SOLUTIONS

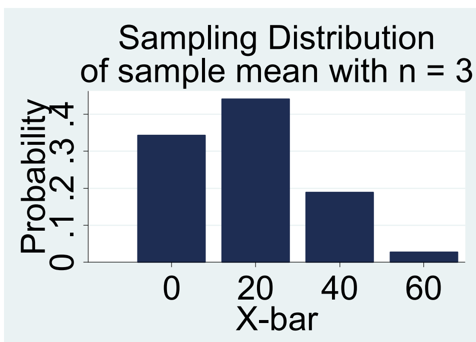
(1)

sample	mean	probability
0, 0, 0	0	$(0.7)^3 = 0.343$
0, 0, 60	20	$(0.7)^2(0.3) = 0.147$
0, 60, 0	20	$(0.7)^2(0.3) = 0.147$
0, 60, 60	40	$(0.7)^1(0.3)^2 = 0.063$
60, 0, 0	20	$(0.7)^2(0.3)^1 = 0.147$
60, 0, 60	40	$(0.7)^1(0.3)^2 = 0.063$
60, 60, 0	40	$(0.7)^1(0.3)^2 = 0.063$
60, 60, 60	60	$(0.3)^3 = 0.027$

Sampling distribution of the sample mean for a sample size of 3:

mean	probability
0	0.343
20	0.441
40	0.189
60	0.027

[EXTRA:



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(2) (a)

$$\frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{n}} = 0.7070909$$

$$2.82849 = \sqrt{n}$$

$$n = 8$$

(b) The sample size is too small to apply the Central Limit Theorem (CLT)—rule of thumb is n is at least 30—and hence we cannot say the sampling distribution will be bell shaped (Normal). The simulation showed us that the shape of the sampling distribution of the sample mean is skewed and it showed us exactly how much skew.

(3) (a)

$$E[\overline{X}] = \mu = \frac{b+a}{2} = \frac{100+0}{2} = 50$$

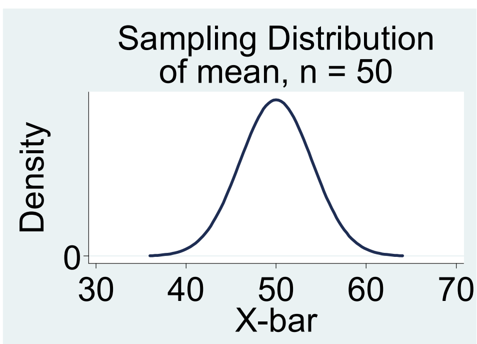
$$V[\overline{X}] = \frac{\sigma^2}{n} = \frac{\frac{(b-a)^2}{12}}{n} = \frac{\frac{(100-0)^2}{12}}{52} = 16.03$$

$$se_{\overline{X}} = \sqrt{16.03} = 4.00$$

Our sample size is 52, which is bigger than 30, so we know that according to the Central Limit Theorem (CLT) the sampling distribution of the sample mean will be Bell shaped (Normally distributed).

Hence the sampling distribution of the sample mean with n = 52 is X-bar ~ N(50, 16.03).

[EXTRA:
When drawing the graph we should make sure that the Empirical Rule holds: about 68% of the density is within 1 s.e.'s, about 95% is within 2 s.e.'s and about 99.7% is within 3 s.e.'s of the mean.



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(b)

$$P(45 < \overline{X} < 55) = P\left(\frac{45-50}{4} < Z < \frac{55-50}{4}\right) = P(-1.25 < Z < 1.25) = 2 * P(0 < Z < 1.25)$$

$$= 2 * 0.3944 = 0.7888$$

$$P(45 < X < 55) = area = 10 * \frac{1}{b-a} = 10 * \frac{1}{100-0} = 0.10$$

(4) (a) To answer this, we need to find P(X ≥ 25). The Normal approximation would be appropriate: rule of thumb is satisfied because the interval 20 ± 3*3.464 does lie within [0, 50].

$$P(X \geq 25) = P\left(Z > \frac{24.5-\mu}{\sigma}\right) = P\left(Z > \frac{24.5-20}{3.464}\right) = P(Z > 1.30) = 0.5 - 0.4032 = 0.0968$$

There is a 9.68% chance of this happening. [This is simple because of sampling error (pure chance) even if what the administration said is true. Sampling error is a plausible explanation for such a large number supporting SU.]

[An alternate and equally correct approach is based on P-hat (=X/n). The answer is mathematically identical.]

(b) To answer this, we need to find P(X ≥ 4). The Normal approximation would be inappropriate: rule of thumb is failed because the interval 2.4 ± 3*1.2 does not lie within [0, 6]. Hence we must use the Binomial probability formula.

$$P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$$
$$P(X \geq 4) = \frac{6!}{4!2!}(0.4)^4(0.6)^2 + \frac{6!}{5!1!}(0.4)^5(0.6)^1 + \frac{6!}{6!0!}(0.4)^6(0.6)^0$$
$$P(X \geq 4) = 0.13824 + 0.036864 + 0.004096$$
$$P(X \geq 4) = 0.1792$$

Yes, sampling error is a plausible explanation for such a large number supporting SU. There is a 17.92% chance of this happening even if what the administration said is true.

(5) (a) It is true that brain activity is an endogenous variable because it is affected by individual choices. It is also true that the researchers cannot randomly set brain activity. While the statement by itself is true, it is the incorrect answer to the question because the dependent variable (Y) is always endogenous and this does not create any problem in measuring causal effects. The problem arises when the independent variable (X) is endogenous.

(b) It is highly likely that the researchers do not observe many variables that affect caffeine consumption. While the statement in (E) is perfectly plausible, it is the incorrect answer to the question posed because it does not explain why we would have a problem measuring the causal effect. Only unobserved variables that affect <u>both</u> the X and the Y variables will explain why the least squares line would not measure the causal effect.