ECO220Y: Homework, Lecture 7 - SOLUTIONS

(1) The number of trials and the probability of success. Go over how this relates to the example and provide the intuition.

(2) The first example would be Binomial but the second case would not. Drawing cards without replacement would lead to a violation of the independence requirement for a Binomial Experiment.

(3) Answer: n = 2. With two tosses you have a 50% chance of getting 50% heads (HT or TH). Each possible outcome has a 25% chance: HH, HT, TH, TT.

For n = to any odd number (1, 3, 5, …) there is a 0% chance of getting 50% heads.

For n = 4 or a larger even number the probability of getting 50% heads goes down. For example: n = 4 has 2^4 = 16 possible outcomes: (HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH THTT, TTHH, TTHT, TTTH, TTTT). Of these 16 possible outcomes only 37.5% (=6/16) have 50% heads. [You could, of course, show this using the combinatorial formula.] The reason is that as the number of tosses goes up there are many more possible outcomes (with close to 50% heads but not exactly 50% heads).

(4) Just like X is a discrete random variable, Y is also a discrete random variable. The fact that Y does not take on integer values is irrelevant. The definition of a discrete random variable is that it takes on a finite (countable) number of values. If, for example, n = 5 then X could be 0, 1, 2, 3, 4, or 5 (six possible values) and Y could be 0, 0.2, 0.4, 0.6, 0.8, 1.0 (six possible values). They are both discrete. Of course, as the number of trials become large, the distribution of X and Y could be approximated using an appropriate continuous distribution.

$E[Y]=E[X/n]=1/n*E[X]=1/n*np=p$
$V[Y]=V[X/n]=1/n^2*V[X]=1/n^2*np(1-p)=p(1-p)/n$

(5) mean = 7 and sd = 1.87. Hence, looking for P(6 ≤ X ≤ 8),
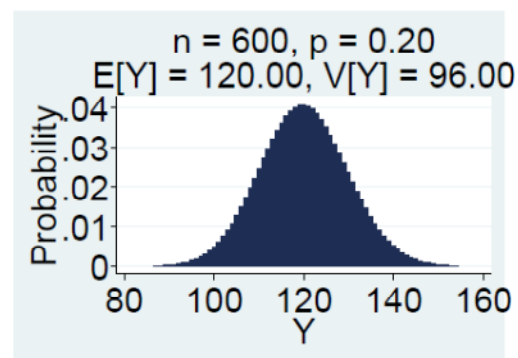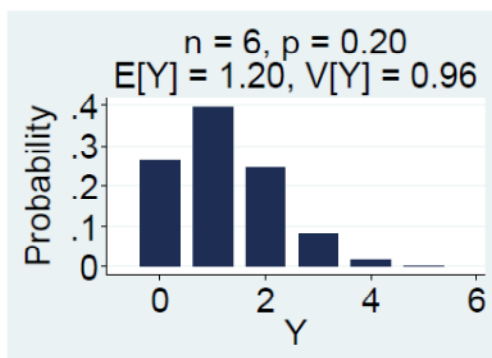
$$P(X=6) = \frac{14!}{6!8!}0.5^6 0.5^8 = 0.1833$$

$$P(X=7) = \frac{14!}{7!7!}0.5^7 0.5^7 = 0.2095$$

$$P(X=8) = \frac{14!}{8!6!}0.5^8 0.5^6 = 0.1833$$

P(6 ≤ X ≤ 8) = 0.5761

(6) If you have questions review your lecture notes & readings. If you still have questions, ask in tutorials.

(7)

(8)

$$P(X = 0) = \frac{25!}{0!25!} 0.12^0 0.88^{25} = 0.0409$$

$$P(X = 1) = \frac{25!}{1!24!} 0.12^1 0.88^{24} = 0.1395$$

$$P(X = 2) = \frac{25!}{2!23!} 0.12^2 0.88^{23} = 0.2283$$

$$P(X = 3) = \frac{25!}{3!22!} 0.12^3 0.88^{22} = 0.2387$$

$$P(X = 4) = \frac{25!}{4!21!} 0.12^4 0.88^{21} = 0.1790$$

$$P(X \geq 5) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4) = 0.1736$$

Yes, it is statistically plausible that our sample would have 20% or more delays caused by mechanical issues even if the claim that only 12% of the delays in the population are caused by mechanical issues is true. Hence, our relatively high number of delays can be explained by sampling error: the probability we would see so many delayed due to pure chance is 0.1736, which is pretty high.