ECO220Y: Homework, Lecture 19 – SOLUTIONS

(1) σ^2 is the variance of the error term (ε_i). You can estimate it by fitting the OLS line using the formulas for the parameter estimates b_0 and b_1. Then you can find the predicted value of y: $\hat{y}$ **.** Then you can find the residual e = y – $\hat{y}$ **.** Then you can find the variance of e (the observed residuals) s^2, which is an estimate of the variance of the unobserved error (ε_i). (Note: There is a degrees of freedom correction in this formula for the variance because we need 2 estimates to compute e: b_0 and b_1.)

$$s^2 = \frac{\sum_{i=1}^{n}(e_i - \bar{e})^2}{n-2} = \frac{\sum_{i=1}^{n} e_i^2}{n-2} = \frac{SSE}{n-2}$$

(2) Test if demand is downward sloping:

$H_0 : \eta = 0$

$H_1 : \eta < 0$

$$t = \frac{-1.47 - 0}{0.38} = -3.87$$

$P(t < -3.87 \mid v = 54) = 0.05$

Hence reject the null hypothesis in favour of the research hypothesis. We do have sufficient evidence to conclude at conventional significance levels that demand is downward sloping (i.e. has a negative elasticity).

Test if demand is elastic:

$H_0 : \eta = -1$

$H_1 : \eta < -1$

$$t = \frac{-1.47 - (-1)}{0.38} = -1.24$$

$P(t < -1.24 \mid v = 54) = 0.11$

Hence fail to reject the null hypothesis. We do not have sufficient evidence to conclude at conventional significance levels that demand is elastic.

(3) **(a)** Plug each observation of X into the following formula: Y-hat = 10.03 + 4.59X. You will have 44 observations.

(b) e = Y-hat – Y = 10.03 + 4.59X - Y. You will have 44 observations.

(c) E[Y-hat] = E[Y]. We know this because according to Assumption #2 E[e] = 0 (average value of e is zero). Since e = Y-hat – Y, it follows from the Laws of Expectation that E[Y-hat] = E[Y].

(d) It is an estimate of σ (the standard deviation of the error ε_i). It is the square root of the variance of the residuals: $(1045.31723/(44-2))^{1/2}$ = 4.9888.

(e) R^2 = SSR/SST. SST = 45.3778*(44-1) = 1951.24558. SSR = SST – SSE = 1951.24558 – 1045.31723 = 905.928357. Hence, R^2 = SSR/SST = 905.928357/1951.24558 = 0.4643. This means that 46.43% of the variation in Y is explained by variation in X. The R^2 is a measure of how well the least squares line fits the data.

(f) (3.05, 6.13). We are 95% confident that the interval from 3.05 to 6.13 includes the unknown true slope β_1.

$$4.59 \pm 2.02 * \frac{4.9888}{\sqrt{(44-1)*1.00}}$$

(g) (i) $H_0 : \beta = 0$
$H_1 : \beta \neq 0$

(ii) $H_0 : \beta = 6$
$H_1 : \beta \neq 6$

(iii) $H_0 : \beta = 3$
$H_1 : \beta \neq 3$

In the first case reject the null hypothesis in favor of the research hypothesis. In other words, conclude that the slope is statistically significant (i.e. statistically different from zero). There is a relationship between X and Y. In the second case, fail to reject the null hypothesis. In the third case, reject the null hypothesis in favor of the research hypothesis. In other words, conclude that the slope estimate is statistically different from 3. (Be careful not to conclude that the slope is greater than 3 because that is not what the research hypothesis says.)

(4) (a) $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

(b) y-hat=23.11+5.35x

(c) SST= 20490.7529, SSE=10237.9926, SSR=10252.7603

(d) 243.937535

(e) Our estimate of the standard deviation of the unobservable error term is 11.106. The standard error of estimate, s, is an estimate of the unobserved population parameter σ, which appears in $\varepsilon_i \sim N(0, \sigma^2)$

(f) σ^2

(g) 50% variation in y is explained by x.

(h) 0.7073896

(i) $H_0 : \beta_1 = 0$
$H_1 : \beta_1 \neq 0$

(j) $t = \dfrac{b_1 - \beta_1^0}{s_{b_1}} = \dfrac{5.346689 - 0}{0.5864525} = 9.12$

(k) (4.180, 6.513) is a 95% confidence interval estimator of β_1. We are 95% confident that interval from 4.180 to 6.513 includes the true slope.

(l) Fail to reject the null; Reject null in favour of alternative; Fail to reject null.

(m) $b_1 \pm t_{\alpha/2} s_{b_1} \Rightarrow 5.347 \pm 1.66 * 0.587 \Rightarrow (4.37258, 6.32142)$

(n) $b_1 \pm t_{\alpha/2} s_{b_1} \Rightarrow 5.347 \pm 2.64 * 0.587 \Rightarrow (3.79732, 6.89668)$