Hypothesis Testing for μ

Lecture 17

Reading: Sections 13.5 – 13.7

1

Sparton Resources of Toronto

- Mini-case, page 384
  - Scarce uranium ore; required for nuclear power
  - Alternate source: coal ash (waste from creating coal power)
  - Concentration of uranium oxide varies widely depending on properties of the coal
- <u>*To profitably exploit this source requires an average concentration of uranium oxide of at least 0.32 pounds (lbs) per tonne of coal ash*</u>
- Sparton randomly selects 10 batches of ash from eight locations: 1 – 4 (China), 5 – 7 (Central Europe), 8 (Africa)

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Sparton: Raw Data

China				Central Europe			S. Africa
1	2	3	4	5	6	7	8
0.32	0.22	0.71	0.33	0.22	0.57	0.41	0.35
0.38	0.28	0.22	0.51	0.21	0.34	0.56	0.31
0.58	0.31	0.78	0.61	0.04	0.59	0.23	0.34
0.61	0.37	0.15	0.11	0.09	0.54	0.09	0.32
0.12	0.39	0.19	0.12	0.25	0.22	0.52	0.33
0.13	0.45	0.88	0.01	0.43	0.89	0.31	0.37
0.48	0.44	0.53	0.07	0.48	0.34	0.18	0.32
0.03	0.13	0.21	0.87	0.39	0.61	0.49	0.36
0.43	0.32	0.33	0.43	0.31	0.53	0.29	0.29
0.17	0.41	0.37	0.29	0.41	0.21	0.75	0.38

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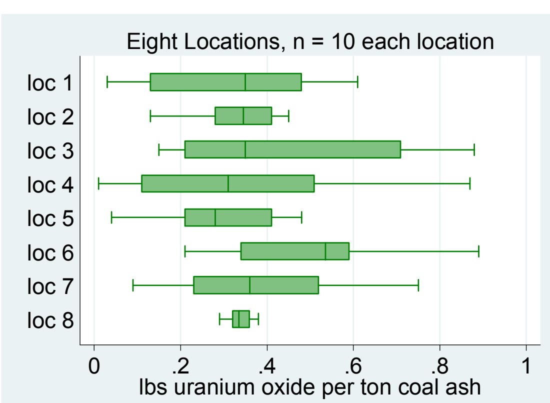

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Review

	n	mean	s.d.
loc 1	10	0.325	0.204
loc 2	10	0.332	0.102
loc 3	10	0.437	0.270
loc 4	10	0.335	0.274
loc 5	10	0.283	0.147
loc 6	10	0.484	0.208
loc 7	10	0.383	0.200
loc 8	10	0.337	0.028

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Hypothesis Testing μ, σ^2 Unknown

- Two approaches to hypothesis testing about μ :
  - Rejection (Critical) Region Approach
  - P-value Approach
- Test statistic: $t = \frac{\bar{X}-\mu}{s/\sqrt{n}}$
  - This test statistic is Student *t* distributed with degrees of freedom $\nu = n - 1$ so long as underlying conditions are met

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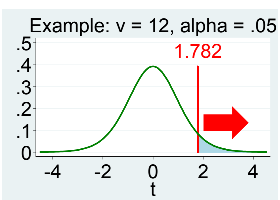
Rejection Region, Right Tailed

- H_0: $\mu = \mu_0$
- H_1: $\mu > \mu_0$
- Test statistic: $t = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$
- Rejection region: (t_α, ∞)
  - Left edge is called the critical value (t_α^*)
    - Depends on degrees of freedom



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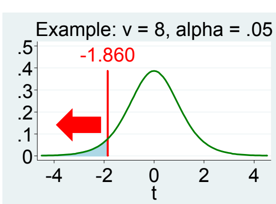
Rejection Region, Left Tailed

- H_0: $\mu = \mu_0$
- H_1: $\mu < \mu_0$
- Test statistic: $t = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$
- Rejection region: $(-\infty, -t_\alpha)$
  - Right edge is called the critical value ($-t_\alpha^*$)
    - Depends on degrees of freedom



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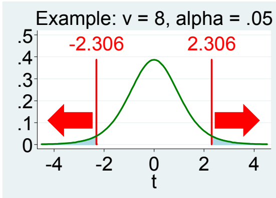
Rejection Region, Two Tailed

- H_0: $\mu = \mu_0$
- H_1: $\mu \neq \mu_0$
- Test statistic: $t = \frac{\bar{X}-\mu_0}{s/\sqrt{n}}$
- Rejection region: (-∞, $-t_{\alpha/2}$) & ($t_{\alpha/2}$, ∞)
  - Edges are called the critical values ($t^*_{\alpha/2}$)
  - Depend on degrees of freedom



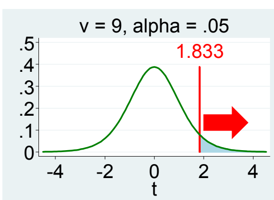
Sparton Ex: Set-up Hypotheses

	n	mean	s.d.
loc 1	10	0.325	0.204
loc 2	10	0.332	0.102
loc 3	10	0.437	0.270
loc 4	10	0.335	0.274
loc 5	10	0.283	0.147
loc 6	10	0.484	0.208
loc 7	10	0.383	0.200
loc 8	10	0.337	0.028

- How to choose from:
  - H_0: μ_i = 0.32
    H_1: μ_i > 0.32
  - H_0: μ_i = 0.32
    H_1: μ_i < 0.32
  - H_0: μ_i = 0.32
    H_1: $\mu_i \neq$ 0.32
  - What does i mean?
  - Where does 0.32 come from?

Sparton Example: Location 8

- Sampled 10 batches of coal ash at Loc. 8
  - Mean conc. of uranium ore is 0.337 lbs/ton
  - S.d. conc. of uranium ore is 0.028 lbs/ton
- H_0: μ_8 = 0.32
- H_1: μ_8 > 0.32



Conclusion?

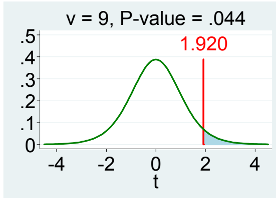
$$t = \frac{\bar{X}_8 - \mu_0}{\frac{s_8}{\sqrt{n_8}}} = \frac{0.337 - 0.32}{\frac{0.028}{\sqrt{10}}} = 1.92$$

P-value Approach

- P-value: Probability of a test statistic at least as extreme (in the direction of H_1) as the one we got presuming that H_0 is true
  - Small P-value means sampling error is a poor explanation of how we got so far from H_0
    - Reject H_0 in favor of H_1 if P-value is small enough
  - For one-tailed test: P-value is area in one tail
  - For two-tailed test: P-value is the sum of areas in two tails

P-value: Location 8

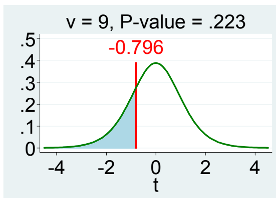
- H_0: $\mu_8 = 0.32$
- H_1: $\mu_8 > 0.32$
- $t = 1.92$
- P-value = $P(t > 1.92 \mid \nu = 9)$
  - With software find exact P-value = 0.044
  - With table find that the P-value is between 0.025 and 0.05



Student *t* table tells us:
$P(t > 2.262 \mid \nu = 9) = 0.025$
$P(t > 1.833 \mid \nu = 9) = 0.050$

Location 5: Confident It's Bad?

- Location 5, n = 10:
  - Mean = 0.283
  - S.d. = 0.147
- How to set-up?
  - H_0: $\mu_5 = 0.32$
    H_1: $\mu_5 > 0.32$
  - H_0: $\mu_5 = 0.32$
    H_1: $\mu_5 < 0.32$
  - H_0: $\mu_5 = 0.32$
    H_1: $\mu_5 \neq 0.32$



$$t = \frac{\bar{X}_5 - \mu_0}{\frac{s_5}{\sqrt{n_5}}} = \frac{0.283 - 0.32}{\frac{0.147}{\sqrt{10}}} = -0.796$$