

Last Name:

First Name:

Student #:

Instructors: M. Pivovarova

Duration: 80 minutes. You may not leave the test room until at least 60 minutes have passed.

Format: 22 multiple-choice questions. Record answers on pink SCANTRON form. Correct answers are worth 5 points each and incorrect answers are worth 0 points. There are 110 total possible points.

Allowed aids: A non-programmable calculator and aid sheets provided.

Instructions:

- Print your **LAST NAME, FIRST NAME** and 9 digit **STUDENT NUMBER** in the boxes above.
- Answers must be properly recorded on the pink SCANTRON form to earn marks.
- Print your **LAST NAME** and **INITIALS** in the boxes AND darken each letter in the corresponding bracket below each box; Sign your name in the **SIGNATURE** box on the SCANTRON form.
- Print your 9 digit **STUDENT NUMBER** in the boxes AND darken each number in the corresponding bracket below each box.
- Your **FORM NUMBER** is **01**
- Use only a pencil or blue or black ball point pen.
 - **Pencil strongly recommended** because it can be erased.
- Make dark solid marks that fill the bubble completely.
- Erase completely any marks you want to change.
- Crossing out a marked box is not acceptable and is incorrect.
- If more than one answer is marked then that question earns 0 points.

DEFEND YOUR ACADEMIC INTEGRITY: Make sure to cover your answers and do not write your letter answers in large font next to each question. Providing assistance to another student writing a test is as bad as receiving assistance and is treated equally harshly. Despite our large numbers, students in our course have an excellent record regarding academic integrity and violations have been rare. Let us continue to behave in a way that is clearly professional and marked by integrity.

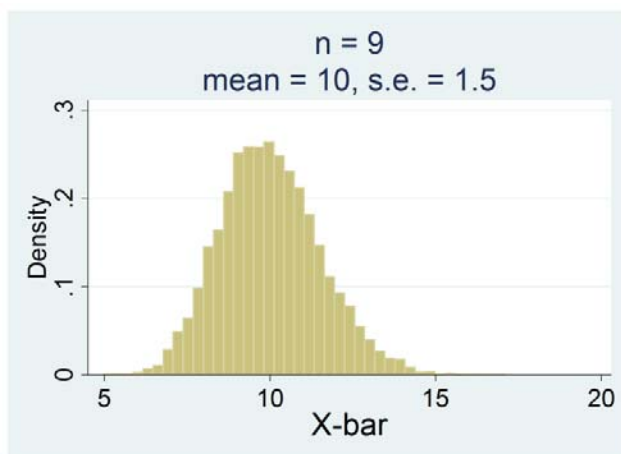
(1) Which of the statements below is true if the slope of the OLS line through the scatter plot of X and Y is negative?

- (A) $V[X-Y] < V[X] + V[Y]$
- (B) $V[X+Y] = V[X] + V[Y]$
- (C) $V[X-Y] = V[X] - V[Y]$
- (D) $V[X+Y] > V[X] + V[Y]$
- (E) $V[X-Y] > V[X] + V[Y]$

(2) You seek to find $P(-3 < X < 5)$ and after standardization you look up in the standard normal table $P(0.25 < Z < 0.75)$. What are the mean and standard deviation of X?

- (A) (-7, 16)
- (B) (-7, 4)
- (C) (-3, 5)
- (D) (0, 1)
- (E) (0.25, 0.75)

► For Question (3): Consider the following histogram of the sampling distribution of sample mean.



(3) Population mean and standard deviation are:

- (A) (10, 0.5)
- (B) (10, 1.5)
- (C) (10, 2.25)
- (D) (10, 4.5)
- (E) (10, 20.25)

(4) Consider taking a multiple choice questions test which consists of 20 questions each with four alternatives. Assume that your choice of the correct answers is purely random. What is your expected score on the test if correct answer earns $1\frac{1}{2}$ points while $\frac{1}{2}$ points is subtracted for incorrect answer?

- (A) -5
- (B) 0
- (C) 0.5
- (D) 5
- (E) 7.5

(5) What are parameters of $W=X+Y$ where X and Y are identically and independently distributed uniform random variables, and $\mu_W = 10$ and $\sigma_W^2 = 6$?

- (A) (2, 8)
- (B) (4, 8)
- (C) (2, 16)
- (D) (4, 16)
- (E) (10, 6)

(6) The weight of oranges is normally distributed. For instance, weight of a navel orange is distributed with mean equal to 300 grams and standard deviation equal to 30, while the average weight of a blood orange is 280 grams with standard deviation equal to 30. If you bought two bags of oranges each containing 25 fruits, what is the chance that the navel bag weighs at least by 400 grams more than the bag of blood oranges?

- (A) 0.0000
- (B) 0.0012
- (C) 0.3192
- (D) 0.6808
- (E) 0.9998

(7) What is the point estimate of the average length of pickles if the 98% confidence interval computed from the sample of size 100 is (11.535, 12.465) and population standard deviation is known to be 2 cm?

- (A) 11
- (B) 11.535
- (C) 11.965
- (D) 12
- (E) 12.465

(8) Random variable W has the following pdf: $f(w) = 3 - 6w$ if $0 \leq w \leq 0.5$ and $f(w) = 2w - 1$ if $0.5 < w \leq 1$. Find $P(0.25 \leq W \leq 0.75)$

- (A) 0.025
- (B) 0.125
- (C) 0.250
- (D) 0.500
- (E) 0.750

► For Question (9): Consider two estimators of the population mean \bar{X}_A and \bar{X}_B such that

$$\bar{X}_A = \left(\frac{1}{n} + \frac{\sum_{i=1}^n x_i}{n} \right) \text{ and } \bar{X}_B = \frac{\sum_{i=1}^n x_i}{n}$$

(9) Which of the following statements about \bar{X}_A and \bar{X}_B is true?

- (A) Both estimators are unbiased and consistent estimators of population mean
- (B) \bar{X}_A is an unbiased and consistent estimator of population mean
- (C) \bar{X}_B is relatively more efficient than \bar{X}_A
- (D) Both estimators are unbiased, but only \bar{X}_B is consistent
- (E) \bar{X}_B is an unbiased and consistent estimator of population mean

► For Questions (10)-(11):

(10) Eleven people have been independently exposed to a serious non-infectious disease. Each one has a 40% chance of contracting the disease. The local hospital has the capacity to handle only 7 cases of the disease. What is the probability that the hospital's capacity will be exceeded?

- (A) 0.03
- (B) 0.07
- (C) 0.40
- (D) 0.93
- (E) 0.97

(11) Refer to the previous problem. The hospital needs to have enough beds available to handle a proportion of all outbreaks. Suppose, a typical outbreak has 100 people exposed, each with a 40% chance of contracting the disease. Which of the statements below is not correct:

- (A) About 95% of the time, between 30 and 50 people will contract the disease.
- (B) On average, about 40 people will contract the disease.
- (C) Almost all of the time, between 25 to 55 people will contract the disease.
- (D) Almost all of the time, less than 40 people will contract the disease.
- (E) The situation described satisfies the assumption of Binomial experiment.

► **For Questions(12)-(13):**

(12) A new headache remedy has been given to a group of 25 subjects who had headaches. Four hours after taking the new remedy, 20 of the subjects reported that their headaches had disappeared. From this information you conclude:

- (A) that the remedy is effective for the treatment of headaches.
- (B) nothing, because the sample size is too small.
- (C) nothing, because there is no control group for comparison.
- (D) that the new treatment is better than aspirin.
- (E) that the remedy is not effective for the treatment of headaches.

(13) Now assume that the chance that the headache disappears in 4 hours without any remedy is equal to 20%. What is the probability that among 25 patients one fifth will report that their headaches disappeared in 4 hours even if all the subjects were given a placebo (a pill without any medicinal components):

- (A) 0.00
- (B) 0.02
- (C) 0.05
- (D) 0.20
- (E) 0.25

(14) Which of the following statements about sampling distribution are correct?

- (I) Sampling distribution of a mean is never identical to the population distribution.
- (II) The mean of sampling distribution is identical to the population mean irrespective of sample size.
- (III) A decrease in sample size increases the variance of sample mean.

- (A) (I)
- (B) (II)
- (C) (III)
- (D) (I) and (II)
- (E) (I) and (III)

(15) The average waiting time in the Sydney Smith's Tim Hortons outlet is uniformly distributed between 1 and 7 minutes. What is the chance that a randomly selected student in line has to wait more than 5 minutes?

- (A)** 0.15
- (B)** 0.17
- (C)** 0.28
- (D)** 0.33
- (E)** 0.72

(16) You are investigating the cases of binge drinking¹ among the freshmen class at the Big State University. You have collected information on the drinking patterns for a random sample of 200 first-year male students. You also know that the population standard deviation is equal to 2.5 drinks. You have found that the 98% confidence interval for the mean number of drinks is (3.8, 5.6). You conclude that:

- (A)** Average consumption of alcoholic drinks on one occasion among male first-year students in the Big State University is between 3.8 and 5.6 drinks.
- (B)** On average, roughly 98 percent of first-year students consume between 3.8 and 5.6 drinks on one occasion.
- (C)** There is 2 percent chance that male first-year students consume less than 4.7 drinks and more than 4.7 drinks on one occasion.
- (D)** There is 98 percent chance that on average male first-year students consume 4.7 drinks on one occasion.
- (E)** There is 2 percent chance that the interval (3.8, 5.6) does not include the average number of drinks consumed by male first-year students.

(17) To get to his friends' home, Harry needs to change two buses. He knows that the waiting time for the first bus is normally distributed with parameters 4 and 1, and the waiting time for the second bus is normally distributed with parameters 7 and 2. Estimate the chance that Harry's waiting time on his way to his friend's home does not exceed 9 minutes, assuming that the schedules of the two buses are independent.

- (A)** 0.2514
- (B)** 0.1867
- (C)** 0.8133
- (D)** 0.3446
- (E)** 0.8849

¹ Binge drinking is defined as an episodic excessive drinking of consuming five or more drinks by males and four or more drinks by females on one occasion.

(18) The distribution of Binomial random variable X is positively skewed. Choose a correct statement from below.

- (A) number of trials is very large and probability of success is close to 0.5.
- (B) number of trials is very small and probability of failure is close to 0.5
- (C) probability of success is close to 1.
- (D) probability of failure is close to 1.
- (E) None of above is correct.

► **For Questions(19)-(20):** Table below is the joint probability distribution of two random variables X and Y (one cell is intentionally left blank):

		X	
		0	1
Y	0	0.1	0.3
	2	0.25	

(19) From the table you conclude that:

- (A) X and Y are positively related.
- (B) Variance of Y is greater than variance of X .
- (C) Expected value of X is greater than expected value of Y .
- (D) X and Y are independent.
- (E) Cannot make any of the conclusions above because of the missing information.

(20) Find variance of $X-Y$.

- (A) -0.080
- (B) 0.6370
- (C) 1.0275
- (D) 1.1875
- (E) 1.3475

(21) Mario runs a pizza restaurant. The restaurant delivers pizza to locations within 4 km. The delivery time is known to be normally distributed with the mean of 30 minutes and standard deviation of 7.5 minutes. Mario is planning to offer pizza for free if the delivery takes longer than advertized. If Mario wants to limit free pizza orders to 8 percent of the time, how many minutes should he set up for free offer in advertisement?

- (A) 19 minutes
- (B) 26 minutes
- (C) 30 minutes
- (D) 34 minutes
- (E) 41 minutes

(22) Choose the cases from following under which the Central Limit Theorem does not apply:

- (I)** Sampling is not independent.
- (II)** Underlying population is not Normal.
- (III)** Underlying population is not continuous.

- (A)** (I)
- (B)** (II)
- (C)** (III)
- (D)** (I) and (II)
- (E)** (I) and (III)

Extra space for rough work:

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Your form number is 01. Make sure to properly mark your name, student number, and signature on the pink SCANTRON form before the end of the exam is announced. No extra time is permitted.

Population Mean:
$$\mu = \frac{\sum_{i=1}^N x_i}{N}$$

Sample Mean:
$$\bar{X} = \frac{\sum_{i=1}^n x_i}{n}$$

Population Variance:
$$\sigma^2 = \frac{\sum_{i=1}^N (x_i - \mu)^2}{N}$$

Sample Variance:
$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n-1} = \frac{1}{n-1} \left[\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i \right)^2}{n} \right]$$

Population s.d.:
$$\sigma = \sqrt{\sigma^2}$$

Sample s.d.:
$$s = \sqrt{s^2}$$

Population coefficient of variation:
$$CV = \frac{\sigma}{\mu}$$

Sample coefficient of variation:
$$cv = \frac{s}{\bar{X}}$$

Population covariance:
$$\sigma_{XY} = \frac{\sum_{i=1}^N (x_i - \mu_X)(y_i - \mu_Y)}{N}$$

Sample covariance:
$$s_{XY} = \frac{\sum_{i=1}^n (x_i - \bar{X})(y_i - \bar{Y})}{n-1}$$

Population coefficient of correlation:
$$\rho = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

Sample covariance (shortcut):
$$s_{XY} = \frac{1}{n-1} \left[\sum_{i=1}^n x_i y_i - \frac{\sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n} \right]$$

Sample coefficient of correlation:
$$r = \frac{s_{XY}}{s_X s_Y}$$

Conditional Probability:

$$P(A | B) = \frac{P(A \text{ and } B)}{P(B)}$$

Complement rule:

$$P(A^C) = 1 - P(A) \quad P(A^C | B) = 1 - P(A | B)$$

Multiplication Rule:

$$P(A \text{ and } B) = P(A | B)P(B)$$

Addition Rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Expected Value:

$$E[X] = \mu = \sum_{\text{all } x} xp(x) \quad \text{Variance: } V[X] = E[(X - \mu)^2] = \sigma^2 = \sum_{\text{all } x} (x - \mu)^2 p(x)$$

Covariance:
$$E[(X - \mu_X)(Y - \mu_Y)] = \sigma_{XY} = \sum_{\text{all } x} \sum_{\text{all } y} (x - \mu_X)(y - \mu_Y)p(x, y)$$

Least squares line / linear regression line:
$$\hat{y} = a + bx \quad b = \frac{\text{cov}(x, y)}{s_x^2} \quad a = \bar{y} - b\bar{x}$$

Expected

Value: $E[X] = \mu = \sum_{\text{all } x} xp(x)$ **Variance:** $V[X] = E[(X - \mu)^2] = \sigma^2 = \sum_{\text{all } x} (x - \mu)^2 p(x)$

Covariance: $E[(X - \mu_X)(Y - \mu_Y)] = \sigma_{XY} = \sum_{\text{all } x} \sum_{\text{all } y} (x - \mu_X)(y - \mu_Y)p(x, y)$

Laws of Expected Value:

$$E[c] = c$$

$$E[X+c] = E[X] + c$$

$$E[cX] = cE[X]$$

$$E[a+bX+cY] = a + bE[X] + cE[Y]$$

Laws of Variance:

$$V[c] = 0$$

$$V[X+c] = V[X]$$

$$V[cX] = c^2V[X]$$

$$V[a+bX+cY] = b^2V[X] + c^2V[Y] + 2bc\text{COV}[X, Y]$$

Laws of Covariance:

$$\text{COV}[X, c] = 0$$

$$\text{COV}[a+bX, c+dY] = bd\text{COV}[X, Y]$$

Combinatorial formula:

$$C_x^n = \frac{n!}{x!(n-x)!}$$

Binomial probability:

$$p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

for $x = 0, 1, 2, \dots, n$

Mean of Binomial: np **Variance of Binomial:** $np(1-p)$

Uniform density function:

$$f(x) = \frac{1}{b-a} \text{ where } a \leq x \leq b$$

Mean of Uniform:

$$\frac{a+b}{2}$$

Variance of Uniform:

$$\frac{(b-a)^2}{12}$$

Sample Mean:

$$\mu_{\bar{X}} = E[\bar{X}] = \mu_X$$

$$\sigma_{\bar{X}}^2 = V[\bar{X}] = \frac{\sigma_X^2}{n}$$

$$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}}$$

Sample Proportion:

$$E[\hat{P}] = p$$

$$\sigma_{\hat{P}}^2 = V[\hat{P}] = \frac{p(1-p)}{n}$$

$$\sigma_{\hat{P}} = \sqrt{\frac{p(1-p)}{n}}$$

Difference Between Two Sample Means:

$$\mu_{\bar{X}_1 - \bar{X}_2} = E[\bar{X}_1 - \bar{X}_2] = \mu_1 - \mu_2$$

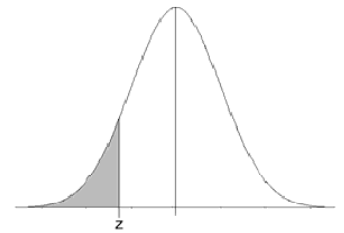
$$\sigma_{\bar{X}_1 - \bar{X}_2}^2 = V[\bar{X}_1 - \bar{X}_2] = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Confidence interval estimator of μ when σ^2 is known: $\bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

Sample size to estimate $\mu \pm \tau$ when σ^2 is known: $n = \left(\frac{z_{\alpha/2} \sigma}{\tau} \right)^2$

Standard Normal Cumulative Probability Table



Cumulative probabilities for **NEGATIVE** z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

