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# THE LOGICAL STRUCTURE OF LINGUISTIC COMMITMENT II: SYSTEMS OF RELEVANT COMMITMENT ENTAILMENT

ABSTRACT. In "The Logical Structure of Linguistic Commitment I" (*The Journal of Philosophical Logic* **23** (1994), 369–400), we sketch a linguistic theory (inspired by Brandom's *Making it Explicit*) which includes an "expressivist" account of the implication connective, $\rightarrow$: the role of $\rightarrow$ is to "make explicit" the inferential proprieties among possible commitments which proprieties determine, in part, the significances of sentences. This motivates reading $(A \rightarrow B)$ as "commitment to $A$ is, in part, commitment to $B$". Our project is to study the logic of $\rightarrow$. LSLC I approximates $(A \rightarrow B)$ as "anyone committed to $A$ is committed to $B$", ignoring issues of whether $A$ is *relevant* to $B$. The present paper includes considerations of relevance, motivating systems of *relevant* commitment entailment related to the systems of *commitment* entailment of LSLC I. We also consider the relevance logics that result from a commitment reading of Fine's semantics for relevance logics, a reading that Fine suggests.

"The Logical Structure of Linguistic Commitment I" (LSLC I) sketches a linguistic theory (inspired by Brandom, 1983, 1985 and 1994) according to which the significance of an expression is, in part, cashed out in terms of its role in the inferential structure of language. This theory emphasizes the importance of *asserting* as a linguistic act: when a person makes an assertion, she undertakes certain commitments – to justify the assertion, and its consequences – and if these commitments are appropriately discharged, she secures prima facie entitlement to the assertion. This motivates consideration of an entailment-like connective "$\rightarrow$", where "$A \rightarrow B$" is to be read as "commitment to $A$ is, in part, commitment to $B$". Given such a connective, to say (correctly) "$A \rightarrow B$" is, in part, to make explicit the inferential moves to which the members of the linguistic community are committed, and thereby to shed light on the meaning or significance of the terms occurring in $A$ and $B$.

Our project in LSLC I and in the present paper is to study the logical structure of "$\rightarrow$". LSLC I notes that a claim of the form "$A \rightarrow B$" suggests two things: "universality of agent", i.e. that *anyone* committed to $A$ is committed to $B$; and "commitment relevance", i.e. that commitment to $A$ is *relevant* to commitment to $B$. LSLC I ignores considerations of relevance, however, and formulates four logics of non-relevant

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commitment entailment. Differences among these logics reflect different intuitions concerning the upshot of embedded commitment claims. The present paper formulates similar logics, this time of *relevant* commitment entailment. (As in LSLC I, we include conjunction, "&", to facilitate axiomatization.)

Although our main object of formal and philosophical concern is the concept of commitment entailment, we report a pleasing connection with earlier work. Anderson and Belnap's original investigation of systems of relevant entailment led them to endorse the system **E** as the correct analysis of relevant entailment and to construct this system by combining, in a certain sense, the modal system **S4** and the relevant system **R**. Later work in the area of relevance logic, however, has tended to diverge more and more from this traditional project.

Many (including Lance, 1988a) have argued against Anderson and Belnap's interpretation of the arrow of **R**. As for **E**, those still endorsing the project of constructing an entailment system from **R** and a modal system have tended to reject **E** in favor of the closely related $\mathbf{R}^N$. Most other researchers in the area have rejected strong relevance systems like **E** altogether, arguing for and investigating weaker systems with little relation to **R**.

Thus **E** and the related project of demonstrating its joint relevance-modal ancestry has been relegated, if not to the trash-heap of history, at least to a provincial sector of the relevance logic commonwealth. The present paper provides a partial rehabilitation of $\mathbf{E}_{\rightarrow\&}$ and **R**. LSLC I motivated the modal system **S4** as one plausible logic of non-relevant commitment entailment, and we show here that adding considerations of relevance to this yields **E** as one possible formalization of the notion of relevant commitment entailment. Further, the corresponding algebraic semantics offers a natural and important interpretation of both **E** and **R**. This rehabilitation is only partial, alas, since our endorsement of the principles of assertional commitment underlying **S4** (and, hence, **E**) is tentative at best (and we will diverge even further when negation is added to the language). Nonetheless, this work should cast new light and generate new interest in an old comrade in the struggle against classical logic as the single legitimate entailment system.

*Note*: for reasons made clear by the main results of LSLC I (which are summarized in §1, below), the present paper uses "$\dashv$" for non-relevant commitment entailment and "$\rightarrow$" for relevant commitment entailment.

## 1. SUMMARY OF LSLC I

### 1.1. *Fitch-style Systems*

LSLC I presents four related Fitch-style natural deduction systems of non-relevant commitment entailment. These systems differ merely in the restrictions placed on the use of modus ponens. In these systems, each line of a proof consists of a formula $A$, prefixed by a (possibly empty) *commitment prefix* $C_1 \cdots C_k$. "$C_1 \cdots C_k A$" is given the intuitive gloss: "person$_1$ is committed to person$_2$ being committed to ... person$_k$ being committed to $A$".

Before we stipulate the rules, note that for any line of a Fitch-style natural deduction proof there is a number of vertical lines to the left of the sentence written at that line. This number is the *rank* of the line. The rules are as follows:

Hyp: A step may be introduced as the hypothesis of a new subproof and each new hypothesis receives a prefix $C_1 \cdots C_k$, where $k$ is the rank of the subproof.

Rep: A sentence occurring at an earlier line may be repeated, retaining the prefix.

Reit: A sentence occurring earlier may be reiterated into hypothetical subproofs, retaining the prefix.

CP: From a proof of $C_1 \cdots C_{n+1} B$ on hypothesis $C_1 \cdots C_{n+1} A$ to infer $C_1 \cdots C_n (A \dashv B)$, $n \geqslant 0$.

MP: From $C_1 \cdots C_n (A \dashv B)$ and $C_1 \cdots C_n \cdots C_{n+m} A$ to infer $C_1 \cdots C_{n+m} B$, where $n \geqslant 0$, and $m$ is restricted according to the table below.

(The following rules for conjunction, "&", are natural and facilitate axiomatization.)

&I: From $C_1 \cdots C_n A$ and $C_1 \cdots C_n B$ to infer $C_1 \cdots C_n (A \& B)$.
&E: From $C_1 \cdots C_n (A \& B)$ to infer either $C_1 \cdots C_n A$ or $C_1 \cdots C_n B$.

LSLC I motivates four ranges for $m$ in MP: $m = 1$; $m = 0$ or $1$; $m \geqslant 1$; and $m \geqslant 0$. These four ranges lead to four Fitch-style systems. The four corresponding sets of theorems are the four corresponding non-relevant commitment logics. Table I indicates the names of these systems and logics. Each commitment logic is the strict implication conjunction fragment of a modal logic (where "$\dashv$" is strict implication), and has a natural axiomatization. Table I also summarizes this information.

TABLE I

The four systems of non-relevant commitment entailment

| Range of $m$ | Fitch-style System | Commitment Logic | Modal Logic | Axiomatization |
|---|---|---|---|---|
| $m = 1$ | **fC1** | **C1** | $\mathbf{K}_{\dashv\&}$ | 1–5 and 8–12 |
| $m = 0$ or 1 | **fC01** | **C01** | $\mathbf{T}_{\dashv\&}$ | **C1** + 6 |
| $m \geqslant 1$ | **fC1+** | **C1+** | $\mathbf{K4}_{\dashv\&}$ | **C1** + 7 |
| $m \geqslant 0$ | **fC0+** | **C0+** | $\mathbf{S4}_{\dashv\&}$ | **C1** + 6 + 7 |

Axioms and rules for axiomatizations:

Basic Axioms:

1. $A \dashv A$
2. $A \& B \dashv A$
3. $A \& B \dashv B$
4. $(A \dashv B) \& (A \dashv C) \dashv .A \dashv (B \& C)$
5. $(A \dashv B) \& (B \dashv C) \dashv (A \dashv C)$

Additional Axioms:

6. $(A \& (A \dashv B)) \dashv B$
7. $(B \dashv C) \dashv .(A \dashv B) \dashv (A \dashv C)$

Rules:

| | | |
|---|---|---|
| 8. Modus Ponens (MP): | from $A$ and $(A \dashv B)$ | to infer $B$ |
| 9. Conjunction: | from $A$ and $B$ | to infer $(A \& B)$ |
| 10. Prefixing: | from $(A \dashv B)$ | to infer $(C \dashv A) \dashv (C \dashv B)$ |
| 11. Relevance destroyer: | from $A$ | to infer $(B \dashv A)$ |

## 1.2. *Algebraic Commitment Semantics*

LSLC I's semantics formalize the notions of:

(1) a person, $\alpha$, being committed to $A$: $\alpha \vDash A$; and
(2) $\alpha_1$ being committed to $\alpha_2$ being committed to $\ldots \alpha_n$ being commit ted to $A$:

$$\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle \models A.$$

In case (2), we say that the *sequence* $\langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$ is committed to $A$.

DEFINITION 1. Let **CL** be **C1, C01, C1+** or **C0+**. A **CL**-*commitment-model* is an ordered pair $M = \langle S, \models \rangle$ where

(1) $S$ is a non-empty set (of persons, or other entities to whom we assign commitments);
(2) $\models \subseteq S^{<\omega} \times Form$, where $S^{<\omega}$ is the set of finite sequences of members of $S$ (including the empty sequence, $\varnothing$) and where *Form* is the set of formulas; and
(2) $\models$ satisfies (i) and (ii) below for $b$ in $S^{<\omega}$ and $A$ and $B$ in *Form*. Here, $bc = b$ concatenated with $c$, and $S_{\mathbf{CL}} \subseteq S^{<\omega}$ depends on **CL** as in the following table:

| **CL**: | **C1** | **C01** | **C1+** | **C0+** |
|---|---|---|---|---|
| $S_{\mathbf{CL}}$: | $S$ | $S \cup \{\varnothing\}$ | $S^{<\omega} \backslash \{\varnothing\}$ | $S^{<\omega}$ |

(i) $b \models (A \& B)$ iff $b \models A$ and $b \models B$; and
(ii) $b \models (A \dashv B)$ iff $(\forall c \in S_{\mathbf{CL}})$ (if $bc \models A$ then $bc \models B$).

*Note*. Suppose that we fix **CL**. Also, suppose that we fix the truth-values of "$b \models p$" for all $b$ and for all atomic formulas $p$. Then, for each $b$ and for each formula $A$, the truth-value of "$b \models A$" is determined. So, if we fix **CL** and if we fix the semantic value of each atomic formula, we can think of clauses (i) and (ii) as providing recursive definitions of validity for complex formulas.

DEFINITION 2. Given an **CL**-commitment-model, $M = \langle S, \models \rangle$, and a formula $A$, we say that $M$ *validates* $A$ ($M \models A$) if $\varnothing \models A$. Otherwise, $M$ *falsifies* $A$. We say that a formula is **CL**-*valid* if it is validated by every **CL**-commitment-model.

THEOREM 1. *For every formula $A$, $A \in$ **CL** iff $A$ is **CL**-valid.*

## 2. RELEVANCE

Anderson and Belnap (1975) distinguish two classes of fallacies that are committed if we take the material or intuitionist conditional to be an entailment connective: "fallacies of relevance" and "fallacies of necessity". Intuitively, the former concern the idea that to say "$A$ entails $B$"

is, in part, to say that $A$ is relevant to $B$. Thus, $A \rightarrow (B \rightarrow B)$ is not a valid entailment: it is not possible for the consequent to be false, but it is not true that the consequent can be derived *from* the antecedent. We can write down $A$ and then prove $(B \rightarrow B)$, but the writing down of $A$ is irrelevant: it does not do any of the work.

The fallacies of necessity concern special features of alethically necessary sentences, but our analysis of commitment entailment does not involve alethic necessity. In its place, LSLC I is concerned with "universality of agent". Now, just as Anderson and Belnap combine a concern with relevance and a concern with necessity (and thus produce **E** from **R** and **S4**), we combine a concern with relevance and LSLC I's concern with "universality of agent".

Anderson and Belnap keep track of relevance by putting subscripts on each hypothesis introduced in a proof – a new unit set of subscripts for each new hypothesis – and then use these to see what premises have been employed in deriving a conclusion. Any step in the proof has as subscript a set consisting of all those numbers corresponding to hypotheses which were used in the step's derivation. Thus, if Modus Ponens (MP) is used with premises which have subscripts $a$ and $b$, then the conclusion has subscript $a \cup b$. Applications of conditional proof are restricted to cases in which the sub-conclusion $B$ has subscript $c$ and the premise of the subproof $A$ has subscript $\{k\}$ where $k \in c$. The conclusion is then $A \rightarrow B$ with subscript $c\backslash\{k\}$. And so, Anderson and Belnap's Fitch-style rules for the $\rightarrow$ & fragment of their relevance system **R** (which is not concerned, as in **E**, with necessity) are:

Hyp: A step may be introduced as the hypothesis of a new subproof and each new hypothesis receives a subscript $\{k\}$, where $k$ is the rank of the subproof. (This "rank" requirement is not necessary, but does not hurt.)

Rep: A sentence occurring at an earlier line may be repeated, retaining the subscript.

Reit: A sentence occurring earlier may be reiterated into hypothetical subproofs, retaining the subscript.

CP: From a proof of $B_b$ on hypothesis $A_{\{k\}}$, to infer $(A \rightarrow B)_{b\backslash\{k\}}$, where $k \in b$.

MP: From $(A \rightarrow B)_b$ and $A_a$ to infer $B_{a\cup b}$.

&I: From $A_a$ and $B_a$ to infer $(A\&B)_a$.

&E: From $(A\&B)_a$ to inter either $A_a$ or $B_a$.

## 3. FITCH-STYLE RELEVANT COMMITMENT ENTAILMENT SYSTEMS

Here, we combine the commitment prefixes and the relevance subscripts.

Hyp: A step may be introduced as the hypothesis of a new subproof and each new hypothesis receives a prefix $C_1 \cdots C_k$, and a subscript $\{k\}$, where $k$ is the rank of the subproof.

Rep: A sentence occurring at an earlier line may be repeated, retaining the prefix and subscript.

Reit: A sentence occurring earlier may be reiterated into hypothetical subproofs, retaining the prefix and subscript.

CP: From a proof of $C_1 \cdots C_{n+1}B_b$ on hypothesis $C_1 \cdots C_{n+1}A_{\{k\}}$ to infer $C_1 \cdots C_n(A \rightarrow B)_{b\setminus\{k\}}$, $n \geqslant 0$ and $k \in b$.

MP: From $C_1 \cdots C_n(A \rightarrow B)_b$ and $C_1 \cdots C_n \cdots C_{n+m}A_a$ to infer $C_1 \cdots C_{n+m}B_{a\cup b}$, where $b \geqslant 0$, and $m$ is restricted according to the table below.

&I: From $C_1 \cdots C_nA_a$ and $C_1 \cdots C_nB_a$ to infer $C_1 \cdots C_n(A\&B)_a$.

&E: From $C_1 \cdots C_n(A\&B)_a$ to infer either $C_1 \cdots C_nA_a$ or $C_1 \cdots C_nB_a$.

As in LSLC I, we consider four ranges for $m$, leading to four Fitch-style systems, whose four correspondings sets of theorems are the four corresponding relevant commitment logics. Table II indicates the names of these systems and logics, and provides axiomatizations. The list of axioms appears after Table II. Finally, Figure 1, which follows the axiom list, indicates where the relevant commitment logics fit in the general context of relevance logics.

Axioms and rules for axiomatizations:

(In Axioms 6–9, $n$ may be 0.)

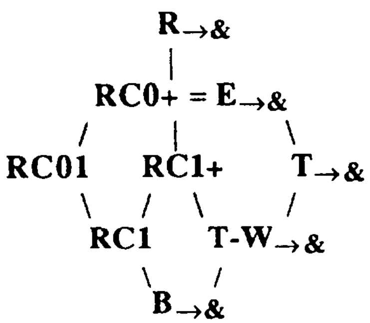


Fig. 1. The place of relevant commitment logics among relevance logics. (The various relevance logics are discussed in Anderson and Belnap 1975, Fine 1974, and Routley *et al.* 1982.)

TABLE II

The four systems of relevant commitment entailment

| Range of $m$ | Fitch-style System | Commitment Logic | Axiomatization |
|---|---|---|---|
| $m = 1$ | **fRC1** | **RC1** | 1–7 and 12–14 |
| $m = 0$ or 1 | **fRC01** | **RC01** | **RC1** + 8–10 |
| $m \geqslant 1$ | **fRC1+** | **RC1+** | **RC1** + 11 |
| $m \geqslant 0$ | **fRC0+** | **RC0+** | **RC1** + 8–11 = 1–5 and 10–13 |

Basic Axioms:

1. $A \to A$
2. $A\&B \to A$
3. $A\&B \to B$
4. $(A \to B)\&(A \to C) \to .A \to (B\&C)$
5. $(A \to B)\&(B \to C) \to (A \to C)$
6. $((A \to A) \to .H_1 \to \cdots \to .H_n \to .B \to C) \to .$
   $(H_1 \to \cdots \to .H_n \to .D \to B) \to .$
   $H_1 \to \cdots \to .H_n \to .D \to C$
7. $((A \to A) \to .H_1 \to \cdots \to .H_n \to .(B \to C)) \to .$
   $(H_1 \to \cdots \to .H_n \to .C \to D) \to .$
   $H_1 \to \cdots \to .H_n \to .B \to D$

Additional Axioms:

8. $((A \to A) \to .H_1 \to \cdots \to .H_n \to .(B \to C)) \to .$
   $(H_1 \to \cdots \to .H_n \to B) \to .H_1 \to \cdots \to .H_n \to C$
9. $((A \to A) \to .H_1 \to \cdots \to .H_n \to B) \to .$
   $(H_1 \to \cdots \to .H_n \to .B \to C) \to .H_1 \to \cdots \to .H_n \to C$
10. $(A\&(A \to B)) \to B$
11. $(B \to C) \to .(A \to B) \to (A \to C)$

Rules:

| | | |
|---|---|---|
| 12. Modus Ponens (MP): | from $A$ and $(A \to B)$ | to infer $B$ |
| 13. Conjunction: | from $A$ and $B$ | to infer $(A\&B)$ |
| 14. Prefixing: | from $(A \to B)$ | to infer $(C \to A) \to (C \to B)$ |

*Note*. Axioms 6–9 look imposing, but make intuitive sense. Consider, as a representative example, Axiom 7 in the case $n = 0$. This can be understood as a weakening of the standard axiom of transitivity: $B \to C \to .(C \to D \to .B \to D)$. According to the latter: if $\alpha$ asserts that commitment to $B$ carries with it commitment to $C$, then $\alpha$ is thereby committed to taking anyone $\alpha$ takes to be committed to $(C \to D)$ to also be committed to $(B \to D)$. As pointed out in LSLC I, this axiom is questionable.

$(A \to A \to .B \to C) \to .(C \to D \to .B \to D)$, on the other hand, merely says that if commitment to $B \to C$ follows from commitment to $(A \to A)$ – on one natural reading this is to say if everyone must be committed to $(B \to C)$ – then commitment to $(B \to D)$ follows from commitment to $(C \to D)$. This axiom is, on the commitment reading, much weaker for it only requires that $\alpha$ assign commitments in accordance with the consequent of the axiom, if $\alpha$ is committed to taking *everyone* to be committed to $(B \to C)$. And if $\alpha$ is committed to taking *everyone*, as a matter of necessity, to be committed to $(B \to C)$, then if $\alpha$ takes $\beta$ to be committed to $(C \to D)$, $\alpha$ takes $\beta$ to be committed to both $(B \to C)$ and $(C \to D)$; and from these, commitment to $(B \to D)$ follows.

Now suppose $n = 1$. Since relevance is not relevant here, we simplify our grammar by reading the axiom as if the arrow were non-relevant. What does Axiom 7 tell us? Suppose that $\alpha$ is committed to the claim that everyone is committed to $H_1 \to (B \to C)$. Axiom 7 tells us that $\alpha$ is committed to $(H_1 \to (C \to D)) \to (H_1 \to (B \to C))$. Does this follow? Suppose that $\alpha$ is committed to $\beta$ being committed to $(H_1 \to (C \to D))$. Does it follow that $\alpha$ is committed to $\beta$ being committed to $(H_1 \to (B \to D))$? Well, suppose that $\alpha$ is committed to $\beta$ being committed to $\gamma$ being committed to $H_1$. We can now represent the situation as follows:

| | | | | |
|---|---|---|---|---|
| 1. | $\alpha$ | is committed to | $(A \to A) \to (H_1 \to (B \to C))$ | |
| 2. | $\alpha\beta$ | is committed to | $H_1 \to (C \to D)$ | |
| 3. | $\alpha\beta\gamma$ | is committed to | $H_1$ | |
| 4. ∴ | $\alpha\beta$ | is committed to | $H_1 \to (B \to C)$ | (from line 1) |
| 5. ∴ | $\alpha\beta\gamma$ | is committed to | $(B \to C)$ | (from lines 3 and 4) |
| 6. and | $\alpha\beta\gamma$ | is committed to | $(C \to D)$ | (from lines 2 and 3) |
| 7. ∴ | $\alpha\beta\gamma$ | is committed to | $(B \to D)$ | (from lines 5 and 6) |

Thus, the case $n = 1$ says the same things as the case $n = 0$ with one more level of nested attribution. In general, the axiom says that if both $(B \to C)$ and $(C \to D)$ are attributed, no matter how far out the string of one's attributions to attributions to attributions, etc., then one is committed to attributing $(B \to D)$ there as well.

Axioms 6, 8 and 9 function similarly. Despite their intuitive content, Axioms 6–9 are rather inelegant, involving, as they do, the sequence "$\cdots H_1 \to \cdots \to .H_n \to \cdots$". §3.1 specifies a sense in which we believe that there are no elegant axiomatizations of **RC1**, **RC01**, and **RC1+**. There are plenty of elegant axiomatizations of **RC0+**, however, since **RC0+** $= \mathbf{E}_{\to\&}$ (as indicated in Figure 1).

### 3.1. *No Elegant Axiomatizations of RC1, RC01 and RC1+*

(This section is an interesting aside, which the reader can skip without losing the flow of the paper.)

What we have listed as axioms are commonly called axiom schemes, to emphasize the fact that they are not single formulas, but sets of formulas. Axiom schemes 1–5 and 10–11 are distinguished by the fact that each of them is *representable* by a single formula in the following sense: we say that a set, $S$, of formulas is *representable* by a single formula, $A$, if $S$ is the set of substitution instances of $A$. One way of noting the inelegance of Axiom schemes 6–9 is to note that none of them is representable by a single formula. Given a logic, we may wonder whether we can characterize it by a finite list of axioms and rules, each axiom being elegant in this sense. In fact, in the case of **RC1**, **RC01** and **RC1+** we can, but at a price. We can replace Axiom 6, for instance, with the following axiom and rule (similar moves can be made for Axioms 7–9):

Axiom 6′. $((A \to A) \to .B \to C) \to .D \to B \to .D \to C.$

Rule 6″. From $((A \to A) \to .H_1 \to \cdots \to .H_n \to .B \to C) \to$
$(H_1 \to \cdots \to .H_n \to .D \to B) \to .$
$H_1 \to \cdots \to .H_n \to .D \to C$

to infer $((A \to A) \to .H_1 \to \cdots \to .H_n \to .$
$H_{n+1} \to .B \to C) \to$
$(H_1 \to \cdots \to .H_n \to .H_{n+1} \to .D \to B) \to$
$.H_1 \to \cdots \to .H_n \to .H_{n+1} \to .D \to C$

The problem is that we have replaced an inelegant axiom with an elegant axiom and an inelegant rule. Intuitively, Rule 6″ is inelegant for the same reason that Axiom 6 is. Presently, we make these intuitions precise.

An *inference* is a non-empty finite sequence of sentences, in which the last sentence is thought of as being derived from the first sentences. The following inference can be thought of as an instance of the rule MP (Rule 13): $\langle p, (p \to q), q\rangle$. A *postulate* is a set of inferences, closed under substitution. Each of our axioms and rules can be thought of as a postulate. Rule 12 is the postulate $\{\langle A, (A \to B), B\rangle$: $A$ and $B$ are formulas$\}$ and Axiom 1 is the postulate $\{\langle(A \to A)\rangle$: $A$ is a formula$\}$. A postulate, $P$, is *representable* by a single inference, $I$, iff $P$ is the set of substitution instances of $I$. Rule 12 is representable by $\langle p, (p \to q), p\rangle$; Axiom 1 is representable by $\langle(p \to p)\rangle$. A postulate is *simple* iff it is representable by a single inference. Axioms 1–5, 6′ and 10–11 are simple; Axioms 6–9 are not. Rules 12–14 are simple; Rule 6″ is not.

Given a set, $S$, of postulates, and a set $L$ of sentences, $L$ is *closed under $S$* iff:

(i) if $\langle A\rangle \in P \in S$ then $A \in L$; and
(ii) if $\langle A_1, \ldots, A_n, A\rangle \in P \in S$ and if $A_1, \ldots, A_n \in L$ then $A \in L$.

$L$ is *characterizable by $S$* if $L$ is the smallest set of sentences closed under $S$.
The following conjecture expresses the sense in which we believe that **RC1**, **RC01** and **RC1+** cannot be elegantly axiomatized.

CONJECTURE 1. **RC1**, **RC01** *and* **RC1+** *are not charactizable by a finite set of simple postulates.*

## 4. RELEVANT COMMITMENT SEMANTICS

The main idea behind the commitment semantics of LSLC I (summarized in §1.2, above) is this: a person $\alpha$ is committed to $A \to B$ ($\alpha \vDash A \to B$) just in case, for every person $\beta$, if $\alpha$ is committed to $\beta$ being committed to $A$ ($\alpha\beta \vDash A$) then $\alpha$ is committed to $\beta$ being committed to $B$ ($\alpha\beta \vDash B$). This is meant to formalize the reading of "$A \to B$" as "anyone committed to $A$ is committed to $B$". Extending this to *sequences of persons* rather than single persons, we find that, for a sequence $b$, of persons,

$$(*)\qquad b \vDash (A \to B) \text{ iff } (\forall \text{ sequences } c)(bc \vDash A \Rightarrow bc \vDash B),$$

where "$\Rightarrow$" is meta-linguistic material implication. (For now we ignore possible restrictions on the quantifier, $(\forall c)$.) For our *relevant* commitment semantics we retain the clause $(*)$, except that we interpret "$\Rightarrow$" as meta-linguistic *relevant* implication. How can we formalize this?

We borrow some of the ideas used to develop a semantics for $\mathbf{R}_{\rightarrow}$ in Urquhart (1972 and 1973). Urquhart begins with the intuitive concept of a *piece of information*: a set of basic sentences concerning a subject about which reasoning is being carried out. He suggests that any two pieces of information, $u$ and $v$, may be conjoined into a new piece of information, $u \cup v$, where the operation $\cup$ must fulfill the laws of set union: $u \cup u = u$; $(u \cup v) \cup w = u \cup (v \cup w)$; and $u \cup v = v \cup u$. He also suggests that we allow for the empty piece of information, 0, for which we have: $u \cup 0 = u$. Noting that the set of pieces of information is a semilattice under the operation $\cup$, with lattice zero 0, Urquhart takes the concept of a semilattice with 0 as a primitive basic for the semantics of relevant implication.

Given a piece of information $u$ and a sentence $A$, $u$ *determines* $A$ ($u \vDash A$) if $A$ may be concluded on the basis of the sentences in $u$. Urquhart interprets this in a rather strong sense: the information in $u$ must all be relevant to $A$. So we do not have: if $u \vDash A$ then $u \cup v \vDash A$.

Under what conditions does $u$ determine "$A$ relevantly implies $B$"? If $A$ is itself a piece of information, then $u$ determines "$A$ relevantly implies $B$" just in case $u \cup A$ determines $B$. More generally, $u \vDash$ ($A$ relevantly implies $B$) iff ($\forall$ pieces of information $v$)(if $v \vDash A$ then $u \cup v \vDash B$).

In the present context, the subject "about which reasoning is being carried out" is the structure of persons' linguistic commitment, and the pieces of information are about that subject. Given the "relevance" reading of $(*)$, the present sequence of equivalences suggests itself, for a piece of information, $u$, and a sequence of persons, $b$:

$u \vDash (b \vDash A \rightarrow B)$ iff
$u \vDash$ ($\forall$ sequences $c$)($bc \vDash A$ relevantly implies $bc \vDash B$) iff
($\forall$ sequences $c$)($u \vDash$ ($bc \vDash A$ relevantly implies $bc \vDash B$)) iff
($\forall$ sequences $c$)($\forall$ pieces of information $v$)(if $v \vDash (bc \vDash A)$ then $u \cup v \vDash (bc \vDash B)$).

Replacing the expression "$u \vDash (b \vDash A)$" with "$\langle u, b\rangle \vDash A$", we now proceed with our formal semantics.

DEFINITION 3. Let **RCL** be **RC1**, **RC01**, **RC1+** or **RC0+**. An **RCL**-*commitment-model* is an ordered quintruple $M = \langle L, \cup, 0, S, \vDash\rangle$ where

(1) $\langle L, \cup, 0\rangle$ is a semilattice with 0;
(2) $S$ is a non-empty set (of persons, or other entities to whom we assign commitments);

(3) $\models \subseteq L \times S^{<\omega} \times Form$, where $S^{<\omega}$ is the set of finite sequences of members of $S$ (including the empty sequence, $\varnothing$) and where *Form* is the set of formulas; for $u \in L$, $a \in S^{<\omega}$ and $A \in Form$, we write "$\langle u, v\rangle \models A$" for "$\langle u, a, A\rangle \in \models$".

(4) $\models$ satisfies (i) and (ii) below for $u$ in $L$, $b$ in $S^{<\omega}$ and $A$ and $B$ in *Form*. Here, $bc = b$ concatenated with $c$, and $S_{\mathbf{RCL}} \subseteq S^{<\omega}$ depends on **RCL** as in the following table:

| **RCL**: | **RC1** | **RC01** | **RC1+** | **RC0+** |
|---|---|---|---|---|
| $S_{\mathbf{RCL}}$: | $S$ | $S \cup \{\varnothing\}$ | $S^{<\omega}\backslash\{\varnothing\}$ | $S^{<\omega}$ |

(i) $\langle u, b\rangle \models (A \,\&\, B)$ iff $\langle u, b\rangle \models A$ and $\langle u, b\rangle \models B$; and

(ii) $\langle u, b\rangle \models (A \to B)$ iff $(\forall c \in S_{\mathbf{RCL}})(\forall v \in L)$(if $\langle v, bc\rangle \models A$ then $\langle u \cup v, bc\rangle \models B$).

*Note*. Suppose that we fix **RCL**. Also, suppose that we fix the truth-values of "$\langle u, b\rangle \models p$" for all $\langle u, b\rangle$ and for all atomic formulas $p$. Then, for each $\langle u, b\rangle$ and for each formula $A$, the truth-value of "$\langle u, b\rangle \models A$" is determined. So, if we fix **RCL** and if we fix the semantic value of each atomic formula, we can think of clauses (i) and (ii) as providing recursive definitions of validity for complex formulas.

DEFINITION 4. Given an **RCL**-commitment-model, $M = \langle L, \cup, 0, S, \models\rangle$, and a formula $A$, we say that $M$ *validates* $A$ ($M \models A$) if $\langle 0, \varnothing\rangle \models A$. Otherwise, $M$ *falsifies* $A$. We say that a formula is **RCL**-*valid* if it is validated by every **RCL**-commitment-model.

THEOREM 2. *For every formula $A$, $A \in$* **RCL** *iff $A$ is* **RCL**-*valid.*

*Note*. The relevant commitment semantics is a result of combining the modal structure of the non-relevant commitment semantics (of LSLC I) with the semilattice structure of Urquhart's semantics for $\mathbf{R}_{\to\&}$. In a similar vein, Urquhart combines the possible worlds structure of Kripke 1959's semantics for **S4** with the semilattice semantics for $\mathbf{R}_{\to\&}$, to get a hybrid semantics for $\mathbf{E}_{\to\&}$. (Indeed, just as our strongest non-relevant commitment logic, **C0+**, coincides with the strict implication-conjunction fragment of **S4**, so our strongest relevant commitment logic, **RC0+**, coincides with $\mathbf{E}_{\to\&}$.) We believe that the additional semilattice structure is better motivated on the commitment reading than on Urquhart's reading, as we suggest in §4.1.

### 4.1. *Another Interpretation of the Semilattice Structure*

So far, we have been interpreting the members of a semilattice as pieces of information. In the context of the development of a semantics for $\mathbf{R}_{\rightarrow}$ it is unclear exactly what a semilattice itself is supposed to represent (other than the structure of these pieces of information). It is not the collection of true pieces of information; after all in Urquhart's canonical model, *every* finite set of sentences occurs as a piece of information, and presumably not every such piece of information is true. Perhaps a semilattice represents the collection of all possible pieces of information which are *about* a certain subject and *relevant* to a particular inquiry concerning that subject. At any rate, it seems that the sentences validated at the 0 point of the semilattice are just the true ones. One might interpret the semilattice structure of our models along similar lines.

On the other hand, we can take a semilattice to represent certain features of a particular person rather than certain features about the world in general. The points of the semilattice represent (possibly linguistic) actions available to the person: actions that might commit her one way or another to various claims. 0 represents the "null action". $u \cup v$ represent the result of combining actions $u$ and $v$. "$\langle u, \langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle\rangle \models A$" is then read as: "if the person performs action $u$, she thereby commits herself to $\alpha_1$ being committed to $\alpha_2$ being committed to $\ldots \alpha_n$ being committed to $A$". In particular:

(1) "$\langle u, \varnothing \rangle \models A$" is then read as: "if the person performs action $u$, she thereby commits herself to $A$";
(2) "$\langle 0, \langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle\rangle \models A$" is then read as: "the person is committed to $\alpha_1$ being committed to $\alpha_2$ being committed to $\ldots \alpha_n$ being committed to $A$"; and
(3) "$\langle 0, \varnothing \rangle \models A$" is then read as: "the person is committed to $A$".

We prefer this reading of the semantics to the "pieces of information" reading for several reasons. First, recall that we do not want to have that $u \cup v \models B$ whenever $u \models B$. In order to achieve this, Urquhart must read $u \models B$ as "the piece of information $u$ implies $B$ in such a way that all elements of $u$ are relevant to $B$". But this involves an awfully small circle, even for an algebraic semantics. Little by way of explanation of the notion of relevance (as opposed to an exhibition of the inferential structure of the concept) can be forthcoming from such a characterization.

On the other hand, our reading of $\langle u, \varnothing \rangle \models B$ as "the person is committed to $B$ in virtue of act $u$" does provide a non-trivial notion. The idea is that the commitment comes with the action $u$, as a part of it. To explicate relevance in terms of this notion is, we claim, at least in the direction of an explanation.

We also do not like to rely on a prior notion of what a piece of information is *about*. After all, our (though not Urquhart's) logical project is part of a larger project which hopes to explicate semantic content in terms of relations of commitment and entitlement. Given this, it would be unfortunate if the algebraic explication of the logical apparatus required a prior reliance on a concept of aboutness. Finally, we simply find our reading of the semantics to be more natural and attractive, though this commitment lies clearly in the logical psychology of the beholder.

[It is worth noting, as Michael Dunn first pointed out to us, that our semantics must generate an interpretation of $R$, simply by suppressing the second element of the pair. The details of an understanding of this interpretation are somewhat complex, however, and are left to the more systematic treatment of Lance and Kremer 1994.]

## 5. EXPRESSING NON-RELEVANT COMMITMENT ENTAILMENT IN THE RELEVANT COMMITMENT LOGICS

Our logics **CL** and **RCL** are closely related by the similarity of their proof theories and of their semantics. The relationship is stronger: if we enrich the **RCL** in a rather minimal way, we can express $\dashv$ in **RCL**. The idea is to add to the language of **RCL** a propositional constant $t$, which is to be interpreted as the conjunction of all theorems (or, more minimally, as the conjunction of all formulas of the form $(A \rightarrow A)$). The idea is not new; Anderson and Belnap 1975, for example, use it at numerous junctures. We then define $(A \dashv B)$ as $((A \& t) \rightarrow B)$. The sense in which we have thereby expressed non-relevant commitment entailment in the relevant systems is given in Theorem 5, below.

In order to add $t$, we enrich the definition of a Fitch-style derivation, the axiomatizations, and the semantics. These enrichments define new Fitch-style systems $\mathbf{fRCL}^t$, new logics $\mathbf{RCL}^t$, and the new concepts of an $\mathbf{RCL}^t$-commitment-model and of $\mathbf{RCL}^t$-validity.

To the natural deduction rules we add:

$t$-intro: $C_1 \cdots C_n t_\varnothing$ may be entered as a line, whenever $n \leqslant$ the rank of the line.

$t$-elim: From $C_1 \cdots C_n t_a$ to infer $C_1 \cdots C_n (A \rightarrow A)_a$.

To the axioms (for each of the **RCL**) we add:

Axiom t1: $t$; and

Axiom t2: $t \to (A \to A)$.

(The rule $t$-elim and Axiom t2 justify, in their respective contexts, the interpretation of $t$ as the conjunction of all formulas of the form $(A \to A)$).

To clause (4) of Definition 3 (the definition of the semantics) in §4, we add these two subclauses:

(iii) $\langle 0, b\rangle \vDash t$; and

(iv) if $(\langle u, b\rangle \vDash t$ and $c \in S_{\mathbf{RCL}}$ and $\langle v, bc\rangle \vDash A)$ then $\langle u \cup v, bc\rangle \vDash A$.

Theorem 2 (§4) still goes through (replacing the expressions "**RCL**" and "**RCL**-valid" with "$\mathbf{RCL}^t$" and "$\mathbf{RCL}^t$-valid", respectively). We also have the following:

THEOREM 3. $\mathbf{RCL}^t$ *is a conservative extension of* **RCL**.

Also, the following theorem, which justifies the interpretation of $t$ as the conjunction of all theorems, is of some interest:

THEOREM 4. $A \in \mathbf{RCL}^t$ *iff* $(t \to A) \in \mathbf{RCL}^t$.

The key definition and theorem are:

DEFINITION 5. If $A$ and $B$ are formulas in the language of $\mathbf{RCL}^t$, $(A \dashrightarrow B) = ((A \& t) \to B)$.

THEOREM 5. *If $A$ is a formula in the language of the* **CL** *(a language in which "$\dashrightarrow$" is a primitive) then $A \in$* **CL** *iff $A \in \mathbf{RCL}^t$. (Here, if* $\mathbf{CL} = \mathbf{C1}$ *then* $\mathbf{RCL}^t = \mathbf{RC1}^t$, *and if* $\mathbf{CL} = \mathbf{C01}$ *then* $\mathbf{RCL}^t = \mathbf{RC01}^t$, *and so on. Also, occurrences of $\dashrightarrow$ in $A$ on the left side of the biconditional are taken to be primitives in the language of* **CL** *while occurrences of $\dashrightarrow$ in $A$ on the right side of the biconditional are taken to be defined as in Definition 5.)*

## 6. FINE'S OPERATIONAL SEMANTICS FOR RELEVANCE LOGICS

Fine 1974 provides a semantics for a large family of relevance logics. Fine hints at a "commitment" interpretation of his semantic primitives, but he does not consider the ramifications of this interpretation. In this section we consider some of these. This leads to another basis for approaching relevant commitment entailment.

The work of this section is interestingly different from the work in the foregoing sections. For one thing, the systems of commitment entailment which we motivate here are distinct from the earlier systems. More importantly, the motivation itself proceeds along quite different lines.

Fine takes his committed entities to be *theories* rather than *persons*. We suggest that the only way to flesh out Fine's commitment interpretation is by taking 'commitment' to have a meaning derivative from that investigated in the foregoing sections of the present paper. In this derivative sense, a person's *theory* may be said to be committed to $P$ even when the person is not (because of facts the person is unaware of – see §6.3). And so the kind of commitment we are interested in may be called "theory commitment" rather than "commitment" *tout court*. Note that this use of "commitment" is secondary, given the linguistic theory that provides the background for our earlier theories of commitment entailment. For a person has a stronger duty to defend claims to which *he* is committed that he does to defend claims to which his *theory* is committed.

§6.1 and §6.2 set the stage for §6.3's discussion of Fine's commitment interpretation of his semantics. We note two interesting results of this discussion. (1) the discussion motivates our system **RTC** which turns out to be equal to the system $\mathbf{R_{123IC}}$ of Urquhart (1973). This provides a philosophical motivation for Urquhart's more technically motivated system. (2) Fine's commitment interpretation is only natural for a small number of logics. For example, this interpretation of the semantic primitives makes no sense if the logic being modeled is **R**. The absence of a natural interpretation of Fine's semantics for **R** reduces the philosophical interest in this semantics *as a semantics for* **R**. On the other hand, the motivatability of other relevance logics on the basis of the commitment interpretation increases the overall philosophical interest of Fine's semantics.

### 6.1. *Fine's Semantics Simplified for* $\rightarrow$ & *Fragments*

DEFINITION 6. A *frame*, $F$, is a quartuple $\langle T, 0, \cdot, \geqslant\rangle$ such that

(1) $T$ is a set, $0 \in T$, $\cdot$ is a binary operation on $T$, and $\geqslant$ is a binary relation on $T$;

(2) $(\forall u, v, w \in T)(\text{if } u \geqslant v \text{ then } u \cdot w \geqslant v \cdot w)$;

(3) $(\forall u \in T)(u = 0 \cdot u)$.

A *Fine-model*, $M$, is a quintuple, $\langle T, 0, \cdot, \geqslant, \phi \rangle$, where $\langle T, 0, \cdot, \geqslant \rangle$ is a frame and $\phi \subseteq T \times Atoms$ (where $Atoms$ is the set of atomic formulas) such that

$$(\forall u, v \in T)(\forall p \in Atoms)(\text{if } \phi vp \text{ and } u \geqslant v \text{ then } \phi up).$$

We say that $M$ is *based on* $F$.

Where there is no possibility of confusion, we write $uv$ for $u \cdot v$, and $uvw$ for $(uv)w$ (to be distinguished from $u(vw)$). We let $u$, $v$, $w$, ... range over members of $T$; $p$, $q$, $r$, ... range over atomic formulas; and $A$, $B$, $C$, ... range over formulas.

DEFINITION 7. Given a Fine-model $M$, the relation, $\vDash$, of *commitment* (this is Fine's terminology) between theories and formulas is defined as follows:

(1) $u \vDash p$ iff $\phi up$;

(2) $u \vDash (A \,\&\, B)$ iff ($u \vDash A$ and $u \vDash B$); and

(3) $u \vDash (A \to B)$ iff $(\forall v)(\text{if } v \vDash A \text{ then } uv \vDash B)$.

We say $M \vDash A$ iff $0 \vDash A$. We say $F \vDash A$ iff ($\forall$ models $M$ based in $F$)($M \vDash A$). We say that $A$ is valid ($\vDash A$) iff ($\forall$ frames $F$)($F \vDash A$). Given a class, $X$, of frames, we say that $A$ is $X$-valid ($X \vDash A$) iff $(\forall F \in X)(F \vDash A)$.

### 6.2. *Results Concerning Fine's Semantics*

Fine shows that the validity of a formula is equivalent to its theoremhood in a certain minimal relevance logic $\mathbf{B}_{\to\&}$, which can be axiomatized as follows:
(Note: we borrow the numbering for axioms and rules from §3.)

Axioms:

1. $A \to A$
2. $A\&B \to A$
3. $A\&B \to B$
4. $(A \to B)\&(A \to C) \to .A \to (B\&C)$

Rules:

| | | |
|---|---|---|
| 12. Modus Ponens (MP): | from $A$ and $A \to B$ | to infer $B$ |
| 13. Conjunction: | from $A$ and $B$ | to infer $A\&B$ |
| 14. Prefixing: | from $A \to B$ | to infer $(C \to A) \to (C \to B)$ |
| 15. Suffixing: | from $A \to B$ | to infer $(B \to C) \to (A \to C)$ |

Fine also considers adding a number of postulates to $\mathbf{B}_{\to\&}$ and a number of corresponding conditions on frames. Table III expands his list with Postulates 6–9. (The sense in which the postulates correspond to the conditions is given in Theorem 6, which follows Table III.)

TABLE III

Additional postulates and conditions (Where possible, we borrow the numbering from §3. Also, if $n = 0$, then $xw_1 \cdots w_n = x$.)

| | *Postulate* | *Condition* |
|---|---|---|
| 5. | $(A \to B)\&(B \to C) \to (A \to C)$ | $uv \geqslant u(uv)$ |
| 6. | $((A \to A) \to .H_1 \to \cdots \to .H_n \to .B \to C) \to (H_1 \to \cdots \to .H_n \to .D \to B) \to .H_1 \to \cdots \to .H_n \to .D \to C$ | $uvw_1 \cdots w_n s \geqslant u0w_1 \cdots w_n(vw_1 \cdots w_n s)$ |
| 7. | $((A \to A) \to .H_1 \to \cdots \to .H_n \to .(B \to C)) \to (H_1 \to \cdots \to .H_n \to .C \to D) \to .H_1 \to \cdots \to .H_n \to .B \to D$ | $uvw_1 \cdots w_n s \geqslant vw_1 \cdots w_n(u0w_1 \cdots w_n s)$ |
| 8. | $((A \to A) \to .H_1 \to \cdots \to .H_n \to .(B \to C)) \to (H_1 \to \cdots \to .H_n \to .B) \to .H_1 \to \cdots \to .H_n \to C$ | $uvw_1 \cdots w_n \geqslant u0w_1 \cdots w_n(vw_1 \cdots w_n)$ |
| 9. | $((A \to A) \to .H_1 \to \cdots \to .H_n \to B) \to (H_1 \to \cdots \to .H_n \to .B \to C) \to .H_1 \to \cdots \to .H_n \to C$ | $uvw_1 \cdots w_n \geqslant vw_1 \cdots w_n(u0w_1 \cdots w_n)$ |
| 10. | $(A\&(A \to B)) \to B$ | $u \geqslant uu$ |
| 11. | $(B \to C) \to .(A \to B) \to (A \to C)$ | $(uv)w \geqslant u(vw)$ |
| 16. | $(A \to B) \to .(B \to C) \to (A \to C)$ | $(uv)w \geqslant v(uw)$ |
| 17. | $(A \to (B \to C)) \to (A\&B \to C)$ | $uv \geqslant (uv)v$ |
| 18. | from $A$ to infer $(A \to B) \to B$ | $u \geqslant u0$ |
| 19. | $A \to ((A \to B) \to B)$ | $uv \geqslant vu$ |

(All these postulates may be referred to as axioms, except Postulate 18, which may be referred to as Rule 18.)
Using methods from Fine 1974, we can show that the additional postulates *correspond* to the additional conditions in the following rather strong sense:

THEOREM 6. *Suppose that* **P** *is a subset of the set of additional postulates and that* **C** *is the corresponding set of conditions, i.e. Postulate* $k \in \mathbf{P}$ *iff Condition* $k \in \mathbf{C}$. *Suppose that* **L** *is the logic which results from adding the postulates in* **P** *to the axiomatization of* $\mathbf{B}_{\to\&}$. *And suppose that* $X$ *is the class of frames for which the conditions in* **C** *hold. Then a formula* $A$ *is* $X$*-valid iff* $A \in \mathbf{L}$.

*Note.* We have available to us another formal (and mathematically useful) semantics for the four systems which we have so far investigated: an **RC1**-frame is one that satisfies the additional conditions 5–7; an **RC01**-frame is one that satisfies the additional conditions 5–10 (and so also 16–18); an **RC1+**-frame is one that satisfies the additional conditions 5–7, and 11; an **RC0+**-frame is one that satisfies the additional conditions 5–11 (and so also 16–18). **RC0+**-frames can also be characterized by conditions 5, 11, 17 and 18.
Note: Conjecture 1 (§3.1) can be reduced to the claim that the set of **RC1**-frames (**RC01**-frames, **RC1+**-frames) cannot be characterized by a finite set of first-order formulas, where we only allow quantification over members of $T$.

### 6.3. *Interpreting Fine's Semantics*

By way of interpretation of the semantic primitives introduced here, Fine (1974) provides us with this (pp. 348–9):

> $T$ is the set of all *theories*, i.e. sets of propositions closed under commitment.
>
> 0 is *logic*, i.e. the theory that comprises all and only the logical truths. (An alternative which Fine does not consider is that 0 is *truth*, i.e. the theory that comprises all and only truths.)
>
> $uv$ the *closure of* $v$ *under* $u$: the set of propositions $P$ such that $u$ commits one to the proposition that $v$ commits one to $P$.

$\geqslant$ is the relation of *inclusion* on theories.

$\phi$ is a *valuation.*

Consider the interpretation of $uv$. We are to interpret $uv \models A$ ($uv$ is committed to $A$) as $u \models (v \models A)$ ($u$ is committed to $v$ being committed to $A$). This is reminiscent of our own reading, in the semantics for non-relevant commitment entailment, of $ab \models A$, where $a, b \in S^{<\omega}$, and, in particular, of our reading of $\langle \alpha, \beta \rangle \models A$, where $\alpha$ and $\beta$ are persons; see §2.2 above.

Now consider the definition of $u \models (A \to B)$, in §6.1, Definition 7. According to this definition, if $u \models (A \to B)$, and if $v \models A$, then $u \models (v \models B)$. Something here seems wrong (even if we are not constrained to giving "$\to$" a commitment reading). Suppose that $u$ is committed to $(p \to q)$ and that $v$ is committed to $p$, but that $u$ has no commitments concerning (or access to) whether $v$ is committed to $p$. Then (supposing for a moment that $u$ is a person) we could not expect $u$ to draw the conclusion that $v$ is committed to $q$. (Indeed, depending upon the circumstances, we might positively insist that $u$ *not* draw such a conclusion.) And so we cannot draw the conclusion that $u$ is committed to $v$ being committed to $q$.

This argument works if we think of our committed entities as *persons.* If person $\alpha$ is committed to $(A \to B)$ then whether $\alpha$ is committed to person $\beta$ being committed to $B$ depends not on $\beta$'s *actual* commitments, but rather on the commitments which $\alpha$ takes $\beta$ to have. On the other hand, there is a sense in which, if $\alpha$ is committed to $(A \to B)$ and $\beta$ is committed to $A$, then $\alpha$'s *theory* is committed to $\beta$ being committed to $B$, though $\alpha$ himself might not be committed to $\beta$ being committed to $B$.

Suppose, for example, that logician $\alpha$ (unwisely) believes that material implication is the correst theory of commitment entailment; so $\alpha$ is committed to $(A \& \sim A) \to B$. Suppose also that there is a contradiction hidden in $\beta$'s theory of class struggle, though $\alpha$ is unaware that $\beta$ even has a theory of class struggle. While it might be too much to say that $\alpha$ is committed to $\beta$ being committed to $B$ (for any $B$), one could deduce "$\beta$ is committed to $B$" from $\beta$'s other commitments and from $\alpha$'s theory. There is a sense in which $\alpha$'s *theory* is committed to "$\beta$ is committed to $B$", although $\alpha$ isn't. We do not claim that this is the *only* sense that could be given to the phrase "$\alpha$'s theory is committed to '$\beta$ is committed to $B$' ". We only claim that it is *a sense* which could be given and that the resulting concept of theoretical commitment might prove to be a useful one. Everything depends upon what the notion of theory commitment is meant to *do*; on what turns on the question of what assertions a theory is committed to.

So, if we think of our committed entities as *theories* rather than *persons*, Fine's definition of $u \vDash (A \rightarrow B)$ makes sense. And Fine does think of his committed entities as theories.

What conditions should be placed on the primitives of Fine's semantics? Here, we want to motivate our choices with his suggested interpretations, rather than with an already predetermined logic which we are trying to model. In what follows, we consider the various conditions already suggested. We include some conditions as obligatory; we rule some out as implausible or unmotivated; and we take some to be plausible but not obligatory.

To begin with, the conditions listed in Definition 6 seem reasonable enough, and so we include them as obligatory.

On the other hand, condition 19 (Table III) seems just wrong: we see no reason to say that if $v$ is committed to $u$ being committed to $p$ then $u$ is committed to $v$ being committed to $p$. This rules out stronger relevance logics like $\mathbf{R}_{\rightarrow\&}$ as logics of the kind of commitment entailment under consideration. Similarly, it is plausible to rule out any conditions which, like condition 19, involve some kind of commutativity. This rules out conditions 7, 9, and 16. Finally there is little to motivate conditions 6 and 8, unless we already have one of **RC1**, **RC01**, **RC1+** or **RC0+** in mind. And so we rule out 6 and 8 as well.

There are some additional conditions which *are* plausible (some even obligatory). First, consider the appropriate readings of $(uv)w \vDash A$ and of $u(vw) \vDash A$. Note: $(uv)w \vDash A$ iff $(uv) \vDash (w \vDash A)$ iff $u \vDash (v \vDash (w \vDash A))$ iff $u \vDash (vw \vDash A)$ iff $u(vw) \vDash A$. And so we include, as obligatory, this condition: $(uv)w = u(vw)$. Now, condition 11 (Table III), is $(uv)w \geqslant u(vw)$. So we include condition 11 *and* its "converse", condition 11′: $u(vw) \geqslant (uv)w$. (Note: this leads us to accept axiom 11 and to reject axiom 16, which are, respectively, a "left-hand" and a "right-hand" version of the strong transitivity axiom. It seems that, despite social predjudice to the contrary, left-handedness is better than right.)

Secondly, consider the appropriate reading of $u0 \vDash A$. $u0 \vDash A \Leftrightarrow u \vDash (0 \vDash A) \Leftrightarrow u \vDash$ ($A$ is a theorem) – or $u \vDash$ ($A$ is true), depending on whether we take 0 to be *logic* or *truth*. In either case, $u0 \vDash A \Rightarrow u \vDash A$. So, we include as obligatory condition 18: $u \geqslant u0$. What about its "converse", condition 18′: $u0 \geqslant u$? If we take 0 to be *logic* then we do not want it: $u \vDash$ ($A$ is a theorem) is not inferrable from $u \vDash A$. If we take 0 to be *truth* then we do want it: $u \vDash$ ($A$ is true) *is* inferrable from $u \vDash A$. And so condition 18' is plausible but not obligatory.

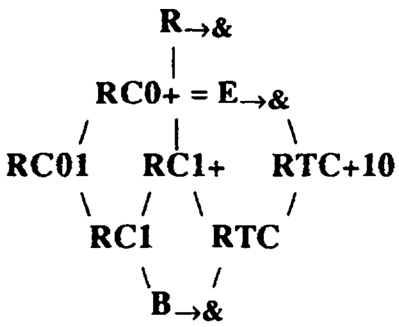


Fig. 2. The relative place of relevant commitment logics and relevant theory commitment logics.

Finally, given conditions 11 and 11′, conditions 5 and 17 follow from condition 10, and lack motivation without it. Note that $(u \vDash A \Rightarrow uu \vDash A)$ just in case commitment to $A$ brings with it commitment to being committed to $A$. Similarly $(uu \vDash A \Rightarrow u \vDash A)$ just in case commitment to being committed to $A$ brings with it commitment to $A$. (These are the two halves of the "C–C thesis" discussed in LSLC I.) So we count both condition 10, $u \geqslant uu$, and condition 10′, $uu \geqslant u$, as plausible but not obligatory.

These considerations provide us with eight semantic accounts of "theory commitment", depending on whether we accept or reject each of conditions 10, 10′ and 18′. We name the minimal such logic suggested by these accounts, **RTC** (for "relevant theory commitment"). That is, **RTC** is the set of formulas validates by all models satisfying conditions 11, 11′ and 18. We simply name the other logics **RTC+10**, **RTC+10′+18′** and so on, depending on which among conditions 10, 10′ and 18′ we insist upon. It turns out that we have fewer than eight distinct logics, given the following theorem.

THEOREM 7. (i) **RTC** = **RTC+18′**

(ii) **RTC** = $\mathbf{R}_{123\mathrm{IC}}$ *of Urquhart (1973).*

CONJECTURE 2. (i) **RTC+10′+18′** = **RTC+10′** = **RTC**.

(ii) **RTC+10** = **RTC+10+10′** = **RTC+10+18′** = **RTC+10+10′+18′**.

(iii) **RTC+10** = $\mathbf{R}_{1235\mathrm{IC}}$ *of Urquhart (1973).*

*Note.* It turns out to be difficult to axiomative **RTC+10**. There are several open questions here.

Axiomatizations:

**RTC** is axiomatized with Axioms 1–4, 11 and Rules 12, 13 and 18.

## 7. CONCLUDING REMARKS

We have been investigating the behaviour of "$\rightarrow$", and its interaction with the rather straightforward connective, "&" (conjunction). The next obvious step is to see how "$\rightarrow$" interacts with such connectives as "$\vee$" (disjunction) and "$\sim$" (negation). There are reasons for trepidation. Firstly, our project is a "relevance" project. Secondly, though our formal motivation has been primarily proof-theoretic, our project has been driven by an intuitive interpretation (and so an intuitive semantics) – which itself has, happily, been amenable to formalization. Unfortunately, in the earlier years of the semantic enterprise in relevance logic, "$\vee$" and "$\sim$" proved particularly recalcitrant. Although formal semantics were developed which could model their behavior, natural intuitive interpretations of these semantics are hard to come by. And so, where informal semantics meets formal proof theory and formal semantics, our intuitions concerning these connectives get fuzzy.

On the positive side, it should not be presumed that these connectives behave in the same way in relevant commitment logics as they do in other relevance logics. For example, while $(A \rightarrow \sim A) \rightarrow \sim A$ is a theorem of many of the stronger relevance logics, it is hard to motivate on our interpretation of "$\rightarrow$". All in all, we expect that upon further investigation of these connectives, our relevant commitment logics will diverge more and more from antecedently studied relevance logics (and our non-relevant commitment logics will diverge from antecedently studied modal logics).

Proof of the technical claims made throughout this article may be obtained from either author.

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