ECO220Y: Homework, Lecture 9 - SOLUTIONS

1. This question is asking what is the probability to observe such a large proportion of Alice's supporters in a sample of 100 voters (or $\hat{p} = \frac{54}{100} = 0.54$)if the true population proportion is 0.5, 0.55 and 0.6 respectively.

   (a) First, check the rule of thumb: $pn = 0.5 * 100 > 10$ and $(1-p)n = 0.5 * 100 > 10$ ✓. Now, let's find $P(\hat{p} \geq 0.54 | p = 0.5, n = 100, \sigma_{\hat{p}} = \sqrt{0.5 * 0.5/100} = 0.05) = P(z \geq \frac{0.54-0.5}{0.05}) = P(z \geq 0.8) = 0.2119$

   (b) First, check the rule of thumb: $pn = 0.45 * 100 > 10$ and $(1-p)n = 0.55 * 100 > 10$ ✓. Now, let's find $P(\hat{p} \geq 0.54 | p = 0.45, n = 100, \sigma_{\hat{p}} = \sqrt{0.45 * 0.55/100} = 0.04975) = P(z \geq \frac{0.54-0.45}{0.05}) = P(z \geq 1.8) = 0.0359$

   (c) First, check the rule of thumb: $pn = 0.4 * 100 > 10$ and $(1-p)n = 0.6 * 100 > 10$ ✓. Now, let's find $P(\hat{p} \geq 0.54 | p = 0.4, n = 100, \sigma_{\hat{p}} = \sqrt{0.4 * 0.6/100} = 0.0489) = P(z \geq \frac{0.54-0.4}{0.05}) = P(z \geq 2.8) = 0.0026$

2. For all parts of this question we know that population proportion $p = 0.3$ and the sample size, n, is 500 for parts (a)-(d). Let's check the rule of thumb to see whether we can use normal approximation for the sampling distribution of sample proportion: $pn = 0.3 * 500 > 10$ and $(1-p)n = 0.7 * 500 > 10$ ✓. Now, we can specify the parameters of the sampling distribution of the sample proportion: .

   (a) Find $P(\hat{p} \geq 0.3 | p = 0.3, n = 500) = P(z \geq \frac{0.3-0.3}{0.02}) = P(z \geq 0) = 0.5$

   (b) Find $P(\hat{p} \geq 0.325 | p = 0.3, n = 500) = P(z \geq \frac{0.3-0.325}{0.02}) = P(z \geq 1.25) = 0.1056$

   (c) Find $P(\hat{p} \geq 0.35 | p = 0.3, n = 500) = P(z \geq \frac{0.3-0.35}{0.02}) = P(z \geq 2.5) = 0.0062$

   (d) Find $P(\hat{p} \geq 0.4 | p = 0.3, n = 500) = P(z \geq \frac{0.3-0.4}{0.02}) = P(z \geq 5) \approx 0$

   (e) In this part, the sample size is not 500, but 2500, and the sampling distribution of sample proportion has different parameters: $\hat{p} \sim N(0.3, \frac{0.3*0.7}{2500})$. Find $P(\hat{p} \geq 0.325 | p = 0.3, n = 5 * 500) = P(z \geq \frac{0.335-0.3}{0.009}) = P(z \geq 2.78) = 0.0027$

3. The best estimate of the proportion of the fish in the lake that are tagged is what we observe in a sample - 10 out of 100 fish are tagged means $\hat{p} = 0.1$. Given that originally 100 fish were tagged, our best estimate of the total number of fish in the lake is $\frac{100}{0.1} = 1000$. We will learn how to estimate the fraction or the number of fish in the lake more precisely on Tuesday, Nov 22nd.

4. This question asks us to first find probability of life for any given tire, and then for an average life of four tires, i.e we are going to deal with the sampling distribution of the sample mean in a sample of four tires.

(a) Let X be the life of a Rolling Rock tire. Then $X \sim N(30000, 5000^2)$. Find $P(X \geq 30000|\mu = 30000, \sigma = 5000) = 0.5$ (Why we do not need to standardize in this case?)

(b) $P(X \geq 40000|\mu = 30000, \sigma = 5000) = P(z \geq \frac{40000-30000}{5000}) = P(z \geq 2) = 0.0228$

(c) Let $\bar{X}$ be a sample mean in a sample of four tires. Then $\bar{X} \sim N(30000, \frac{5000^2}{4})$. $P(\bar{X} \geq 30000|\mu_{\bar{X}} = 30000, \sigma_{\bar{X}} = 2500) = 0.5$

(d) $P(\bar{X} \geq 40000|\mu_{\bar{X}} = 30000, \sigma_{\bar{X}} = 2500) = P(z \geq \frac{40000-30000}{2500}) = P(z \geq 4) \approx 0$

5. Let X denote height of the students. Then $X \sim N(18.8, 1.7^2)$. We need to find $P(\bar{X} \leq 18|\mu_{\bar{X}} = 18.8, n = 25, \sigma_{\bar{X}} = \frac{1.7}{5})$.
$P(\bar{X} \leq 18) = P(z \leq \frac{18-18.8}{0.34}) = P(z \leq -2.35) = 0.0094$