

ECO220Y: Homework, Lecture 7

Readings: Chapter 9, sections 9.4 and 9.6

Exercises: 3, 5, 6, 7, 9, 13, 16, 18, 23, 25-28

Problems:

- (1) What two factors affect the probability of any particular number of successes in a Binomial experiment? For example, what would affect the $P(X = 1)$? Make sure to explain the intuition.
- (2) Would the following be good examples of a Binomial random variable: number of hearts drawn in 5 draws with replacement from a standard 52 card deck; number of hearts drawn in 5 draws with replacement from a standard 52 card deck without replacement?
- (3) For which (finite) number of tosses of a fair coin is the probability of getting exactly 50% heads the biggest? [Hint: Your answer should be of the form: $n = \#$, where $\#$ is a number.] Explain and show work.
- (4) Consider doing a Binomial Experiment and defining a new random variable Y as the fraction of successes in the n trials. For example, if there are 3 successes in 5 trials, Y would be 0.60 whereas the Binomial random variable X that we learned in Lectures 13-14 would be 3. Is Y a discrete random variable? Explain. What are the mean and variance of Y ?
- (5) For a Binomial random variable with $n = 14$ and $p = 0.5$ what is the probability of obtaining a value of X within one standard deviation of the mean?
- (6) Describe the shape of the Binomial distribution in the following cases. For Parts (c) and (e) illustrate your answer by choosing parameter values and graphing the distribution by hand.
- (a) p is 0.5 and n is small
 - (b) p is 0.5 and n is large
 - (c) p is less than 0.5 and n is small [Reminder: Illustrate with a graph]
 - (d) p is less than 0.5 and n is large
 - (e) p is more than 0.5 and n is small [Reminder: Illustrate with a graph]
 - (f) p is more than 0.5 and n is large
- (7) Draw a graph of a Binomial distribution if $n = 6$ and $p = 0.2$. Carefully label it. What is the mean and variance? Explain how the shape would change if $n = 600$? What would be the new mean and variance?
- (8) You are told that 12 percent of airline delays are caused by mechanical issues with the airplane. You randomly sample 25 flights and find that 5 (20%) are delayed by mechanical issues with the airplane. What is the chance that you observed this many or more delayed for a mechanical reason if the original claim is true? Do you think that your sample is statistically plausible? [Hint: Be careful: The question is not asking for the probability that exactly 5 out of 25 are delayed.]