ECO220Y: Homework, Lecture 6 - SOLUTIONS

(1)

(a) Conditional. P(work|woman)=0.65

(b) Conditional. P(woman|work)=0.45

(c) Conditional. P(do not have English|Commerce program)=0.66

(d) Conditional. P(minor Philosophy|major Economics)=0.25

(e) Marginal.P(part-time)=0.15

(f) Conditional. P(part-time|Uof T student)=0.15

(g) Conditional. P(male|tenured)=0.90

(h) Joint. P(fail ECO100 and the qualifying exam)=0.05

(i) Conditional. P(fail qualifying exam|failed ECO100)=0. 90

(j) Marginal. P(dislike green)=0.10

(k) Conditional. P(dislike green | dislike red)= 0.05

(l) Conditional. P(ECO200 | ECO220)=0.60

(m) Joint. P(ECO220 and ECO200)=0.70

(2)

(a) Yes, that would be a correct interpretation of these conditional probabilities. Conditional on spending smaller amounts, customers are more likely to pay by cash than by a credit card. We can put it differently by saying that for a random customer who spent under $20 dollars the chance that he/she paid by cash is much higher than that he/she paid by a credit card.

(b) No, that would be an incorrect interpretation of these conditional probabilities. These conditional probabilities show that those who paid by a credit card are less likely to spend smaller amounts than those who paid by cash. That does not mean that of those customers who spent under $20 fewer paid by a credit card.

(3) Suppose you draw 4 cards from deck and all are ♥'s.

(a) Yes because events are independent.
(b) No, because events are not independent.

(4) Let's first solve for the probabilities of each of two outcomes Chevalier de Mere used to bet on.

(a) Probability of rolling <u>at least</u> 1 six in 4 throws is equivalent to probability of rolling six in only one throw or in two of the throws, or in three of the throws, or in all four throws of the dice. In terms of notation we learned in Lecture 6, it means that we are interested in the union of 4 events. The addition rule tells us to add up marginal probability of each of 4 events and subtract the joint probability. This is quite a cumbersome task. But we should realize that rolling a 6 and rolling any other number are two complementary events. So that P(rolling 6)=1-P(not 6)=1-P(rolling any other number) and can be found by the subtraction rule. We know that probability of rolling any number other than 6 is 5/6, and each roll of a die is independent of

another roll (dice do not have memory, right?). Thus, we can use multiplication rule for independent events:
P(Not 6 and Not 6 and Not 6 and Not 6)= $(5 / 6)^4$ =0.4823.
P(at least one 6)=1 – P(Not 6 and Not 6 and Not 6 and Not 6)=1-0.4823=0.5177.
So, that the odds are in favour of the bettor.

(b) Probability of rolling at least one double-six in 24 throws of two dice is equivalent to rolling one double-six in one throw of two dice, or in two throws, or in three throws, ..., or in twenty four throws of two dice. This is a union of events and we need to use addition rule. If we were to use the addition rule without realizing that we can use instead complement rule, the computation would be even worse than in the first case. If we realize that P(at least one double-six)=1-P(not double-six in all 24 rolls). Probability of "Not double-six" is 35/36 (all other outcomes). Probability "not double-six in all 24 rolls" is then $(35/36)^{24}$=0.5086. The complement of this event, "at least one double-six in 24 rolls" is 1-0.5086=0.4914.

So, in this case the odds are not in favour of the bettor. And that is why the Chevalier de Mere used to lose badly on the second bet.

(c) The Chevalier de Mere knew that the probability to roll six was 1/6 in one single throw. That is why he supposed that his chance to roll six in four rolls was 1/6*4=4/6=2/3. He also knew that the probability to roll double-six in a single roll of two dice was 1/36 (recall that the number of outcomes for two-dice roll is 36). So he presumed his chance to win 1/36*24=24/36=2/3 which was equal to his chance in the first game. The Chevalier de Mere made a mistake in his presumption that the winning probability was the same for both dice games.