Scaling Properties of Well-Tiled PFCAs

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Abstract: Planar Fourier Capture Arrays (PFCAs) are imaging devices made from unmodified CMOS Angle-Sensitive Pixels (ASPs). PFCAs require no external imaging optics to photograph distant objects. Here, we explore PFCAs miniaturization in two analyses. First, we show an efficient method of tiling Fourier space with ASPs. Second, we show that the area of an optimally-tiled PFCA scales as the square of the effective number of pixels.

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1. Introduction

The Planar Fourier Capture Array [1,2] (PFCA) is an imager composed of angle-sensitive pixels [3–6] (ASPs) capable of photographing arbitrarily distant objects without resorting to any focusing element or moving part (Figure 1). ASPs individually have a sinusoidal sensitivity to light as a function of incident angle along their optically-active axis. As each ASP relates the intensity of an image filtered by a sinusoid, the correct ensemble of ASPs yields Fourier-complete information about the far-away scene.

One of the strengths of PFCAs as imagers is their tiny size, since they need no focal distance and can be manufactured in unmodified CMOS. Indeed, the first prototype PFCA [1] is a factor of 10^5 smaller than the smallest focusing camera [7] by volume. Here, we demonstrate a method for tiling Fourier space more efficiently, then we derive the scaling properties of the area of a PFCA needed to capture images of a given resolution.

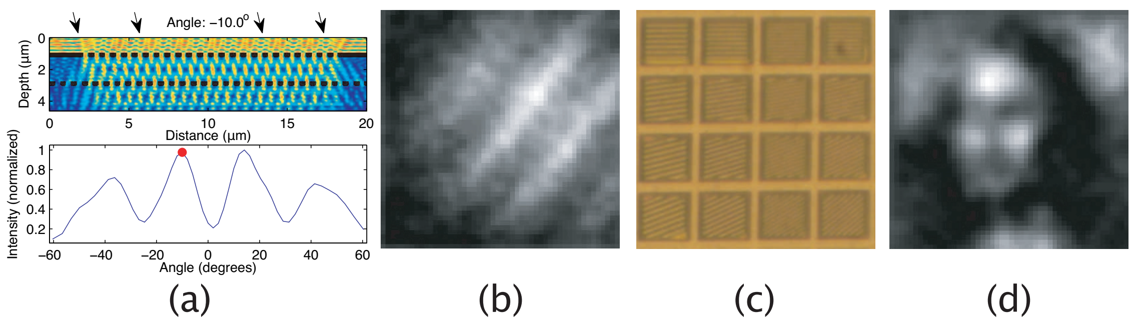

Fig. 1. PFCA Overview. (a) A simulated ASP with two gratings (in black) passes light (intensity in color) to a photodiode below with a sensitivity that is sinusoidal in incident angle. (b) Measured transfer function of an ASP from [1]. (c) Light micrograph showing ASP diversity in a portion of a PFCA. (d) Sample reconstructed image from [1].

2. Tiling Fourier Space Efficiently

Individually, ASPs have a sensitivity to incoming light that can be modeled as follows [2]:

$$R = I_0(1 - m\cos(b\theta + \alpha))F(\theta)(1 + \eta), \quad (1)$$

where R is the photocurrent observed by the ASP, I_0 is proportional to the photon flux at the ASP, θ is the incident angle (relative to the normal) along the sensitive axis, b is the angular sensitivity of the ASP, m is the modulation depth of the ASP, α is a designable phase offset caused by a displacement between the top and bottom gratings, $F(\theta)$ is a slowly-varying aperture function and η is multiplicative noise. By considering the difference signal between 2 ASPs with α differing by π, one can isolate the sin or the cos term through subtraction. A "quadruplet" is 2 pairs of ASPs of a given b and orientation with $\alpha = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$, relating full quadrature information at a given b and orientation. Quadruplets share a common top grating to save space, as reported previously [6].

Here, we report a way of selecting ASPs such that they tile Fourier space more efficiently (see Figure 2) than our manufactured prototype [1, 2]. Our improved procedure consists of four steps. First, we select manufacturable ASP designs [2] with an acceptable m and an evenly-spaced range[1] of bs. Second, we select a target ASP count, or equivalently a target device area. In this example we chose a 13×13 grid of ASP quadruplets, with the center device replaced by 4 low-b devices characterized elsewhere [8]. Third, we allocate the 168 ASP quadruplets among our 23 ASP designs[2] such that a design's count is proportional to the swath of Fourier space it must cover. Fourth, we rotate the sensitive axis of each quadruplet individually so as to cover Fourier space optimally.

Optimizing orientation is a non-convex problem. It can be solved satisfactorily by gradient descent on the sum of the inverse fourth powers of all inter-ASP distances in Fourier space given fixed radius but variable θ. Specifically, if θ_i is the orientation of the i^{th} ASP quadruplet and b_i is its b, we minimize:

$$\min_{\{\theta\}} \left(\sum_{i=1}^{167} \sum_{j=i+1}^{168} \frac{1}{\left[(b_i \cos(\theta_i) - b_j \cos(\theta_j))^2 + (b_i \sin(\theta_i) - b_j \sin(\theta_j))^2\right]^2} \right). \tag{2}$$

We use a momentum term [9] to speed convergence, fix $\theta_1 = 0$ to account for rotational symmetry, and select the best

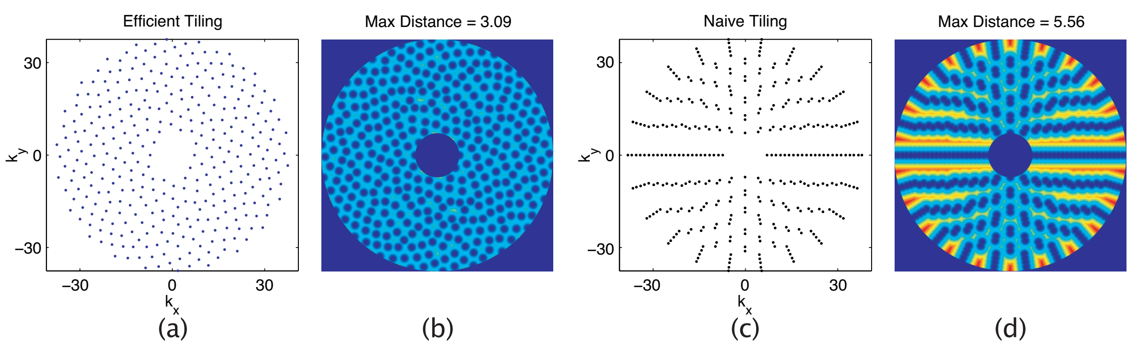

Fig. 2. Optimized tiling covers Fourier space efficiently. (a) Spatial frequencies in a PFCA whose orientations have been optimized to reduce gaps in Fourier space. (b) Map of distances to the nearest ASP in (a); maximum 3.09. (c) Spatial frequencies in a PFCA identical to that in (a) except with all ASP types including one common orientation (0°) and evenly-spaced orientation differences. (d) Map of distances to the nearest ASP in (c); maximum 5.56. Color scale identical to that in (b).

of 100 optimizations of (2) with random, reasonable initial starting conditions: each device type has equally-spaced ASPs with a random angular offset.

A PFCA is Fourier complete if the basis functions from neighboring ASPs have at most a one-cycle difference over the range of observable incident angles. Here, we refine the equation reported in [1] relating the maximum half-angle $h < 90^\circ$ over which the PFCA can be Fourier-complete to the following:

$$h = \frac{90^\circ}{\Delta b_{\text{max}}} \tag{3}$$

where Δb_{max} is the maximum distance from any point in Fourier space in the range reported by the PFCA to the nearest coverage by an ASP. Using (3), we see that with naive tiling h is 16.2° while with optimized tiling h is 29.1°. The Nyquist limit on the effective number of pixels n depends on b_{max} and h in the following way:

$$n \approx b_{\text{max}}^2 \left(\frac{h}{90^\circ}\right)^2. \tag{4}$$

[1] Specifically, we chose 23 ASP designs with bs of 7.4, 8.9, 10.4, 11.8, 13.3, 14.7, 16.2, 17.6, 19.1, 20.5, 22.0, 23.4, 24.8, 26.3, 27.7, 29.1, 30.6, 32.1, 33.5, 34.9, 36.3, 37.8, and 39.0.

[2] The counts of ASP quadruplets for each b were 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 7, 8, 8, 9, 9, 10, 10, 11, 11, 11, 11, and 11.

From (4), the naively-tiled PFCA has $n = 49$ effective pixels, while the optimally-tiled PFCA has $n = 159$ effective pixels. In this example, careful tiling more than triples the effective number of pixels. In general, n in a well-tiled PFCA (here 159) approximately equals[3] the number of ASP quadruplets (168). If each quadruplet fits into a 10×10 micron area, this PFCA would occupy less than 0.017 mm^2 on the die.

3. Scaling Properties of PFCAs

In Section 2 we saw that the effective number of pixels n measured by a well-tiled PFCA is approximately equal to the number of ASP quadruplets. We will now derive an expression for the scaling of the area of a PFCA as a function of n. In order to ensure the light hitting the analyzer grating has passed through the diffraction grating [3], the width of an ASP quadruplet of thickness no more than z_{max} should be

$$w = s z_{\mathrm{max}} \tan(h) \tag{5}$$

where $s > 1$ is a safety margin. Devices with a high b require an increased z, and z_{max} can be found in terms of b_{max} [3]: $z_{\mathrm{max}} = \frac{p_{\mathrm{min}} b_{\mathrm{max}}}{2\pi}$ where p_{min} is the minimum pitch used. Substituting into (5),

$$w = s \frac{p_{\mathrm{min}} b_{\mathrm{max}}}{2\pi} \tan(h). \tag{6}$$

From (4) expressed in radians, $b_{\mathrm{max}} = \sqrt{n}\frac{\pi}{2h}$, so

$$w = s \frac{p_{\mathrm{min}}\sqrt{n}}{4} \frac{\tan(h)}{h}. \tag{7}$$

For $h < \frac{\pi}{4}$, assume $s\frac{\tan(h)}{h} = 4$, so (7) becomes $w = p_{\mathrm{min}}\sqrt{n}$. The total area A of the PFCA is $w^2 n$, so total area of a PFCA scales with n and p_{min} as:

$$A \approx p_{\mathrm{min}}^2 n^2. \tag{8}$$

The area of a well-tiled PFCA therefore scales as the square of the effective number of pixels reported, indicating that small PFCAs are particularly space-efficient.

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[3] That each quadruplet yields 2 pieces of information but only 1 pixel implies that demanding Fourier completeness sacrifices information. In contrast, compressed sensing techniques [2] recover compressible images with *more* effective pixels than observations; h for the well-tiled PFCA under these techniques could be as high as 60°, close to the limit where surface and reflection effects significantly attenuate incoming light.