# ECO220Y
## Introduction to Probability
## Readings: Chapter 6

Fall 2011

Lecture 6
Part 1 of 1

# Historical Roots of Probability

- Probability was used to study the games of chance
- The story of the Chevalier de Mere
  - Tried luck with two games
  - Roll a single die 4 times and bet on getting a six
  - Roll two dice 24 times and bet on getting a double six
  - The Chevalier ended up loosing badly on the second gamble
- Blaise Pascal (1623-1662) discovered a fundamental principle for assessing the probability of a certain event
- 

# Definitions

- A random experiment is the process of observing an outcome of a chance event
- An outcome is a realization of a random experiment
  - Mutually Exclusive
  - Exhaustive
- Sample space, $S$, is a collection of *all* possible outcomes
- An event is a collection of particular outcomes

# Examples

- Roll a Die
  - A random experiment $\longrightarrow$ Roll a Die!
  - All elementary outcomes$\longrightarrow$1, 2, 3, 4, 5, 6
  - Sample space, $S \longrightarrow \{1, 2, 3, 4, 5, 6\}$
  - Event A = "More than 4"$=\{5,6\}$
- Toss a Coin (Fair Coin!)
  - A random experiment $\longrightarrow$ Toss a Coin!
  - All possible outcomes $\longrightarrow$ Head and Tail
  - Sample space, $S \longrightarrow \{H,T\}$
  - Event "Win if Head"$=\{H\}$

# Different Types of Probability - Part I

- Subjective probability - an individual's assessment of the likelihood of a certain event
  - Based on one's own experience
  - The less accurate of all types
- Theoretical probability
  - Based on mathematical model, $P(\text{Event A}) = \frac{\#\text{ of outcomes in A}}{\text{Total }\#\text{ of outcomes}}$
  - Fair coin - equal chances of head and tail
  - Deck of card - can compute probability of randomly selecting each card
- Empirical probability - relative frequency of event's occurrence in the long-run
  - Based on repeatedly observing the event's outcome
  - We observe that from year to year the fraction of second-year students who take ECO220 is 70%.
  - Can write it as a fraction $\frac{70}{100}$ or a decimal, 0.7

# Probability Rules

1. For any event A, $0 \leq P(A) \leq 1$
   - Probabilities are never negative.
   - A probability of zero means an event cannot happen. Less than zero would be meaningless.
   - If event is certain to happen, we assign it probability 1.
2. $P(S) = 1$
   - The total probability of the sample space must be 1.
   - If we conduct an experiment, something is bound to happen.
3. $P(A) = 1 - P(A^C)$
   - Complement rule
   - $A^C$ is a complement of $A$, or $A$ is not occurring, or "not" $A$

# Event

- Recall: An event is a set of elementary outcomes.
- The probability of event is the sum of the probabilities of the elementary outcomes in the set.
- We can combine events to make other events!
- For instance, given events A and B, we can make new events:
  1. A OR B $\longrightarrow A \cup B$ - "union" of A and B
     - Addition Rule: $P(A \cup B) = P(A) + P(B)$, if events are disjoint
     - General Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  2. A AND B $\longrightarrow A \cap B$ - "intersection" of A and B
     - Multiplication Rule: $P(A \cap B) = P(A) \times P(B)$ if $A$ and $B$ are independent

# Different Types of Probabilities - Part II

1. Joint Probability
   - Given two events, $A$ and $B$, we would like to know what is the probability that both $A$ and $B$ occur
   - Notation - $P(A \cap B)$
2. Marginal Probability
   - Probability of a single event
   - Notation - $P(A)$
3. Conditional Probability
   - Probability of event $A$ given that event $B$ has already occurred
   - Notation - $P(A|B)$

# Probability - Joint

| | **Cash** | **Credit Card** Event A | **Debit Card** |
|---|---|---|---|
| Under \$20 | .09 | .03 | .04 |
| \$20 - \$100 | .05 | .21 | .18 |
| Over \$100 Event B | .03 | .23 | .14 |

⇑

We call it joint probability

# Probability - Marginal

| | **Cash** | **Credit Card**<br>Event A | **Debit Card** | Marginal Pr |
|---|---|---|---|---|
| Under \$20 | .09 | .03 | .04 | .16 |
| \$20 - \$100 | .05 | .21 | .18 | .44 |
| Over \$100<br>Event B | .03 | .23 | .14 | .40 |
| Marginal Pr | .17 | .47 | .36 | 1 |

$\Uparrow$

We call it marginal probability of event A

# Probability - Conditional

| | **Cash** | **Credit Card** Event A | **Debit Card** | Marginal Pr |
|---|---|---|---|---|
| Under \$20 | .09 | .03 | .04 | .16 |
| \$20 - \$100 | .05 | .21 | .18 | .44 |
| Over \$100 Event B | .03 | .23 | .14 | .40 |
| Marginal Pr | .17 | .47 | .36 | 1 |

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- P(A|B): For a customer who spent over \$100 what is the probability that he/she paid with a credit card?
- $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.23}{0.40} = 0.575$
- P(B|A): For a customer who paid with a credit card what is the probability that he/she spent over \$100?
- $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.23}{0.47} = 0.489$

# Union of Events

| | **Cash** | **Credit Card** | **Debit Card** | Marginal |
|---|---|---|---|---|
| Under $20 | .09 | .03 | .04 | .16 |
| $20 - $100 | .05 | .21 | .18 | .44 |
| Over $100 | .03 | .23 | .14 | .40 |
| Marginal | .17 | .47 | .36 | 1 |

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B) = 0.47 + 0.40 - 0.23 = 0.64$$

# Independent Events

- Events are independent when occurrence of one is independent of another
- Example: toss two coins - are the outcomes related?
- Probability definition: events A and B are independent if:
- $P(A|B) = P(A)$ and $P(B|A) = P(B)$
- What does that imply for the joint probability of $A$ and $B$?

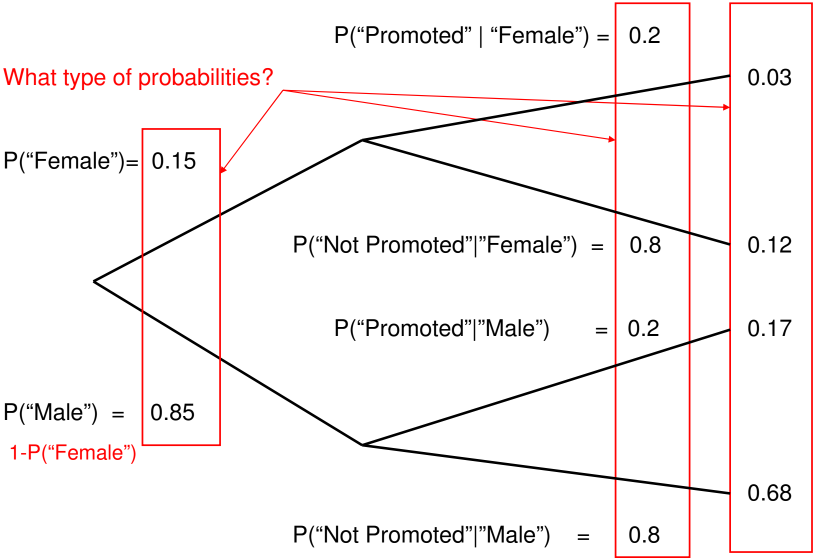
# Gender and Promotion Related?

| | **Promoted** | **Not Promoted** | Marginal |
|---|---|---|---|
| Female | .03 | .12 | .15 |
| Male | .17 | .68 | .85 |
| Marginal | .20 | .80 | 1 |

- Based on this joint probability table, can we conclude that promotion is independent of gender?
- P(Male and Promoted) = .17 (from Table)
- P(Male)*P(Promoted) = .85*.20 = .17 = .17

# Extending Independence to More than 2 Events

- Often, we are interested in a joint probability of more than two events
- We still can apply the multiplication rule for independent events
- $P(A_1 \cap A_2 \cap A_3 \cap ... \cap A_n) = P(A_1) \times P(A_2) \times P(A_3) \times ... \times P(A_n)$
- What is the chance to get ten straight heads tossing a fair coin?
- $P(\text{ten heads in a row}) = P(H_1) \times P(H_2) \times ... \times P(H_{10}) = 0.5^{10} = 0.00098$



What type of probabilities?
P("Female")= 0.15
P("Male") = 0.85
1-P("Female")
P("Promoted" | "Female") = 0.2
P("Not Promoted"|"Female") = 0.8
P("Promoted"|"Male") = 0.2
P("Not Promoted"|"Male") = 0.8
0.03
0.12
0.17
0.68