric equilibrium is also more efficient in terms of boosting customers' willingness to pay.

When customers "coordinate" on purchasing from the same firm, their decisions collectively enhance others' utilities all together. This probably explains why market dominance can emerge among seemingly identical products, such as Dropbox and OneDrive. In addition, the result also implies that when investors invest in startup firms in industries with strong network effects, the outcome tends to be binary and riskier.

The "asymmetry" in the equilibrium is regarding the strategy of the firms—that is, the market share. It does not directly imply that the dominating firm is earning more revenue, because it could be retaining the position by setting very low prices. Our next result demonstrates the power of network effects: The leading firm is able to charge a higher price while gaining a majority of the market at the same time.

**Corollary 1.** *In the asymmetric Nash equilibrium of Proposition 3, the firm dominating in market share charges higher prices.*

**4.2.3. Nonexistence of Symmetric Equilibrium.** Our third result features two asymmetric equilibria (and no symmetric equilibrium) when the quality is low and the network effects are neither too weak nor too strong.

**Proposition 4.** *Suppose Assumptions 1–3 hold. There exist constants $\alpha_0$, $\underline{\Gamma}(\alpha)$, and $\overline{\Gamma}(\alpha)$ such that if $\alpha < \alpha_0$ and $\gamma \in (\underline{\Gamma}(\alpha), \overline{\Gamma}(\alpha))$, then there are only two asymmetric equilibria.*

Somewhat paradoxically, Proposition 4 documents the possibility that even if the firms are ex ante symmetric, the resulting market equilibrium is *never* symmetric. This statement remains true, even if we allow for unstable equilibria. In this sense, our result demonstrates a strong rebuttal to the conventional, and perhaps naïve, intuition that symmetric equilibria shall exist for symmetric firms.

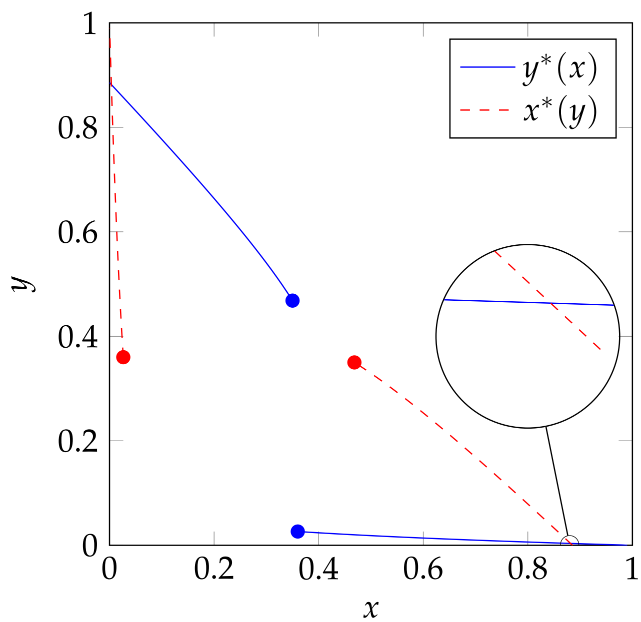
To help visualize the nonexistence of symmetric equilibrium, the best responses and the Nash equilibria are illustrated in Figure 3. In the online appendix, we provide the exact expressions of $\alpha$ and the interval. We verify that for such $\alpha$ and $\gamma$ in the interval, $\bar{y}(x)$ is *not monotone.* Because of this, the best response $x^*(y)$ involves selecting the global maximum from the multiple solutions (local maxima) to $\bar{y}(x) = y$, which is virtually intractable.

To circumvent this difficulty, we directly deal with $\bar{y}(x)$. First, we show that $\bar{y}(x) = x$ has only one solution $x_0$, which implies that if a symmetric equilibrium exists, it must be $(x_0, x_0)$. Second, we show that $\bar{y}(x) = x_0$ has three solutions, including $x = x_0$. Then, the best response of firm $X$ when firm $Y$ plays $x_0$ is selected from the three solutions. Furthermore, we prove that $x_0$ is not a global maximum. In other words, $x_0$ cannot be the best response among the three solutions. This observation *rules out the existence of any symmetric equilibria*. Third, we analyze the behavior of the function $\bar{y}(x)$ when $x$ is approaching 0 and 1 and, hence, establish the existence of two asymmetric equilibria.

Having articulated the mathematical reasoning for the nonexistence of symmetric equilibrium, we now turn to the economic rationale for Proposition 4. To decipher the counterintuitive phenomenon, it is crucial to understand why the firms' strategies change dramatically in response to the competitor's aggressiveness (the *discontinuity* of the best-response function). It can only happen when the network effects are neither too strong nor too weak ($\gamma$ is in a bounded interval) and the quality is low:

- When the opponent acts passively—that is, takes a small market share—then the best response is to take a large market share to capitalize the (not too weak) network effects—that is, $x^*(y) \approx 1 - y$ when $y$ is small.
- When the opponent acts aggressively—that is, takes a substantial market share—then the best response is to virtually quit the market—that is, $x^*(y) \approx 0$ when $y$ is above a threshold (in Figure 3, this happens when $y \geq 0.36$). This is because the remaining market share does not create strong enough network effects to overcome the low quality. Therefore, customers are unwilling to pay much for the product, and virtually exiting the market turns out to be a better option for the firm.

**Figure 3.** (Color online) Two Asymmetric Nash Equilibria Corresponding to Proposition 4




*Note.* In this example, we set $\alpha^X = \alpha^Y = -2.5$ and $\gamma = 15$.

As a result, only asymmetric equilibria arise due to the discontinuity. Our analysis, therefore, indicates a concrete operating regime in which such "anomaly" occurs.

**Remark 1.** It is worth pointing out that our methodology can be generalized to oligopoly competitions. When there are $n$ symmetric firms competing with each other in our model, there does not exist any symmetric equilibrium when the quality is low and the network effects are neither too weak nor too strong.

**Remark 2.** Propositions 2–4 do not exhaust all the possible values of parameters—for example, when $\alpha$ and $\gamma$ are moderate. In theory, more complex equilibrium structures may emerge. However, based on the numerical study in Section 4.4, the three types of equilibria demonstrated from Propositions 2–4 cover all the cases.

### 4.3. Differentiated Products

In this section, we consider differentiated products—that is, $\alpha^X \neq \alpha^Y$. One firm's product is inherently superior to the competitor's. Our first result extends Proposition 3: There are two asymmetric Nash equilibria that are close to a market dominance when the network effects are strong enough, regardless of their quality difference.

**Proposition 5.** *Suppose Assumptions 1 and 2 hold. For given $\alpha^X, \alpha^Y$, and $\delta \in (0, 1/4)$, there is a constant $\Gamma(\alpha^X, \alpha^Y, \delta)$ such that for $\gamma > \Gamma$, there are three Nash equilibria:*

- *Two stable Nash equilibria $(x_1^*, y_1^*)$ and $(x_3^*, y_3^*)$, in which $x_1^*, y_3^* \in (1-\delta, 1)$ and $y_1^*, x_3^* \in (0, \delta)$, and*
- *An unstable Nash equilibrium $(x_2^*, y_2^*)$ such that $x_1^* > x_2^* > x_3^*$ and $y_1^* < y_2^* < y_3^*$.*

It is worthwhile to elaborate on the procedure to prove Proposition 5. Because the firms (products) are heterogeneous, we cannot apply the same approach as in the proof of Proposition 4 (which requires symmetry). In the first step, we show that when $\gamma$ is sufficiently large, both $\bar{y}(x)$ and $\bar{x}(y)$ are decreasing and have inverse functions. Moreover, there are, at most, three Nash equilibria. Next, we compare the functions $\bar{y}(x)$ and $y^*(x)$ in a neighborhood of $x = 0$. We conclude that $\bar{y}(0) = 1 > y^*(0)$, but $\bar{y}(\cdot)$ is decreasing slightly faster than $y^*(\cdot)$, which leads to an intersection (a Nash equilibrium) in that neighborhood. By the same token, a Nash equilibrium exists close to $y = 0$. Finally, a Nash equilibrium always exists between the two equilibria mentioned because the best response is continuous in this case.

Proposition 5 states that, even though one product, say, product $X$, is far inferior to its competitor, it can still retain market dominance due to the strong network effects. In the battle between the QWERTY keyboard and others, QWERTY is widely acknowledged to be inferior to the Dvorak typewriter. Its victory could be partially explained by our theory, as the strong network effects lead to the dominance of the QWERTY keyboard over the Dvorak typewriter and other alternatives.

Although Proposition 5 hints at the possibility that an inferior product may dominate the market, the next result shows that the quality superior firm can secure its dominating position if the quality difference is sufficiently large.

**Proposition 6.** *Suppose Assumptions 1 and 2 hold. For given $\alpha^Y$ and $\gamma$, and $\delta \in (0, 1/4)$, there is a constant $\alpha_0(\alpha^Y, \gamma, \delta)$ such that for $\alpha^X > \alpha_0$, there exists one and only one Nash equilibrium $(x^*, y^*)$, where $x^* \in (1-\delta, 1)$.*

Proposition 6 does some justice to the firm that invests substantially in quality: It suggests that if the qualities are not "in the same league," then the high-quality firm will unambiguously become the market leader. This may explain why Gmail was able to overtake Hotmail and Yahoo Mail as the largest global email service, despite the huge customer base the others have.

### 4.4. Parameter Ranges and Equilibrium Types

To give a concrete view of the connection between parameters and equilibrium types, we conduct a numerical study assuming identical qualities (Assumption 3) to complement our theoretical studies. More precisely, we sample $\alpha$ and $\gamma$ from the range $[-6, 6] \times [0, \infty)$. For a particular combination of $\alpha$ and $\gamma$, we investigate the resulting equilibrium type after numerically solving the best-response function. We do not sample $|\alpha| > 6$ because $\exp(\pm 6) \approx 10^{\pm 2}$ already has a different order of magnitude from the utility of the outside option 1 in the Choice Probability (1). Thus, $\alpha \in [-6, 6]$ seems to cover the practical range of parameters.

For all the combinations, the numerical results of the equilibrium structure can always be categorized into three types: type I (one symmetric Nash equilibrium; e.g., Figure 1), type II (two Nash equilibria; e.g., Figure 3), and type III (three Nash equilibria; e.g., Figure 2). The ranges of the parameters corresponding to the equilibrium types are listed in Table 1. For $\alpha \geq -1$, a type-II equilibrium structure does not emerge. The types are not monotone in the parameters: For example, a type-III equilibrium structure emerges for two disjoint intervals of $\gamma$, $[8.5, 9.0)$ and $[13.7, +\infty)$, when $\alpha = -2$.

## 5. Heterogeneous Customers

In this section, we investigate the general setting and analyze a market of customers who might be heterogeneous in their price sensitivities and network effects. We first show that when the network effects are weak, then there exists a unique Nash equilibrium, extending Proposition 2.

**Table 1.** Ranges of Parameters and the Equilibrium Types

| $\alpha$ | Type I | Type II | Type III |
|---|---|---|---|
| 6 | $\gamma \in [0, 200.3)$ | — | $\gamma \in [200.3, +\infty)$ |
| 4 | $\gamma \in [0, 112.2)$ | — | $\gamma \in [112.2, +\infty)$ |
| 3 | $\gamma \in [0, 43.2)$ | — | $\gamma \in [43.2, +\infty)$ |
| 2 | $\gamma \in [0, 17.9)$ | — | $\gamma \in [17.9, +\infty)$ |
| 1 | $\gamma \in [0, 8.8)$ | — | $\gamma \in [8.8, +\infty)$ |
| 0 | $\gamma \in [0, 6.0)$ | — | $\gamma \in [6.0, +\infty)$ |
| −1 | $\gamma \in [0, 6.5)$ | — | $\gamma \in [6.5, +\infty)$ |
| −2 | $\gamma \in [0, 8.5)$ | $\gamma \in [9.0, 13.7)$ | $\gamma \in [8.5, 9.0) \cup [13.7, +\infty)$ |
| −3 | $\gamma \in [0, 11.1)$ | $\gamma \in [11.2, 19.4)$ | $\gamma \in [11.1, 11.2) \cup [19.4, +\infty)$ |
| −4 | $\gamma \in [0, 13.5)$ | $\gamma \in [13.6, 24.5)$ | $\gamma \in [13.5, 13.6) \cup [24.5, +\infty)$ |
| −6 | $\gamma \in [0, 18.2)$ | $\gamma \in [18.2, 34.1)$ | $\gamma \in [34.1, +\infty)$ |

*Note.* We use bisection search to find the cutoff values of $\gamma$, which are rounded to one digit.

**Proposition 7.** *Given $\alpha^X$, $\alpha^Y$ and $\beta_i$ for all $i$, there exists a constant $G(\alpha^X, \alpha^Y, \beta_1, \ldots, \beta_n)$ such that when the network effects $g_{ij} < G$ for all $i$ and $j$, there exists a unique pure-strategy Nash equilibrium.*

In the proof, we are able to obtain an explicit condition. Let $c = (1 + h^{-1}(\alpha^X) + h^{-1}(\alpha^Y))^{-1}$. If

$$\sum_{i=1}^{n} \left( \frac{g_{ij}}{\beta_i} + \frac{g_{ji}}{\beta_j} \right) \le \min\left\{ 1, \frac{c^2}{\beta_j(1-c)} \right\} \quad \text{for all } j,$$

then the Nash equilibrium is unique.

## 5.1. Symmetric Nash Equilibrium

By Proposition 7, if the network effects are weak and qualities of the two products are identical ($\alpha^X = \alpha^Y = \alpha$; i.e., Assumption 3), then the unique Nash equilibrium must be symmetric—that is, $x_i^* = y_i^*$ for all $i$ in the equilibrium. Naturally, we are interested in the following question: To which customer are the firms eager to sell their products? The following proposition gives a sufficient condition.

**Proposition 8.** *Suppose Assumption 3 holds. If $\beta_j g_{ij} + \beta_i g_{ji}$ is decreasing in $i$ for all $j = 1, 2, \ldots, n$, then in any symmetric Nash equilibrium, $x_i^* = y_i^*$ is decreasing in $i$.*

When customers can be ranked in the same order by the quantity $\beta_j g_{ij} + \beta_i g_{ji}$ for all $j$, the firms are vying for the customers who rank high—that is, their equilibrium choice probabilities for both products are large. To better interpret the quantity, consider a homogeneous network.

**Corollary 2.** *Suppose Assumption 3 holds. If $g_{ij} > 0$ are equal for all $i$ and $j$ and $\beta_1 \ge \beta_2 \ge \ldots \ge \beta_n$, then in any symmetric Nash equilibrium, $x_i^* = y_i^*$ is decreasing in $i$.*

The corollary states that customers who are more price-sensitive turn out to be more likely to purchase either product in equilibrium. To understand this seemingly counterintuitive result, note that an extremely price-sensitive customer is unwilling to pay even a small amount for the products. Therefore, neither firm can make much revenue from him. The only purpose for the firms to attract such a customer is to benefit from his network effects on other customers. The benefit from network effects is always increasing in the choice probability. Thus, the firms are willing to set very low prices and forgo the revenue. For customers who are less price-sensitive, the firms attempt to earn revenue and utilize the network effects at the same time. The former objective is not always increasing in the choice probability. Facing such trade-off, the choice probabilities are relatively low for such customers (so that higher prices can be charged) in equilibrium. This is common in the online game industry. For example, Hearthstone, a popular collectible card game, is F2P (free to play). For price-sensitive players, they can spend more time to grind and wait for the needed cards to drop, while VIP players can skip the grinding and directly buy their dream decks.

If the customers are equally price-sensitive, then Proposition 8 implies:

**Corollary 3.** *Suppose Assumption 3 holds. If $\beta_i \equiv \beta$ for all $i$ and $g_{ij} + g_{ji}$ is decreasing in $i$ for all $j = 1, 2, \ldots, n$, then in any symmetric Nash equilibrium, $x_i^* = y_i^*$ is decreasing in $i$.*

If the social influences of the customers can be ranked unanimously, then the firms tend to increase their market share for customers with higher network effects in equilibrium. This is consistent with the intuition: The market share for those customers is of higher value because it significantly increases the choice probabilities of all customers and, thus, the total revenue. As a special case, for a network with a single "star"—that is, $g_{i1} \equiv \gamma > 0$ and $g_{ij} = 0$ for $j \ge 2$—the equilibrium market share for the star is higher than the rest of the market. This is usually implemented through the referral program, in which the influential customers earn bonuses and are happy to consume more at a lower or even negative price. The firm makes up for the revenue loss by monetizing their influences in the network.

## 5.2. Two Communities

In this section, we consider a network with two communities. Suppose $n = n_1 + n_2$, where $n_1$ and $n_2$ are the sizes of the two communities. The network effects are specified as follows:

**Assumption 4** (Two Communities)**.** *The network matrix $\{g_{ij}\}$ satisfies:*

$$g_{ij} = \begin{cases} \gamma_1 & i, j \in \{1, \ldots, n_1\} \\ \gamma_2 & i, j \in \{n_1 + 1, \ldots, n\} \\ \gamma_{12} & i \in \{1, \ldots, n_1\}, j \in \{n_1 + 1, \ldots, n\} \\ \gamma_{21} & j \in \{1, \ldots, n_1\}, i \in \{n_1 + 1, \ldots, n\}. \end{cases}$$

Under Assumption 4, the connectivity inside each community is homogeneous, which is different from the externality between communities. This network structure encompasses many special networks that are investigated in the literature, such as star graphs and complete bipartite graphs. In practice, it can capture a segmented market structure whose customers in the two groups differ significantly in their locations, ages, or other characteristics. We assume $\beta_i \equiv \beta$ for all customers and focus on firms' strategies that induce the same market share *inside a community*.

#### 5.2.1. Market Segmentation.

The next result is concerned with market segmentation under network effects: Product $X$ is dominating in one community, whereas product $Y$ is popular in the other community.

**Proposition 9.** *Suppose Assumption 4 holds and $\beta_i \equiv \beta, \forall i$.*

- *For any $\delta \in (0, 1/4)$, there is a constant $\Gamma$ that may depend on $\alpha^X$, $\alpha^Y$, $\beta$, $n_1$, $n_2$, $\gamma_{12}$, $\gamma_{21}$, and $\delta$, such that if $\gamma_1 > \Gamma$ and $\gamma_2 > \Gamma$, then market segmentation arises in the Nash equilibrium—that is, there exists a Nash equilibrium $(x_1^*, y_1^*, x_2^*, y_2^*)$ satisfying $x_1^* > 1 - \delta$ and $y_2^* > 1 - \delta$.*
- *Suppose the two communities are identical—that is, $n_1 = n_2$, $\gamma_1 = \gamma_2$. If $\gamma_{12} + \gamma_{21} \geq \gamma_1 + \gamma_2$, then any Nash equilibrium $(x_1^*, y_1^*, x_2^*, y_2^*)$ satisfies $x_1^* = x_2^*$ and $y_1^* = y_2^*$.*

This proposition gives a potential explanation for market segmentation. It can arise if the network effects inside communities are relatively strong in comparison with those between communities. This is parallel to the equilibrium behavior in Bimpikis et al. (2016), where in the context of duopoly-targeted advertising, each of the two firms chooses to target a "hub" in one of the two clusters in the graph. This may explain why different messaging apps dominated different regions and developed their own ecosystems—that is, WeChat in China, Facebook's Messenger in North America, and WhatsApp in other regions.

On the other hand, when the connections between communities are stronger, we find that the aforementioned segmentation is no longer feasible. Under the symmetric-community assumption, each firm will obtain the same market share in the two communities. Notably, market dominance may still exist, in the sense that one firm dominates the other in both communities.

## 6. Dynamic Game

The game studied in the previous sections is static. It can be viewed as the market outcome when all customers have perfect foresight: They can *predict* the consumption of connected customers and are willing to *coordinate* with each other to realize the Choice Probabilities (1) and (2). In reality, network effects usually build up over time: Customers cannot foresee the potential network effects in the distant future. Rather, they make consumption based on the *current* network effects. This assumption is made in other dynamic competitions (Mitchell and Skrzypacz 2006, Cabral 2011). It also better explains the first-mover advantage, as early entrants are able to build stronger current network effects and attach customers, which potentially blocks later entrants that may have better quality. In this section, we extend the game to such a dynamic setting.

### 6.1. Open-Loop Nash Equilibrium

Next, we introduce the dynamic game and the associated equilibrium concept. Suppose $x_t$ and $y_t$ are the market shares of firm $X$ and $Y$ in period $t$. The current network effects are, thus, $\sum_{j=1}^n g_{ij} x_{j,t}$ and $\sum_{j=1}^n g_{ij} y_{j,t}$ for customer $i$. In period $t+1$, the customers observe and enjoy the network effects generated in period $t$, and their market shares are given by the following choice model:[7]

$$x_{i,t+1} = \frac{\exp\left\{\alpha^X - \beta_i p_{i,t+1}^X + \sum_{j=1}^n g_{ij} x_{j,t}\right\}}{1 + \exp\left\{\alpha^X - \beta_i p_{i,t+1}^X + \sum_{j=1}^n g_{ij} x_{j,t}\right\} + \exp\left\{\alpha^Y - \beta_i p_{i,t+1}^Y + \sum_{j=1}^n g_{ij} y_{j,t}\right\}}, \tag{9}$$

$$y_{i,t+1} = \frac{\exp\left\{\alpha^Y - \beta_i p_{i,t+1}^Y + \sum_{j=1}^n g_{ij} y_{j,t}\right\}}{1 + \exp\left\{\alpha^X - \beta_i p_{i,t+1}^X + \sum_{j=1}^n g_{ij} x_{j,t}\right\} + \exp\left\{\alpha^Y - \beta_i p_{i,t+1}^Y + \sum_{j=1}^n g_{ij} y_{j,t}\right\}}, \tag{10}$$

where the network effects are based on the market share in period $t$. Such dynamic setting with myopic expectation of other customers' consumption was originally proposed in the seminal paper of Brock and Durlauf (2001) to understand the limiting behavior of a dynamic game in the network context. When customers have perfect foresight and can anticipate the network effects in the next period (replacing $x_{j,t}$ and $y_{j,t}$ by $x_{j,t+1}$ and $y_{j,t+1}$), it is equivalent to the static game (1) and (2).

The dynamic game proposed in this section is more suitable to describe the market evolution of nondurable services/products with network effects, such as Netflix subscriptions. The customers repeatedly make purchases in each period based on the network effects experienced in the last period. The model does not apply to durable network goods that customers only purchase once, such as video game consoles. The dynamic game of competing durable goods requires new formulations that depart from the static game and is, thus, out of the scope of this paper.

In the static MNL model, we implicitly assume that customers generate random utilities for both products, as well as the outside option. In a dynamic model, however, it is not reasonable to assume that customers independently redraw their random utilities over time for the same product. Although this setup has appeared in the literature of dynamic mechanism design and structural models for dynamic discrete choices (Aguirregabiria and Mira 2010, Pavan et al. 2014), we acknowledge that this is a weakness of the model when extended to the dynamic setting, and a dynamic model with microfoundations remains our priority in future research.

Again, we consider Cournot competition in the dynamic setting: Given initial market share $x_0$ and $y_0$, firm $X$ is deciding $x_t$ for $t = 1, 2, \ldots$ to maximize

$$\begin{aligned}
&\max_{\{x_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} r^t \sum_{i=1}^{n} x_{i,t} p_{i,t}^X \\
&\text{s.t. } p_{i,t}^X = \frac{1}{\beta_i}\left(\alpha^X + \sum_{j=1}^{n} g_{ij} x_{j,t-1} - \log\left(\frac{x_{i,t}}{1 - x_{i,t} - y_{i,t}}\right)\right) \\
&\quad 0 \le x_t \le 1 - y_t, \quad t = 1, 2, \ldots,
\end{aligned}$$

where $r \in (0, 1)$ is the discount factor, and $p_{i,t}$ is solved from (9) and (10). The payoff function of firm $Y$ can be defined symmetrically.

We investigate the equilibrium-concept *open-loop Nash equilibrium* (OLNE) (see, e.g., Basar and Olsder 1999) arising from the dynamic game. Given the initial market share at $t = 0$, $\boldsymbol{x_0}$, and $y_0$, firm $X$ ($Y$) is deciding $\{x_t\}_{t=1}^{\infty}$ ($\{y_t\}_{t=1}^{\infty}$) to maximize the total discounted revenue $\sum_{t=1}^{\infty} r^t \sum_{i=1}^{n} x_{i,t} p_{i,t}^X$ ($\sum_{t=1}^{\infty} r^t \sum_{i=1}^{n} y_{i,t} p_{i,t}^Y$) subject to the constraint $x_t + y_t \le 1$. This is in contrast to the *feedback Nash equilibrium* (FNE), in which the firms are deciding $x_{t+1}$ ($y_{t+1}$) dynamically based on the market condition $(x_t, y_t)$ at $t$. In general, the two concepts do not lead to the same equilibrium strategy. In this paper, we adopt OLNE instead of FNE because of two reasons: (1) We focus on the industries where the infrastructure and capacity have to be planned ahead; and (2) technically, the Bellman equations arising from FNE are intractable because of the lack of structure in the objective function.

### 6.2. Existence

We first establish the existence of such an OLNE. Similar to Proposition 1, we first transform the game to a supermodular game and apply the lattice theory. Note that the transformation is crucial: Firms $X$ and $Y$ are clearly selling substitutable products, and the dynamic game is *not* supermodular/complementary. After the transformation, we replace firm $X$ by another player $X'$, which turns out to be complementary to firm $Y$. This allows us to use the theory of supermodular games and establish the existence. Because there is a one-to-one correspondence between the Nash equilibria of the transformed game and the original game, we can establish the existence of the OLNE of the original game as well.

**Proposition 10.** *There exists a pure-strategy OLNE.*

There are other methods commonly used to establish existence, including fixed-point theorems (Brouwer's or Kakutani's). However, as we have seen, the best-response functions exhibit discontinuity and nonconcavity, even in the static case. Thus, they cannot be applied to the dynamic game.

### 6.3. Necessary Conditions

Suppose $\{x_t\}_{t=1}^{\infty}$ and $\{y_t\}_{t=1}^{\infty}$ form an OLNE. Given $\{y_t\}_{t=1}^{\infty}$, $\{x_t\}_{t=1}^{\infty}$ maximizes the payoff function of firm X—that is, $\sum_{t=1}^{\infty} r^t \sum_{i=1}^{n} x_{i,t} p_{i,t}^X$. Therefore, for a given $t$, $x_t$ must maximize the terms in the payoff function that involve $x_t$. More precisely,

$$\begin{aligned}
x_t &= \arg\max_{0 \le x \le 1 - \boldsymbol{y_t}} \sum_{s=1}^{\infty} r^s \sum_{i=1}^{n} x_{i,s} p_{i,s}^X \\
&= \arg\max_{0 \le x \le 1 - \boldsymbol{y_t}} \sum_{i=1}^{n} \frac{x_i}{\beta_i}\Bigg(\alpha^X + \sum_{j=1}^{n}\left(g_{ij} x_{j,t-1} + \frac{r g_{ji} \beta_i}{\beta_j} x_{j,t+1}\right) \\
&\qquad - \log\left(\frac{x_i}{1 - x_i - y_{i,t}}\right)\Bigg).
\end{aligned}$$

Similarly, $y_t$ satisfies

$$\begin{aligned}
y_t = \arg\max_{0 \le y \le 1 - x_t} \sum_{i=1}^{n} \frac{y_i}{\beta_i}\Bigg(\alpha^Y + \sum_{j=1}^{n}\left(g_{ij} y_{j,t-1} + \frac{r g_{ji} \beta_i}{\beta_j} y_{j,t+1}\right) \\
- \log\left(\frac{y_i}{1 - x_{i,t} - y_i}\right)\Bigg).
\end{aligned}$$

The necessary conditions are satisfied by any OLNE.

From the necessary condition, it is clear that $x_t$ and $y_t$ form a Nash equilibrium of the following static Cournot game: Firm $X$ ($Y$) is choosing the market share $\boldsymbol{x} = \boldsymbol{x_t}$ ($\boldsymbol{y} = \boldsymbol{y_t}$) to maximize $\sum_{i=1}^{n} x_i p_i^X$ ($\sum_{i=1}^{n} y_i p_i^Y$), where the relationship between $x_i$ and $p_i^X$ ($y_i$ and $p_i^Y$) is given by the vanilla MNL model:

$$\begin{aligned}
x_i &= \frac{\exp\{\tilde{\alpha}^X - \beta_i p_i^X\}}{1 + \exp\{\tilde{\alpha}^X - \beta_i p_i^X\} + \exp\{\tilde{\alpha}^Y - \beta_i p_i^Y\}}, \\
y_i &= \frac{\exp\{\tilde{\alpha}^Y - \beta_i p_i^Y\}}{1 + \exp\{\tilde{\alpha}^X - \beta_i p_i^X\} + \exp\{\tilde{\alpha}^Y - \beta_i p_i^Y\}},
\end{aligned}$$

where $\tilde{\alpha}^X \triangleq \alpha^X + \sum_{j=1}^{n}(g_{ij} x_{j,t-1} + \frac{r g_{ji} \beta_i}{\beta_j} x_{j,t+1})$ and $\tilde{\alpha}^Y \triangleq \alpha^Y + \sum_{j=1}^{n}(g_{ij} y_{j,t-1} + \frac{r g_{ji} \beta_i}{\beta_j} y_{j,t+1})$. In particular, if we treat $x_{t-1}$, $x_{t+1}$, $y_{t-1}$, and $y_{t+1}$ as exogenous variables, then $x_t$ and $y_t$ are the Nash equilibrium of a classic MNL Cournot game, in which the qualities are enhanced by the network effects in the neighboring periods.

### 6.4. Connection Between the Dynamic and Static Games

Dynamic games are generally intractable, especially given the complexity of the static game. A crucial question in this section is, thus, whether the equilibrium structures and insights derived in the static case continue to hold in the dynamic game. We are able to provide a positive answer to this question:

**Proposition 11.** *If the dynamic game converges, then the limit $\boldsymbol{x}_\infty$ and $\boldsymbol{y}_\infty$ satisfy the Nash equilibrium first-order conditions (5) and (6) of the following static game (denoted by $\tilde{\cdot}$): $\tilde{\alpha}^X = \alpha^X$, $\tilde{\alpha}^Y = \alpha^Y$, and $\sum_{j=1}^n(\tilde{g}_{ij} + \frac{\tilde{\beta}_i\tilde{g}_{ji}}{\tilde{\beta}_j}) = \sum_{j=1}^n(g_{ij} + \frac{r\beta_i g_{ji}}{\beta_j})$ for all $i$.*

Proposition 11 connects the static and the dynamic games. When both firms are myopic ($r \to 0$), the dynamic game is essentially equivalent to the static game as the evolving market reaches a steady state. For $r > 0$, the relationship $\sum_{j=1}^n(\tilde{g}_{ij} + \frac{\tilde{\beta}_i\tilde{g}_{ji}}{\tilde{\beta}_j}) = \sum_{j=1}^n(g_{ij} + \frac{r\beta_i g_{ji}}{\beta_j})$ implies that the static game discounts the network effects by $r$, and the qualitative characteristics of the equilibrium still hold. For example, in the homogeneous case (Assumptions 1–3) $g_{ij} = \gamma/2n$ and $\tilde{g}_{ij} = \tilde{\gamma}/2n = (1+r)\gamma/4n$, and Propositions 2–4 can be used to predict the equilibrium outcome of the dynamic game. This connection makes the theoretical results derived for the static game applicable to the dynamic game.

**Remark 3.** The proposition has two restrictions. First, we have to assume that the OLNE has a steady state. This is not a merely technical simplification. In general, because the dynamic game has an infinite number of decision variables and the payoffs are highly nonconcave, OLNEs may well be cyclic, in which case we cannot draw useful insights. It also a common assumption adopted in the literature (Mitchell and Skrzypacz 2006). Second, the connection between the dynamic and static games is based on the first-order conditions, instead of the Nash equilibria. Again, the complexity arises from the nonconcavity of the payoff functions, as the first-order conditions do not necessarily lead to Nash equilibria.

### 6.5. First-Mover Advantage

One appealing feature of dynamic games is the capability to capture the phenomena emerging from the market evolution, which cannot be explained by the static game. For example, it is widely believed that an earlier entrant to a market with strong network effects is able to enjoy the first-mover advantage and dominate the market eventually without having superior quality. To simplify the analysis, we consider a two-period model in this section: The choice model is given by (9) and (10). The game lasts for two periods, and the payoff function of firm $X$ is given by

$$\max_{x_1,x_2} \sum_{t=1}^{2} r^t \sum_{i=1}^{n} x_{i,t}p_{i,t}^X$$
$$\text{s.t. } p_{i,t}^X = \frac{1}{\beta_i}\left(\alpha^X + \sum_{j=1}^{n} g_{ij}x_{j,t-1} - \log\left(\frac{x_{i,t}}{1-x_{i,t}-y_{i,t}}\right)\right)$$
$$0 \le x_t \le 1 - y_t, \quad t = 1,2.$$

The benefit of considering a two-period model is to avoid the complexity and potential ill behavior (see Remark 3) of a full dynamic game, while retaining the dynamic feature.[8]

If firm $X$ has the first-mover advantage, then $x_0 > y_0$. That is, at the beginning of the game (possibly the time of entry of firm $Y$), firm $X$ gains more market share than firm $Y$. Then, a natural question is, under which conditions the advantage is retained—that is, $x_2 > y_2$ for any OLNE $(x_1, x_2)$ and $(y_1, y_2)$? For example, if firm $X$ has inferior quality and the network effects are weak, then firm $Y$ may end up being the market leader. The next proposition provides such a sufficient condition.

**Proposition 12.** *For any $\epsilon, \delta > 0$, there are constants $G(\epsilon, \delta, \alpha^X, \alpha^Y, \{\beta_i\}_{i=1}^n)$ and $R(\epsilon, \delta, \alpha^X, \alpha^Y, \{\beta_i\}_{i=1}^n, \{g_{ij}\}_{i,j=1}^n)$ such that if $g_{ij} > G$ for all $i, j$ and $r < R$, then $x_{i,0} - y_{i,0} > \delta$ for all $i$ implies $x_{i,2} - y_{i,2} > (1-\epsilon)\delta$ for all $i$ in any OLNE. Moreover, firm $X$ makes more revenues than firm $Y$.*

Proposition 12 states that firm $X$ is able to retain the market-leader position over periods, regardless of the quality differences, if (1) the network effects are strong, and (2) the discount factor is small. The first point is straightforward, as strong network effects are necessary to persuade customers to keep using product $X$, even though its quality could be inferior. To see the second point, consider firm $Y$'s strategy to break the first-mover advantage of the opponent: It can subsidize the customers heavily to counter the network effects of firm $X$. In the long run, the subsidy may secure the market-dominance position for firm $Y$ and makes the investment worthwhile. Therefore, first-mover advantage remains valid when future revenues are greatly discounted and subsidizing customers is costly.

## 7. Conclusion

In this paper, we consider two firms that sell their substitutable products to a market of network-connected customers, who make their purchasing decisions through the MNL model. We demonstrate the different market structure when the strength of network effects varies. The results can be used to predict the competitive outcome under various market conditions. They can also consistently explain real-world market phenomena.

Our analysis hopefully opens up potential research directions related to firms' competitive strategies when confronted with network-connected customers. In particular, we believe that (1) investigating the strategic interaction between firms and customers in a dynamic setting can generate valuable insights and explain market phenomena; and (2) analyzing complicated network structures may reveal the link between competitive outcomes and social interactions.

## Endnotes

[1] The QWERTY keyboard is widely regarded to be inferior to the Dvorak typewriter (David 1985). However, when a majority of users have already gotten used to typing using the QWERTY keyboard, they can easily obtain support and help from other QWERTY keyboard users. Ultimately, this leads to the dominance of the QWERTY keyboard over the Dvorak typewriter and other alternatives. Similarly, in the video-recordings industry, Betamax was considered to be superior to VHS at equal network sizes (Gabel (1991)), but VHS was more widely adopted, and, eventually the VCR market tipped in favor of VHS (Farrell and Klemperer 2007). See also Cusumano et al. (1992) for a detailed review of VHS–Beta battle.

[2] We can also interpret $x_i$ and $y_i$ as customer $i$'s fraction of consumptions in $X$ and $Y$.

[3] For the applications we have in mind, such as Dropbox and OneDrive, customers typically cannot purchase/subscribe to the same product twice. Therefore, negative prices do not induce multiple purchases from the same customer.

[4] We have assumed homogeneous network effects for simplicity of notation. The subsequent analysis extends naturally to a network with symmetric, but heterogeneous customers—for example, when the customers form a "cycle" network—that is, $g_{12} = g_{23} = \ldots = g_{n-1,n} = g_{n1} > 0$, while all other $g_{ij} = 0$.

[5] Du et al. (2016) consider a monopoly pricing problem with network effects. Under certain circumstances, the firm is optimal to promote a single product, although the set of products is homogeneous. Such asymmetry may also arise in our problem.

[6] Some useful properties of $h(\cdot)$ and $h^{-1}(\cdot)$ can be found in Lemma EC.1 in the online appendix. In fact, $h^{-1}$ is closely related to the Lambert $W$ function as $h^{-1}(x) = W(e^x)$.

[7] In the dynamic game, it is more natural to interpret $x$ and $y$ as market share instead of binary choices. As a result, the dynamics describes the mean field of the game.

[8] The two-period model adopted in this section is not a special case. In fact, the main result Proposition 12 can be generalized to any $T$-period models.

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