ECO220Y: Homework, Lecture 11 - SOLUTIONS

(1) That statement is wrong because the population mean is a constant and hence probability statements do not make sense: we don't talk about the probability that 5 is between some values. We have to be careful to make clear that it is the sample mean (a random variable) about which probabilistic statements make sense. To correct the statement we could say: "The probability that the interval estimate contains the population mean is 0.95." The interval depends on the sample mean, which is a random variable. The sample mean is always at the center of the interval. In this example the point estimate (sample mean) is 11.

(2) First thing to recognize in this question is that we are dealing with the Binomial distribution. Here is why: when we construct confidence interval, there are two mutually exclusive outcomes – confidence interval will include population parameter or confidence interval won't include population parameter. By choosing confidence level 1- α (or significance level α), we assign a constant probability of missing the population parameter – remember, α measures the chance that the confidence interval won't include the population parameter. Hence, two conditions of the Binomial experiment are satisfied: constant probability of "success" and "failure" and two distinct outcomes. We also know that the number of trials is 20 and the trials are independent.

Now, let X be a Binomial random variable which counts the number of times when the confidence interval will miss population mean: X~B(20, 0.05). Probability that a statistician will miss population parameter at least once can be written as:

$$P(X \geq 1) = 1 - P(X = 0) = 1 - C_0^{20} 0.05^0 0.95^{20} = 1 - 0.3585 = 0.6415$$

Probability that he will miss population mean only once:

$$P(X = 1) = C_1^{19} 0.05^1 0.95^{19} = 0.377$$

Probability that he will not miss at all:

$$P(X = 0) = C_0^{20} 0.05^0 0.95^{20} = 0.3585$$

(3) First, find the standard error of the distribution of the sample mean when n=10:

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{0.1}{\sqrt{10}} = 0.03$$

X and standard error will be the same for all confidence intervals (a)-(c):

Confidence level, $1-\alpha$	Significance level, α	Critical value, $z_{\alpha/2}$	Margin of error, $z_{\alpha/2}\frac{\sigma_X}{\sqrt{n}}$	Lower confidence limit	Upper confidence limit
90%	10%	1.645	1.64*0.03	15.94	16.04
95%	5%	1.96	1.96*0.03	15.93	16.05
99%	1%	2.58	2.58*0.03	15.91	16.07

(d) A 99 percent confidence interval is wider than 90 percent confidence interval – we can see it from the table above. We allow ourselves to miss the population mean only one percent of cases and to cover for that we need to include more values in the interval, or to make it wider. The only way we can do that – is to go more standard deviations from the mean to cover the longer distance.