## ECO220Y: Homework, Lecture 19

Readings: Chapter 18, Sections 18.3 -- 18.5
Exercises: Chapter 18, \#3, 6, 7, 16, 18, 24
Problems:
(1) In a simple linear regression model, what is the meaning of the parameter $\sigma^{2}$ ? How you can estimate it?
(2) Consider this constant-elasticity demand model: $\ln \left(Q_{i}\right)=\alpha-\eta \ln \left(P_{i}\right)+\varepsilon_{i}$

To get around the usual endogeneity problem, suppose that experimental data is available: 56 consumers were faced with randomly assigned prices (and they couldn't trade with each other to get deals). With this unusual experimental data, we obtain the following OLS parameter estimates and standard errors.

$$
\begin{equation*}
\ln (\hat{Q})=6.01-1.47 * \ln (P)+\varepsilon_{i} \tag{1.23}
\end{equation*}
$$

Test whether we can infer that demand is downward sloping. Test whether we can infer that demand is elastic.
(3) Suppose that using the Least Squares Method you estimate the linear relationship between $Y$ and $X$. The sample size is 44 . You obtain an intercept estimate of 10.03 and a slope estimate of 4.59. You also compute the following:
$\sum_{i=1}^{44}\left(e_{i}\right)^{2}=1045.31723$
$\mathrm{V}[\mathrm{Y}]=45.3778$
$\mathrm{V}[\mathrm{X}]=1.0000$
(a) How would you obtain Y-hat? How many observations will you have of it?
(b) How would you obtain e (where e is the residual of the estimated regression)? How many observations will you have of it?
(c) Is $\mathrm{E}[\mathrm{Y}$-hat $]=\mathrm{E}[\mathrm{Y}]$ ? What is $\mathrm{E}[\mathrm{e}]$ ?
(d) What is the standard error of estimate? Compute it.
(e) Compute the R-squared. What is its interpretation? What is the R-squared a measure of?
(f) Find the 95\% confidence interval estimate of the slope. Interpret it.
(g) Conduct the following hypothesis tests at the $5 \%$ significance level.
(i) $\begin{aligned} & H_{0}: \beta=0 \\ & H_{1}: \beta \neq 0\end{aligned}$
(ii) $\begin{aligned} & H_{0}: \beta=6 \\ & H_{1}: \beta \neq 6\end{aligned}$
(iii) $\begin{aligned} & H_{0}: \beta=3 \\ & H_{1}: \beta \neq 3\end{aligned}$
(4) Consider the following Stata output for a simple regression:
regress y x

| Model | 10252.7603 | 1 | 10252.7603 |
| :---: | :---: | :---: | :---: |
| Residual | 10237.9926 | 83 | 123.349309 |
| Total | 20490.7529 | 84 | 243.937535 |


| Number of obs | $=$ | 85 |
| :--- | ---: | ---: |
| F (1, 83) | $=83.12$ |  |
| Prob $>$ | $=0.0000$ |  |
| R-squared | $=0.5004$ |  |
| Adj R-squared | $=0.4943$ |  |
| Root MSE | $=11.106$ |  |


| Y | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. Interval] |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| x | 5.346689 | . 5864525 | 9.12 | 0.000 | 4.180258 | 6.513119 |
| _cons | 23.10653 | 3.107632 | 7.44 | 0.000 | 16.92557 | 29.28748 |

(a) Write down the theoretical model that is being estimated.
(b) Write down the estimated regression line using conventional notation.
(c) According to the results, what is the SSE? SST? SSR?
(d) According to the results, what is the variance of $y$ ?
(e) According to the results, what is the standard error of estimate? What is the interpretation of it?
(f) What is 123.349309 an estimate of?
(g) According to the results, what is the R-squared? Interpretation?
(h) What is the coefficient of correlation between $x$ and $y$ ?
(i) What is the hypothesis test about the slope that is reported?
(j) What is the formula used to calculate 9.12? Check that it gives the right answer.
(k) According to the results, what is the $95 \%$ confidence interval estimator of the slope? Which parameter is this an estimate of? What is the interpretation?
(I) Using your answer to Part (k), conduct the following hypothesis tests:
$H_{0}: \beta=5, H_{1}: \beta \neq 5$
$H_{0}: \beta=4, H_{1}: \beta \neq 4$
$H_{0}: \beta=6, H_{1}: \beta \neq 6$
(m) Compute the 90\% confidence interval estimator of the slope.
(n) Compute the $99 \%$ confidence interval estimator of the slope

