In a Monte-Carlo simulation, there is an exploratory gradual refinement of some possible solution, via small perturbations, that optimizes a quantity. Fluctuations about local optimum are allowed, this lets the exploration to "tunnel" or move around local maxima in the energy landscape. The problem here is convex, I believe, and there is only 1 minimum, so the obtained curve does approximate the actual solution. There are, as suggested by thermodynamics, *no* Markov or random procedures which can optimize for all initial conditions. Here, we attempt to find the curve which optimizes the time of travel of a particle between two fixed points. The particle being influenced by Earth's gravity. This is called the *brachistochrone* problem.

We note that the brachistochrone string, or ramp does no work on the bead. The force acting on the bead is normal, perpendicular to the displacement at all times. This is an example of an holonomic constraint. So, due to the conservation of energy in Earth's gravity, we arrive at the geometric relation,

$$a \times dl = g \times dh$$

where dl is the (infinitesimal) distance the bead travels on the curve, and a is the constant acceleration along that length, as it descends a(n infinitesimal) distance dh, in Earth's gravity g. In computer simulations, discretizing makes dl and dh finite. One recognizes from basic dynamics of inclined planes or ramps, $a = \frac{dh}{dl} g = \sin(\theta) g$, where θ is the angle of incline measured from the horizontal.

We are free to choose our ground potential. So we begin at $h = 0$, with zero mechanical energy. This allows us to solve for the kinetic energy,

$$v(h) = \sqrt{2gh}$$

We seek a convenient form for us to use in Monte Carlo optimization. The time required to accelerate is,

$$dt = \frac{dv}{a}$$

When discretized, for finite distance between some vertices $1, 2$,

$$\Delta t_{1,2} = \frac{\Delta v_{1,2}}{a} = (v(h_2) - v(h_1)) \frac{\Delta l_{1.2}}{g \Delta h_{1,2}}$$

where $\Delta l_{1,2}$ is the Euclidean distance between two vertices as computed by the Pythagorean theorem, and $\Delta h_{1,2}$ is the height difference between two vertices or the difference in y-coordinates.

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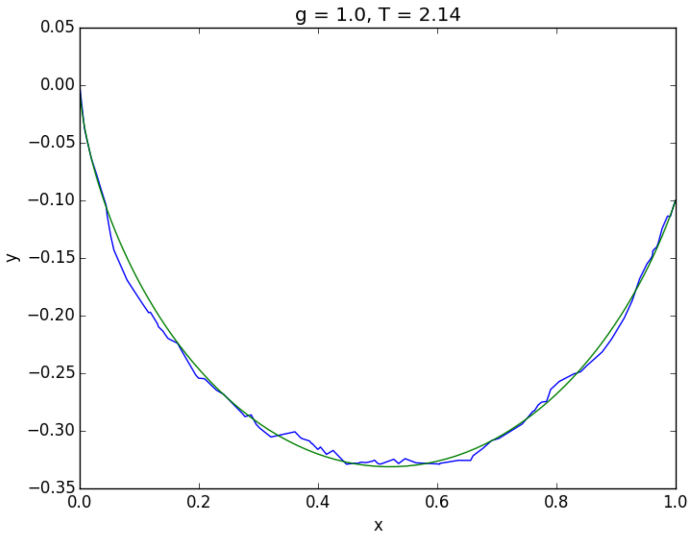
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0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
−0.35
y
0.0
0.2
0.4
0.6
0.8
1.0
x