ECO220Y: Homework, Lecture 14 - SOLUTIONS

Problems:

(1) Let me postpone the discussion of this question until we have learned Type I and Type II errors in detail.

(2) The probability of Type I error here is not a significance level, α set at the beginning of the test, but a p-value computed from the test.

H_0: p=0.5
H_A: p>0.5

The p-value of this test is computed as probability of observing the value of test statistic as extreme as the one we computed from the data. Translating into the statistical notation,

P($\hat{P} > test - statistic$) =0.05

If we standardize values inside the brackets, this is what we get:

$$P\left(\frac{\hat{P}-p_0}{\sqrt{p_0(1-p_0)/n}} > \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}\right)=0.05$$

$$\frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)n}} = 1.645$$

$$\hat{p} = 1.645\sqrt{0.5*0.5/50} + 0.5 = 0.616$$

$$\hat{p} = \frac{X}{n} \rightarrow X = \hat{p} \cdot n = 0.62 \cdot 50 = 31$$

The subject must predict correctly 31 outcomes.

(3)

H_0: p=1/6, or 0.167
H_A: p≠1/6

Significance level, $\alpha = 0.05$

n= 18000

$$\hat{p} = \frac{X}{n} = \frac{3126}{18000} = 0.17367$$

$$\text{Test-statistic}=\frac{0.174-0.167}{\sqrt{0.167(1-0.167)/18000}} = \frac{0.007}{0.0028} = 2.5$$

$$\text{P-value}= P(z > 2.5) + P(z < -2.5) = 2*0.0062 = 0.0124$$

P-value of the test is 0.0124, which means that there is about 1% chance that the observed proportion of outcomes is due to a pure chance. This is a very small chance compared to the significance level of the test, 5%. We should reject the null hypothesis and conclude that the alternative is true. In other words, we infer that there is statistically significant evidence that the wooden die is indeed lopsided. However, this difference is unlikely of any practical importance because the difference between the observed and predicted number of times the number 6 comes up is relatively small. Please note that this is a very subjective conclusion. Each time you need to decide about the practical importance of your statistically significant findings, all available information should be taken into consideration.