ECO220Y: Homework, Lecture 10 - SOLUTIONS

Problems:

(1) The point estimate of the fraction of the tagged fish in the lake is 10%. The confidence interval estimate is a range of values: $\hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$. Let's say we want to estimate a 95% confidence interval. Then $z_{0.025} = 1.96$ and $\sqrt{\hat{p}(1-\hat{p})/n} = 0.03$ and CI=(0.10-1.96*0.03; 0.10+1.96*0.03)=(0.0412, 0.1588). We are 95% confident that the interval (0.0412, 01588) will land or cover the true population proportion. We could estimate the total number of fish in the lake as follows: since we know that 100 fish were tagged, and interval (0.0412, 0.1588) represents the lower and upper bounds of the estimate of the population proportion, then the total number of fish could be anywhere between 100/0.1588=630 and 100/0.0412=2427

(2) The sample proportion is 999/1000=0.999. The 95% confidence interval is $\hat{p} \times z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$ The lower and upper bounds are : 0.999-1.96*0.0009995=0.997 and 0.999+1.96*0.0009995=1. (Actually, the rule of thumb doesn't hold for this sample, but given that the sample size is big enough to assume normal approximation, we can go ahead and assume the sampling distribution to be normal)

(3) The answer is simpler that you thought: the margin for the sampling error is 0 because we are given a point estimate, not a confidence interval estimate.

(4) False. We can use the formula for computing the sample size to prove our point: Let's say, we started with a 95% confidence interval, the margin of error or half of the interval is equal to $z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$ If we want to halve the size of the confidence interval, then we must actually have the sample size four times bigger than before! $ME/2 \; = z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/4n} = \frac{1}{2} z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$

(5) The margin of error in this example is 1%.To find how many cars Chisler Corporation must test , use the formula: $n = \frac{1.96^2}{0.01^2} 0.5 * 0.5 = 9604$. Seems like too many. That's because we do not know the chance a car will require a major repair within three years. Perhaps, statisticians at Chisler Corporation have better guess of that chance. If they estimate the chance that a car will require a major repair within 3 years as 0.10, then they would need $n = \frac{1.96^2}{0.01^2} 0.1 * 0.9 = 3458$ cars, and if the chance is estimated to be 0.02, then $n = \frac{1.96^2}{0.01^2} 0.02 * 0.98 = 753$.