ECO220Y

Continuous Probability Distributions: Uniform and Triangle

Readings: Chapter 9, sections 9.8-9.9

Fall 2011

Lecture 8 Part 1

Probability Distributions

Discrete ↙

- Binomial (n, p) ✓
- Poisson
- Bernoulli (p) ✓

Continuous ↘

- Uniform (a, b)
- Triangle
- Normal/Standard Normal
- Student t
- F distribution

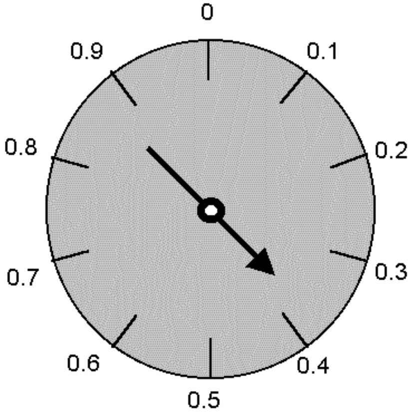
Probability Distributions

- In a discrete probability distribution, the possible outcomes are countable. We use a discrete random variable X and discrete probability distribution $p(x)$. Each of the possible outcomes has a nonzero probability.

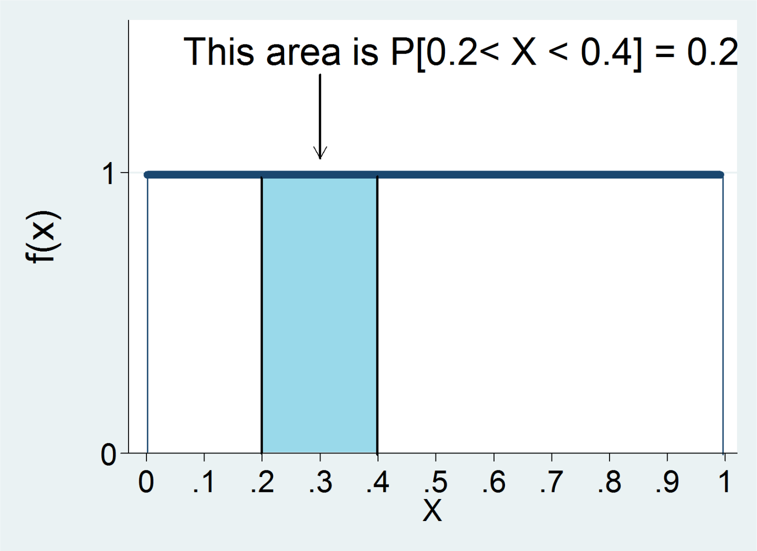
- In a continuous probability distribution, the possible outcome are not countable. We use a continuous random variable X and continuous probability distribution $f(x)$. Each possible outcome has zero probability, while an interval of possible outcomes has a nonzero probability.

A Spinner

A spinner randomly selects a point on a circle. How many points are there on this circle?

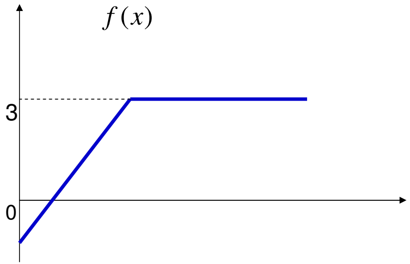


A Spinner

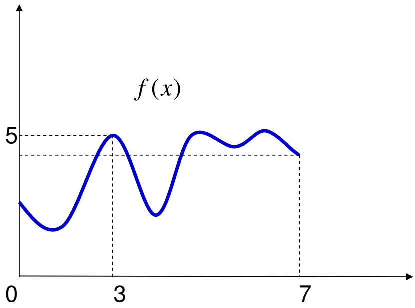


Probability Density Function

- For continuous RV, area under the curve f(x) is the probability of a range of values.
- Height of the function f(x) is not probability! To find probability, need to use calculus to find area under the curve ($\int f(x)dx$).
- Probability density function (pdf) satisfies two conditions:
  1. $f(x) \geq 0$ for all possible values of X.
  2. The total area under the curve is 1 ($\int f(x)dx = 1$)



Is this a valid probability density function?



Is this a valid probability density function?

Uniform Distribution

- All outcomes are equally likely.
- All values have equal chance - 0. (Why?)
- Often referred as Rectangular distribution because the graph of the pdf has the form of a rectangle.
- $P(X < x) = P(X \leq x)$. Why?

Uniform Probability Distribution

Uniform PDF:

$$f(x) = \frac{1}{b-a}$$

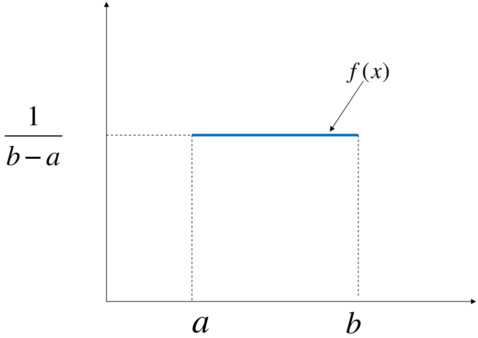
where $a \leq x \leq b$

are parameters and

$[a, b]$ - bounded support

Intuition for

the formula of f(x)?



Uniform Probabilities

$P(X = 2) =$

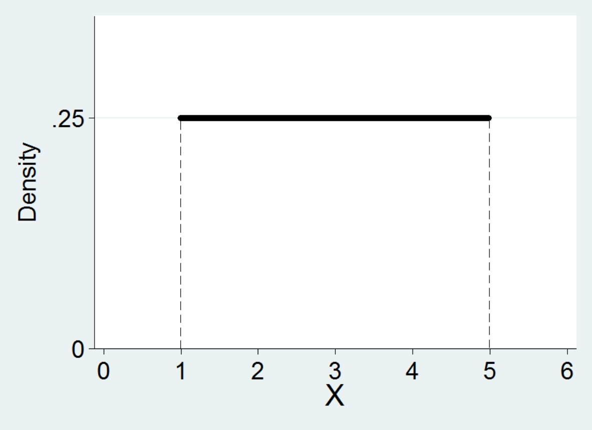
$P(X \leq 2) =$

$P(X < 2) =$

$P(X \geq 5) =$

$P(X \geq 4) =$

$P(3 \leq X \leq 4) =$



Uniform Probabilities

$P(X = 2)=0$

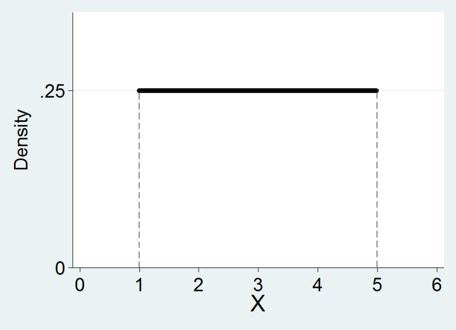
$P(X \leq 2)=1*0.25=0.25$

$P(X < 2)=1*0.25=0.25$

$P(X \geq 5)=0$

$P(X \geq 4)=1*0.25=0.25$

$P(3 \leq X \leq 4)=1*0.25=0.25$



Mean and Variance of Uniform RV

For Uniform RV X $\sim$ U[a,b]

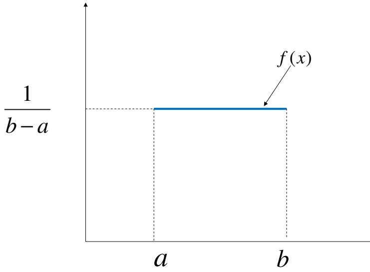
($\sim$ in statistics means "distributed")

$\mu = \frac{a+b}{2}$

$\sigma^2 = \frac{(b-a)^2}{12}$

Note: To derive μ and σ^2,

we need to use integral calculus.



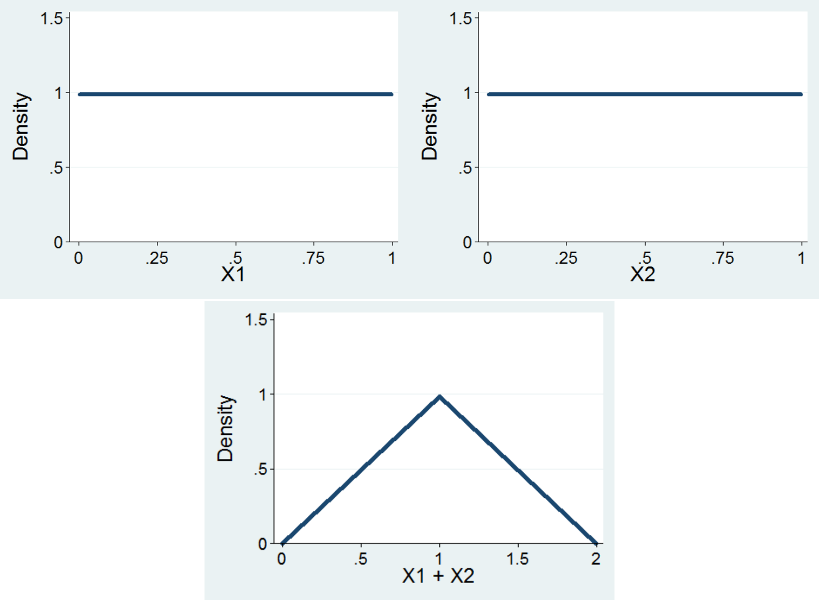
Triangle Distribution

We can create triangle distribution by adding up two independent and identically distributed uniform random variables.

Why do we need independence?
What does identically distributed mean?

$X1 \sim U[a, b]$
$X2 \sim U[a, b]$
X1 and X2 are independent
$T = X1 + X2$
$T \sim T[2a, 2b]$

Triangle Distribution



Triangle Distribution: Mean and SD

Mean and Variance for Uniform Distribution U[a, b]:

$$\mu = \frac{a+b}{2} \qquad \sigma^2 = \frac{(b-a)^2}{12}$$

Mean and Variance for Triangle Distribution T[2a, 2b]?

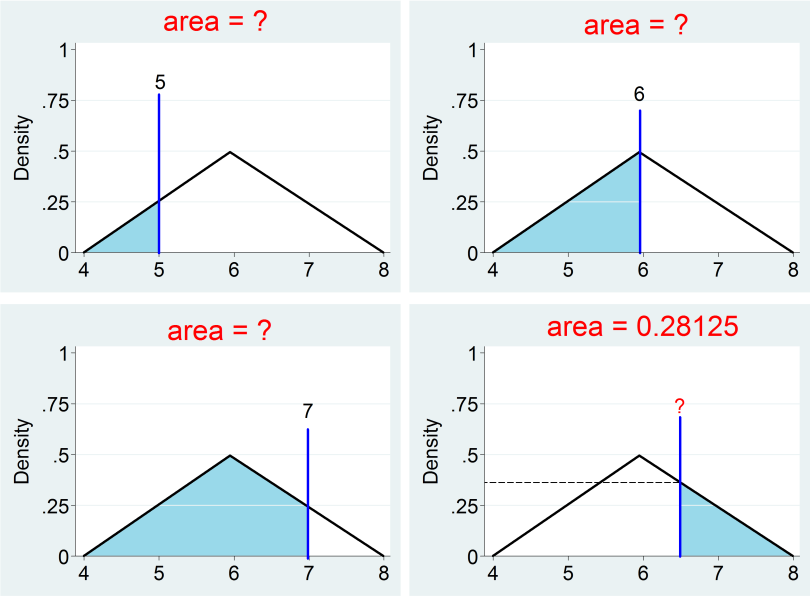
$$\mu = ? \qquad \sigma^2 = ?$$

Let X_1 and X_2 be two identically and independently distributed random variables such that $X_1 \sim U[a, b]$.

$$\mu_{x_1+x_2} = E[X_1 + X_2] = E[X_1] + E[X_2] = \frac{a+b}{2} + \frac{a+b}{2} = a + b$$

$$\sigma^2_{x_1+x_2} = V[X_1 + X_2] = V[X_1] + V[X_2] = \frac{(b-a)^2}{12} + \frac{(b-a)^2}{12} = \frac{(b-a)^2}{6}$$

What property of two uniformly distributed random variables have we used to derive the mean and variance of triangle distribution? Have we used laws of expectation and variance?


area = ?
area = ?
area = ?
area = 0.28125
Density
5
6
7
?
0
.25
.5
.75
1
4
5
6
7
8

Summary: Uniform and Triangle

1. Uniform
   - Symmetric, rectangle-shaped, even density
   - Parameters: a and b
   - Bounded support [a, b]
   - Find probabilities P(x_1 <X< x_2) with A=base*height
   - $\mu = \frac{a+b}{2}$ and $\sigma^2 = \frac{(b-a)^2}{12}$
2. Triangle
   - Symmetric, triangle-shaped, more density around the mean
   - Parameters: 2a and 2b
   - Bounded support [2a, 2b]
   - Find probabilities P(x_1 <X< x_2) with A=1/2*base*height
   - $\mu =$ and $\sigma^2 =$

Consider two identically and independently distributed random variables X and Y, such that X$\sim$U[-2,4]. What is the mean and standard deviation of X+Y?

(A) -4 and 5.65
(B) -4 and 8
(C) -2 and 4
(D) 2 and 2.45
(E) 2 and 6

Consider two identically and independently distributed random variables X and Y, such that X∼U[-2,4]. What is the mean and standard deviation of X+Y?

(A) -4 and 5.65
(B) -4 and 8
(C) -2 and 4
(D) 2 and 2.45
(E) 2 and 6

X+Y∼ T[-4,8]
E[X+Y]=a+b=-2+4=2
V[X+Y]=$\frac{(b-a)^2}{6} = \frac{(4+2)^2}{6} = 6$
SD[X+Y]=$\sqrt{V[X+Y]} = \sqrt{6} = 2.45$