# Topics for the Final Exam

## MAT 1128: Topics in Probability

## Autumn 2013

Here is a list of concepts you should be familiar with for the final exam. You should have an understanding of the main steps in the proofs of the theorems we have seen in class. The exception to this is anything measure-theoretic that we have delegated to the handout.

I have added references to Durrett, but note that his treatment of measure theory is slightly more general and organised in a different way.

1. **Measure theoretic construction of probability theory:** Probability measures, probability spaces, random variables, their laws and distribution functions. $\sigma$-algebras generated by events or random variables; the Borel $\sigma$-algebra on $\mathbb{R}^d$. This material is covered in sections 1.1-1.3 of Durrett.

2. **Expectation/integration:** Indicator functions of measurable sets and simple functions. Definition of $\mathbb{E}X$ for $X \geq 0$ measurable and measurable $X$ such that $\mathbb{E}|X| < \infty$. Properties of the integral (linearity etc). Definition of $L^p(\omega, \mathcal{F}, \mathbb{P})$ for $p > 0$. Change of variables formula. Jensen's, Hölder's and the Cauchy–Schwarz inequality. This corresponds roughly to sections 1.4, 1.5 and 1.6.1 of Durrett.

3. **Product measures:** product spaces, existence and uniqueness of product measure (statement only). Fubini's theorem. Section 1.7 of Durrett.

4. **Limit theorems:** Fatou's lemma, bounded convergence theorem, MCT and DCT. Covered in parts of section 1.5 and in section 1.6.1.

5. **Independence:** Definition for events, random variables and $\sigma$-algebras. Sufficient conditions for independence. Relation to probability laws and distributions and to the product measure. Section 2.1 of Durrett.

6. **Laws of large numbers:** Convergence a.s., in $L^2$ and in probability. You should be familiar with the various weak and strong laws we proved, what their assumptions and their conclusions are. Covered in section 2.2-2.4 of Durrett.

7. **Central limit theorems:** The main themes here are characteristic functions and weak convergence and how to use these concepts to establish the various CLTs we discussed. This corresponds to sections 3.1-3.4 of Durrett. We also informally discussed Poisson convergence, stable laws and infinitely divisible distributions, these are discussed in greater detail in sections 3.6-3.8. In the exercises we considered certain extensions to $\mathbb{R}^d$.

8. **Random walks:** We covered sections 4.1 and 4.2. You should be familiar with the concepts of the tail and exchangeable $\sigma$-algebras, recurrence, transience (and the connection to the dimension of the state space) and stopping times.

9. **Conditional expectation and martingales:** You should be familiar with the concept and properties of conditional expectation and martingales, their interpretations and the main results regarding stability, convergence and the connection to stopping times. See sections 5.1,5.2 and 5.7.