Topics for the Final Exam

APM 384 Partial Differential Equations

Autumn 2013

Here is a list of concepts you should be familiar with for the final exam.

1. **Definitions and basic classification of differential equations:** You should be familiar with the difference between ordinary and partial differential equations (ODEs and PDEs) and know what the order of a differential equation is. You should also be able to distinguish linear and non-linear PDEs and know when a PDE (or an associated boundary or initial condition) is homogeneous. You should also know what Δ, ∇ and the directional derivatives are.

2. **The method of characteristics:** You should be able to use the method of characteristics in order to solve first order linear PDEs with two arguments. Handout 1 covers this. You may wish to look at the relevant parts of Chapter 12 in Haberman for a slightly different (but essentially equivalent) approach and many practice questions.

3. **Separation of Variables:** We have seen in class how to use separation of variables to solve second order PDEs by reducing then to ODEs via the method of separation of variables. You should be familiar with this technique and able to apply it. You should be able to solve the resulting second-order ODEs of the type we discussed in class and use any homogeneous boundary condition. You should also be able to combine this with the method of superposition of solutions and Fourier series (see below) to deal with a non-homogeneous boundary or initial condition. This is covered in Chapter 2 of Haberman

4. **Complex differentiable and harmonic functions:** You should be comfortable with switching viewpoints between a function of one complex and two real variables and know about the connection between complex differentiable and harmonic functions. You should also be aware of the properties of complex differentiable and harmonic functions that we discussed in class. They are summarised in Handout 2, an informal discussion about qualitative properties of harmonic functions can be found in Section 2.5.4 of Haberman.

5. **Fourier Series:** We covered Chapter 3 in Haberman, except for section 3.5 and the subsection of 3.4 titled 'the method of eigenfunction expansion'. You should be comfortable with this material, in particular you should be able to compute Fourier series for a given function and be able to apply what you know about Fourier Series to solving differential equations.

6. **Sturm–Liouville problems:** Sections 5.1 - 5.7 in Haberman were discussed in class, with the exception of some worked examples (working through which would constitute great exam preparation). We also discussed higher order SL problems, corresponding to sections 7.2 – 7.6, with the exception of the appendix on the Gram–Schmid method. You should know what the definition of a SL problem is and how it arises when solving PDEs. You should also be comfortable with recalling and applying the properties of SL problems (including in higher dimensions) that we derived in class.

7. **Green's functions and the Fredholm alternative.** You should be comfortable with the material in sections 9.3 to 9.5, including the two methods of obtaining Green's functions (using SL problems and the Dirac delta), the Fredholm Alternative and generalised Green's functions. We did not explicitly discuss variation of parameters, although the ideas are familiar to us.

8. **The Fourier transform.** You should be familiar with the definition of the Fourier transform, its basic properties and how it can be used to find general solutions to PDEs. Our discussions followed the first four sections of Chapter 10 in Haberman. In the sixth problem sheet you saw how to define higher-order Fourier transforms, and you should be comfortable with this extension. Section 10.6 provides many worked examples, some of which we discussed in class.

9. **Numerics**. We will discuss the finite difference and finite element methods, but only the material on the finite difference method is examinable. Sections 6.2, 6.3.1,6.3.3 and 6.4 in Haberman are relevant for the exam.

10. **PDEs and probability**. This material will not feature on the final exam.