ECO220Y

Hypothesis Testing: Population Mean, μ

Readings: Chapter 13, 13.5-13.6

Winter 2012

Lecture 16

Building Blocks of Hypothesis Testing: Review

- Test-statistic - numerical value from sample
- Critical value - numerical value from table
- P-value - probability computed from table
- Significance level - probability set up before test

Idea of Formal Testing of μ

- H_0 is initial presumption about population
  - H_0 is not based on any evidence
  - Is sample plausible if H_0 is true?
    - If sample is not plausible, reject H_0 and infer H_A is true
- Test-statistic - A random variable created using sample statistic
  - When testing μ, test-statistic is a function of $\bar{X}$
  - Recall that when testing p, test-statistic is a function of $\hat{p}$

Example: Parking Fees

Hoping to lure more shoppers downtown, a city builds a new public parking garage in the central business district. The city plans to pay for the structure through parking fees. The city would break even only if the average parking revenues are greater than $125. For a random sample of 44 weekdays, daily fees collected averaged $126. Assume that the standard deviation is known to be $15.

$H_0 : \mu = 125$
$H_A : \mu > 125$

n=44

population standard deviation, $\sigma = 15$

significance level, $\alpha = 0.05$

sample mean, $\bar{X} = 126$

Rejection Region

Unstandardized

Critical value when $\alpha = 0.05$

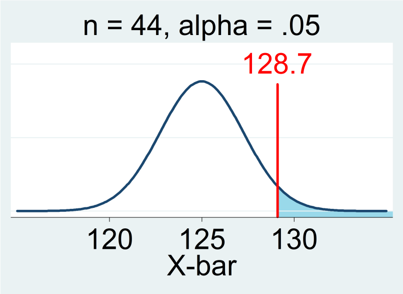
$\bar{X}_{critical} =$

$\bar{X}_{critical} =$

Rejection region:

Conclusion if $\bar{X} = 126$?

What if $\bar{X} = 129$?



Rejection Region

Standardized

Critical value when $\alpha = 0.05$

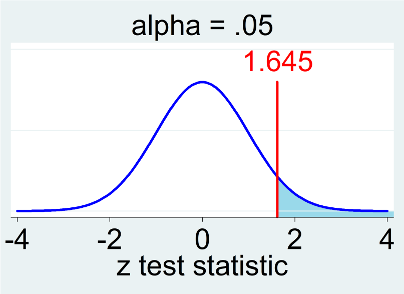
$z_\alpha =$

z test statistic$=\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$

z test statistic$=$

Rejection region:

Conclusion if z t-statistic=0.44?



For what values of z we should reject H_0?

P-value

Significance level, $\alpha = .05$

P-value=P($\bar{X}$ >sample statistic)

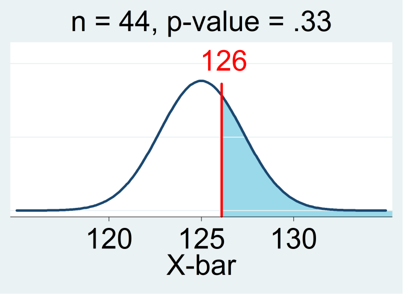
P-value=P($\bar{X} > 126$)

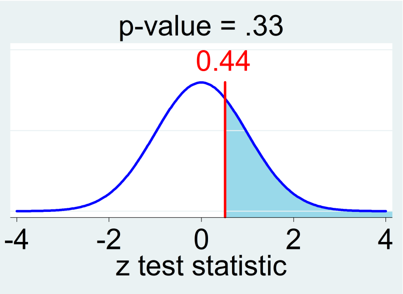
P-value=

P-value=

Conclusion?

What if $\bar{X} = 129$?





Probability of Type II error

Significance level, $\alpha = .05$

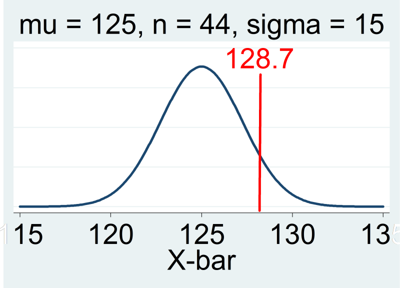
Parameter value under H_A=129

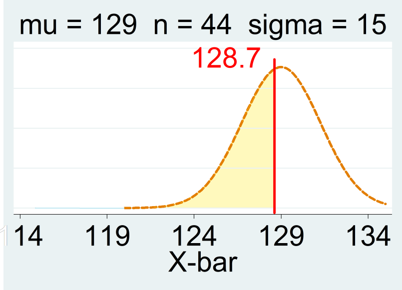
P(Type II error)=

P($\bar{X}$ <critical value$|\mu_0 = 129$)

β=

Power=1-β=





Link CI Estimator and Hypothesis Testing

- Confidence interval is centered at $\bar{X}$
- 90% confidence interval, $\alpha = .10$
- CI: $\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
- LCL:
- UCL:
- Parameter value under H_0

What if σ is unknown?

- If population standard deviation, σ is unknown, we use our best guess to estimate it - sample standard deviation, s
- We can no longer rely on normal distribution as a sampling distribution of sample mean
- Instead, we use Student t distribution (Review Lecture 12)
- Test statistic is distributed according to Student t, not standard normal

$$\text{Test statistic} = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

Rejection region with t

One-sided test

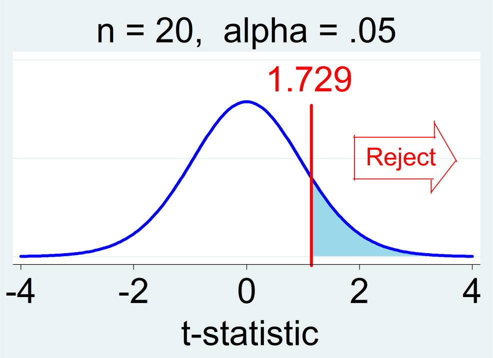
$H_0 : \mu = \mu_0$

$H_A : \mu > \mu_0$

Rejection region:

$t > t_\alpha$

Test-statistic$=\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$



Rejection region with t

One-sided test

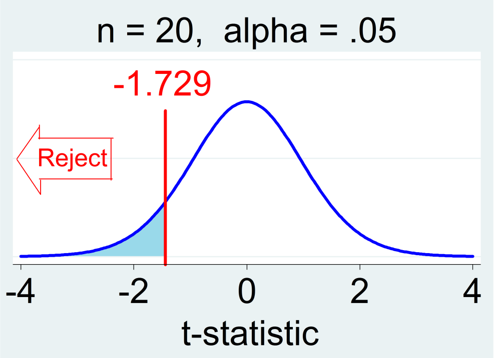
$H_0 : \mu = \mu_0$

$H_A : \mu < \mu_0$

Rejection region:

$t < -t_\alpha$

Test-statistic$=\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$



Rejection region with t

Two-sided test

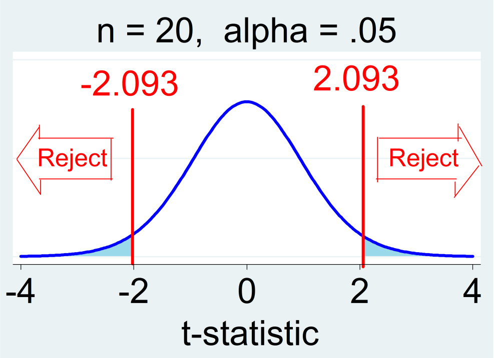
$H_0 : \mu = \mu_0$

$H_A : \mu \neq \mu_0$

Rejection region:

$t < -t_\alpha$ and $t > t_\alpha$

Test-statistic$=\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$



p-value with t

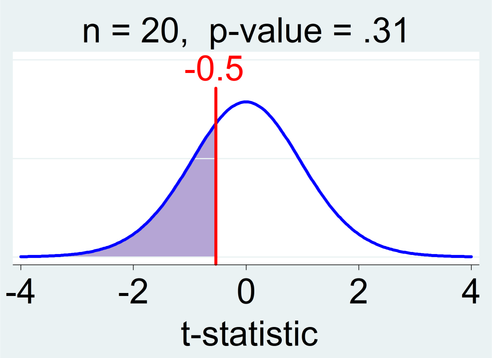
One-sided test

$H_0 : \mu = \mu_0$

$H_A : \mu < \mu_0$

Test-statistic$=\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$

P-value$=P\left(t < \frac{\bar{X}-\mu_0}{s/\sqrt{n}}\right)$



p-value with t

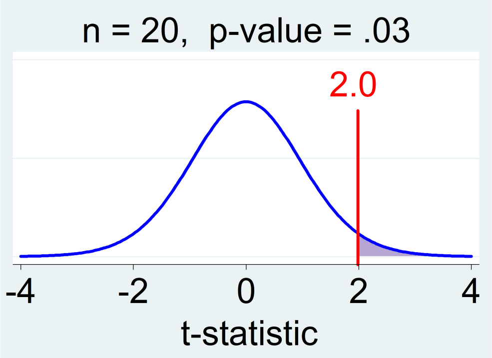
One-sided test

$H_0 : \mu = \mu_0$

$H_A : \mu > \mu_0$

Test-statistic$=\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$

P-value$=P\left(t > \frac{\bar{X}-\mu_0}{s/\sqrt{n}}\right)$



p-value with t

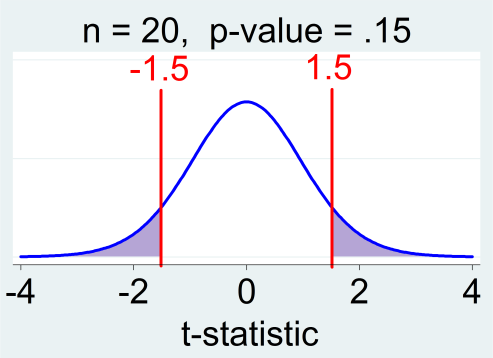
One-sided test

$H_0 : \mu = \mu_0$

$H_A : \mu \neq \mu_0$

Test-statistic$=\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$

P-value$=2 * P\left(t > \frac{\bar{X}-\mu_0}{s/\sqrt{n}}\right)$



Example: Parking Fees

$H_0 : \mu = 125$
$H_A : \mu > 125$

n=44

sample standard deviation, $s = 15$

significance level, $\alpha = 0.05$

sample mean, $\bar{X} = 126$

How is the test-statistic distributed in this example?

Rejection region with t

One-sided test

$H_0 : \mu = \mu_0$

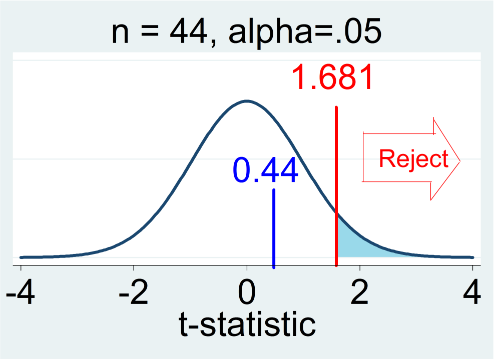
$H_A : \mu > \mu_0$

Rejection region: $t > t_\alpha$

$t_{\alpha,\nu} = t_{.05,43} = 1.681$

Test-statistic $= \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$

Test-statistic $= \frac{126-125}{15/\sqrt{44}} = 0.44$



p-value with t

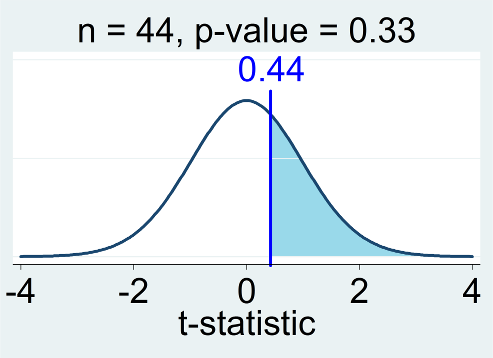
One-sided test

$H_0 : \mu = \mu_0$

$H_A : \mu > \mu_0$

P-value$= P\left(t > \frac{\bar{X} - \mu_0}{s/\sqrt{n}}\right)$

P-value$= P(t > 0.44) = 0.33$



Can find p-value from our table?