

PART 1

Last Name:

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First Name:

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Student #:

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Instructor: M. Pivovarova

Duration: 120 minutes.

Allowed aids: A non-programmable calculator and aid sheets provided.

Format: This test consists of two parts and a SCANTRON form. For both parts combined there are a total of 120 possible points.

BEFORE we announce the end of the test, enter your name and student # on BOTH graded pieces:

- (1) The pink SCANTRON form
- (2) Part 1

Part 2: 16 multiple choice questions worth 3 points each for a total of 48 points

Part 1: 5 written questions worth a total of 72 points

- For each question I give a guide for your response in brackets. It indicates what is expected: a quantitative analysis, a graph, and/or a written response. For example, "Is sampling error a plausible explanation for the result? [Analysis & 2 – 3 sentences]"
- Make sure to focus on answering the specific questions asked. Extraneous analysis does not earn positive marks even if it is correct and may earn negative marks if incorrect.
- Show your work and answer clearly, concisely, and completely. You do not have to fill all of the blank space: a generous amount is provided for your convenience.

	Q1	Q2	Q3	Q4	Q5	Part 1	Part 2	Total	% Mark
<i>Point Value</i>	<i>18</i>	<i>12</i>	<i>12</i>	<i>16</i>	<i>14</i>	<i>72</i>	<i>48</i>	<i>120</i>	
Points Earned									

(1) [18 points] Suppose, the producer claims that Cheerios is a power breakfast: eating Cheerios for breakfast would boost test scores on average by 4 percentage points and decrease the variance among students by 1 percentage point. The average on the national test score is known to be distributed with the mean of 59 and standard deviation of 21. To find out whether a Cheerios is a power breakfast, a study is done in a large elementary statistics class; 499 students agree to participate; 250 are randomized to the treatment group, and 249 to the control group. The treatment group is fed Cheerios for breakfast 7 days a week. The control group gets Captain Crunch.¹

(a) [12 points] If the producer's claim is true, what is the chance that the control group outperforms treatment group by at least 1 percentage point? [Analysis&1-2 sentences]

¹ Cheerios and Captain Crunch are registered trademarks. The study is hypothetical.

(b) [4 points] Explain whether the sampling error would be a plausible explanation for such a disappointing result of the study? [1 – 2 sentences]

(c) [2 points] What type of study is this? What is an exogenous variable in this study? Explain [2 – 3 sentences]

(2) [12 points] A random variable X has the following continuous probability density function:

$$f(x) = 2 - 2x, \quad 0 \leq x \leq 1$$
$$f(x) = 0, \quad \text{otherwise}$$

(a) [6 points] Graph this probability distribution. Carefully label the graph. How do you know that this is a valid probability density function? [Graph & 1-2 sentences]

(b) [6 points] Find $P(X \geq 0.5)$ and $P(X \leq 0.25)$. [Analysis only]

(3) [12 points] The researcher wants to learn more about whether the number of years of education is a good predictor of future income. A sample of 150 30-old men and women has been randomly drawn and each respondent has been asked how many years of formal education he or she had completed and his or her income (in thousands of \$) for previous 12 months. The STATA summary for the sample is presented below. Coefficient of correlation between years of education and current income is 0.66.

Education					

	Percentiles	Smallest			
1%	5	5			
5%	8	5			
10%	9	6	Obs		150
25%	11	6	Sum of Wgt.		150
50%	13		Mean		13.17333
		Largest	Std. Dev.		3.335284
75%	16	20			
90%	17	20	Variance		
95%	19	20	Skewness		
99%	20	20	Kurtosis		

Income					

	Percentiles	Smallest			
1%	26	19			
5%	47	26			
10%	53	33	Obs		150
25%	69	35	Sum of Wgt.		150
50%	80		Mean		78.12667
		Largest	Std. Dev.		20.92598
75%	88	112			
90%	99.5	116	Variance		
95%	108	174	Skewness		
99%	174	181	Kurtosis		

(a) [6 points] Find the least squares line (round your answer to the nearest tenth).
[Analysis & 1 equation]

(b) [6 points] Interpret coefficients of the OLS line. [3 – 4 sentences]

(4) [16 points] In a small town of Cape d'Azur, all households live either in 1- , 2-, or 3-storey houses. Fraction of those who live in 1-storey houses is 45%, those who live in 2-storey houses – 35%, and 20% of the households live in 3-storey houses. Define X to be a random variable which denotes the number of floors in a house of a randomly selected household.

(a) [8 points] Derive probability distribution of \bar{X} (sample mean) if sample size is 2 ($n=2$). Show your work. [Analysis only]

(b) [8 points] Graph probability distribution of \bar{X} . Carefully label the graph. Find the mean and standard deviation of \bar{X} . [Analysis & graph]

(5) [14 points] The number of coffee cups consumed by the second year students during exam period is normally distributed with population standard deviation of 4 cups per day. The number of cups per day is recorded for a random sample of 60 second year students. Tabulation below summarizes these data.

coffee_cups	Freq.	Percent	Cum.
0	4	6.67	6.67
1	6	10.00	16.67
2	9	15.00	31.67
3	20	33.33	65.00
4	8	13.33	78.33
5	5	8.33	86.67
6	4	6.67	93.33
10	3	5.00	98.33
12	1	1.67	100.00
Total	60	100.00	

(a) [10 points] Compute the 99% confidence estimator of the average number of cups per day for all second year students. [Analysis only]

(b) [4 points] Interpret confidence interval [1 – 2 sentences]

Extra Space: If you need to use this space, it is *your responsibility* to clearly indicate for which question(s) and which part(s) AND to clearly indicate at the end of the space specifically provided for that question and part that you have also used this extra space.

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Instructor: M. Pivovarova

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ENTER YOUR NAME AND STUDENT # ON BOTH THE PINK SCANTRON FORM AND PART 1 BEFORE THE END OF THE EXAM IS ANNOUNCED.

Part 2: 16 multiple choice questions worth 3 points each for a total of 48 points

- A correct answer is worth 3 points and an incorrect answer is worth 0 points
- Answers must be properly recorded on the pink SCANTRON form to earn marks
- Print your **LAST NAME** and **INITIALS** in the boxes AND darken each letter in the corresponding bracket below each box; Sign your name in the **SIGNATURE** box
- Print your 9 digit **STUDENT NUMBER** in the boxes AND darken each number in the corresponding bracket below each box
- Your **FORM NUMBER** is **01**
- Use only a pencil or blue or black ball point pen
 - **Pencil strongly recommended** because it can be erased
- Make dark solid marks that fill the bubble completely
- Erase completely any marks you want to change
- Crossing out a marked box is not acceptable and is incorrect
- If more than one answer is marked then that question earns 0 points

(1) Consider two random variables X and Y which are distributed with means 4 and 5 and standard deviations 1 and 4 respectively. If standard deviation of $W=X+Y$ is 3.5, what can we conclude about the relationship between X and Y ?

- (A) X and Y are weakly positively related
- (B) X and Y are independent
- (C) X and Y are negatively linearly related
- (D) X and Y are negatively related
- (E) Not enough information

(2) In a large company, 25% of employees work for more than 15 years, 30% work for more than 10 and less than 15 years, and 45% work less than 10 years. What is the chance that both of 2 randomly selected employees work more than 10 years?

- (A) 0.0625
- (B) 0.0750
- (C) 0.0900
- (D) 0.3025
- (E) 0.5500

(3) The heights of the 20,000 men at Big State University (BSU) are normally distributed with a mean of 70 inches and a standard deviation of 2.5 inches. How many of these men are at least 6 feet 5 inches tall? (1 foot=12 inches)

- (A) 0
- (B) 28
- (C) 52
- (D) 65
- (E) 96

► **Questions (4)-(5):** Tweedle Dee and Tweedle Dum are running for President of Wonderland. Half of the voters prefer Dee and half prefer Dum.

(4) If Mad Hatter randomly polls 10 voters, what is the probability that at least 60 percent of those polled will prefer Mr. Dee?

- (A) 0.016
- (B) 0.205
- (C) 0.377
- (D) 0.465
- (E) 0.623

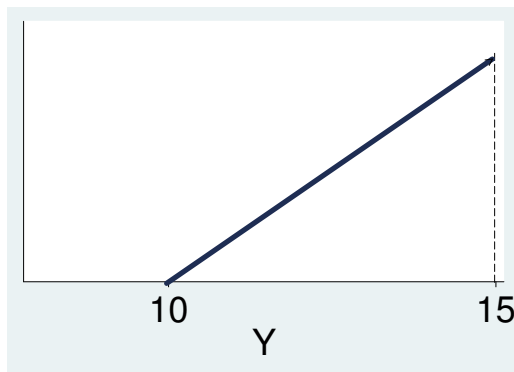
(5) If Mad Hatter randomly polls 3 voters and X records the proportion of those who prefer Mr. Dee, which best describes the shape of the probability distribution of X ?

- (A) Triangle
- (B) Bell shaped
- (C) Positively skewed
- (D) Bimodal
- (E) Symmetric

(6) Consider two identically and independently distributed random variables Z and W , and $Z \sim U[-2, 4]$. What is the mean and standard deviation of $Z+W$?

- (A) -4 and 5.65
- (B) -4 and 8
- (C) -2 and 4
- (D) 2 and 2.45
- (E) 2 and 6

► **For Question (7)** consider probability density function for continuous random variable Y (y-label was intentionally erased):



(7) Find $P(Y \leq 12)$

- (A) 0.10
- (B) 0.15
- (C) 0.16
- (D) 0.20
- (E) 0.35

► **Questions (8)-(9):** Consider STATA summary of random variable X :

x				

	Percentiles	Smallest		
1%	4.05276	4.000406		
5%	4.270007	4.001718		
10%	4.549523	4.011818	Obs	1278
25%	5.447782	4.012512	Sum of Wgt.	1278
50%	6.920566		Mean	6.94036
		Largest	Std. Dev.	1.742317
75%	8.45915	9.994036		
90%	9.394897	9.998101	Variance	
95%	9.7288	9.998448	Skewness	
99%	9.953926	9.999187	Kurtosis	

(8) What is the 95th percentile of $Y = 2.5 + 3X$?

- (A) 32.43
- (B) 31.66
- (C) 27.56
- (D) 23.32
- (E) 9.73

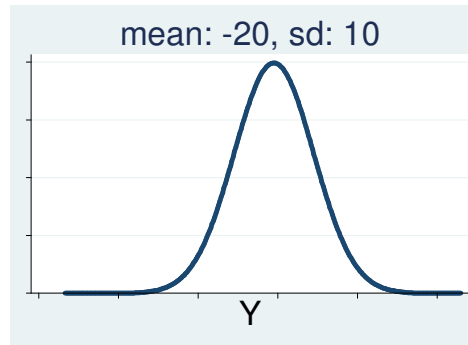
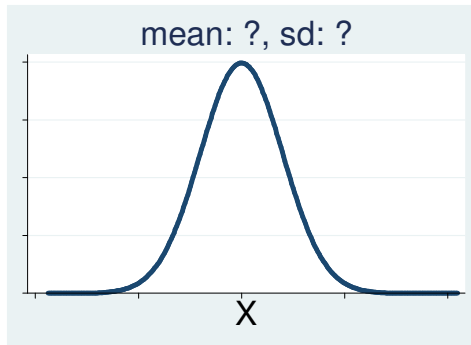
(9) Which best describes the shape of the distribution of X?

- (A) uniform
- (B) bell-shaped
- (C) unimodal
- (D) positively skewed
- (E) bimodal

(10) Cans of salmon have a nominal net weight of 250 g. However, due to variation in the canning process, the actual net weight has an approximate normal distribution with a mean of 255 g and a standard deviation of 10 g. According to Consumer Affairs, a sample of 16 tins should have less than a 5% chance that the mean weight is less than 250 g. What is the actual probability that a sample of 16 tins will have a mean weight less than 250 g?

- (A) 0.0500
- (B) 0.0228
- (C) 0.1915
- (D) 0.3085
- (E) 0.4792

(11) If $Y = 4 - 5X$, what are the mean and standard deviation of X?



- (A) mean: -5, sd: 2
- (B) mean: -2, sd: 5
- (C) mean: 4.8, sd: 2
- (D) mean: 4.8, sd: 5
- (E) mean: 5, sd: 2

► **Questions (12)-(14):** Web-site of Duke University reports following facts about the number of students:

“Number of undergraduates: 6,244 (5,225 in arts and sciences and 1,019 in engineering)”

(12) If 2 students are selected at random, what is the chance that one is from arts and sciences and one is from engineering?

- (A) 0.027
- (B) 0.134
- (C) 0.160
- (D) 0.269
- (E) 0.500

(13) If 10 students are selected, what is the chance that half of them are from engineering?

- (A) 0.000
- (B) 0.011
- (C) 0.026
- (D) 0.110
- (E) 0.134

(14) If 400 students are randomly selected, what is the chance that more than 80 are from engineering?

- (A) 0.0000

- (B) 0.0228
- (C) 0.2280
- (D) 0.4772
- (E) 0.9772

► **Questions (15) – (16):** The Library at the Big State University (BSU) records the number of books per month checked out by students. Below is the joint probability table. The librarian studies the relationship between the number of books checked out and the student status: undergraduate or graduate.

Number of books per month	Undergraduate Students	Graduate Students
0	0.10	0.05
1	0.05	0.05
2	0.10	0.15
3	0.10	0.20
4	0.05	0.15

(15) Which is the expected number of books for each type of student: undergraduate and graduate (round answer to nearest tenth)?

- (A) Undergraduate: 0.4; Graduate: 0.6
- (B) Undergraduate: 0.6; Graduate: 0.4
- (C) Undergraduate: 0.8; Graduate: 1.6
- (D) Undergraduate: 2.3; Graduate: 2.3
- (E) Undergraduate: 1.9; Graduate: 2.6

(16) If 1% of all checked out books get lost, and it costs \$145 to replace one book, how much money does the librarian expect to allocate for monthly replacements if there are 5,000 students who regularly use library services (round answer to nearest dollar)?

- (A) \$16,675
- (B) \$16,313
- (C) \$14,500
- (D) \$7,250
- (E) None of the above

Extra space for rough work:

Your form number is 01. Make sure to properly mark your name, student number, signature, and answers for Part 2 on the pink SCANTRON form before the end of the exam is announced. No extra time is permitted.