INTRODUCTION TO JEAN BURIDAN'S LOGIC

1. Jean Buridan: Life and Times

Buridan is best-known to philosophers for the example of "Buridan's Ass," starving to death between two equidistant equally tempting bales of hay, who appears in Spinoza, *Ethica* II, scholium to Proposition 49. But this poor fragment of Buridan's great reputation is as apocryphal as his supposed amorous adventures with the Queen of France, famous from François Villon's poem "La testament," or his founding the University of Vienna: Buridan's ass is not to be found in Buridan, though his examples are studded with asses.[1]

Our knowledge of Buridan's life is sketchy.[2] We know that he was French, but little else about his background; various examples in his writings suggest a man familiar with Picardy. Just as we do not know where Buridan was born, we do not know when he has born. He must have been born by 1300, but this is the only reliable inference we can make.

Buridan is first glimpsed in the pages of history in 1328, the rector of the University of Paris, *vir venerabilis et discretus*, presiding over a debate that took place on February 9. The next year, on 30 August 1329, he received a benefice from Pope John XXII; on 2 November 1330 he received another benefice from the same Pope, who addressed him each time as a Master of Arts. We then lose sight of him until 25 September 1339, when Buridan was a signatory to a condemnation of certain doctrines (supposedly including those of William of Ockham); during this period he received an expectation of a prebend from Pope Benedict XII. In 1340 he was again rector of the University of Paris. The last time he graces the pages of history is as a figure signing a border treaty under the authority of the University, on 12 July 1358.

* All translations are mine. See the Bibliography for abbreviations, editions, and references; when citing Latin texts I use classical orthography and occasionally alter the given punctuation and capitalization.

[1] Unless one identifies Plato as Buridan's Ass: "He [Plato] said that if I am indifferent and able to go to the left or right, for whatever reason I go to the right by the same reason I go to the left, and conversely; therefore, either I go to each [direction], which is impossible, or I go to neither until another determining sufficient cause comes along" (QM 6.5 ff. 35vb–36ra).

[2] The classic biographical work is Edmond Faral [1949], the source of the factual knowledge of Buridan's life described in the succeeding paragraphs.

From several remarks (*e. g.* TC 3.4.14) we know Buridan spent his life as a career Master in the Faculty of Arts—a rarity, for the Faculty of Arts was generally made up of students who were going on to advanced study in theology, and there was a fast turnover.

These few facts are all we know of Buridan's life. Yet we possess his works (in large measure), and in them there is a wealth of material for the philosopher. Buridan's influence and reputation were immense, both during his life and for centuries afterwards. He was known for his contributions to ethics, physics, and, perhaps most important, for his philosophy of logic. It is the latter to which this introduction and the translations are devoted. Buridan's mediæval voice speaks directly to modern concerns: the attempt to create a genuinely nominalist semantics; paradoxes of self-reference; the nature of inferential connections; canonical language; meaning and reference; the theory of valid argument. It is to be hoped that Buridan can reclaim his lost reputation among contemporary philosophers for his penetrating and incisive views on these and other matters.

2. Buridan's Treatises

The "Treatise on Supposition" [TS] is the fourth treatise of a much longer work known as the *Summulae de dialectica*, the contents of which Buridan himself describes in the beginning of the first chapter of the first treatise:

> We divide this work into nine treatises, of which the first will be about sentences and their parts and passions; the second about the predicables; the third about the categories; the fourth about supposition; the fifth about the syllogism; the sixth about dialectical lock; and the seventh about fallacies. An eighth [treatise] about division, definition, and demonstration is added, which our author did not deal with in his book; the ninth will be about the practice of sophisms—but in my lectures I shall not follow the other lectures with this last treatise.

"Our author" is Peter of Spain,[3] and "his book" is the *Summulae logicales*. Buridan adopted this work as the basis of his lectures on logic, for the reason he states in the last sentence of the Proemium to the *Summulae de dialectica*:

> Wishing to say certain general things about the whole of logic without excessively painstaking investigation, I shall particularly rely

3 The edition of Peter of Spain I have used to check against Buridan's text—the points of comparison are virtually nonexistent—is De Rijk [1972].

> upon the brief treatise of logic the venerable Doctor, master Peter of Spain, has already composed, analyzing and supplementing what he wrote and said in another way when at times it seems to me opportune.

The *Summulae de dialectica* is written as a commentary on Peter of Spain. But there were topics Peter of Spain said little or nothing about; and sometimes what he did say Buridan regarded as hopeless. In the former case Buridan wrote an independent treatise, as he remarks for the eighth and ninth treatises listed. In the latter case Buridan simply jettisons the text written by Peter of Spain and substitutes his own text, commenting upon it. Such a case is the fourth treatise, the treatise on supposition.

The *Summulae de dialectica* is one of two or three major logical works we have of Buridan's: the ninth treatise seems to have been considered an independent work, called the *Sophismata*.[4] The other work is the *Treatise on Consequences* [TC], translated here, an advanced independent investigation in logic. If TS was the textbook for Buridan's introductory course on logic, TC is a handbook to the logic graduate seminar. The rest of Buridan's works on logic are *quaestiones* on the standard logical corpus: Porphyry's *Isagoge*, and Aristotle's *Categories*, *De interpretatione*, *Prior Analytics*, *Posterior Analytics*, *Topics*, and *Rhetoric*.[5]

For TS I have used the edition of the text given by Maria Elena Reina in "Giovanni Buridano: Tractatus de suppositionibus," *Rivista critica di storia della filosofia* (1959) 175–208 and 323–352. For TC I have used the edition of the text given by Hubert Hubien, *Iohannis Buridani tractatus de consequentiis: Édition critique*, in the series *Philosophes médiévaux* Vol. 16, Université de Louvain 1976. Divergences from these texts are noted in the translation where they occur.

3. Meaning and Mental Language

3.1 Levels of Language

Buridan and other logicians of the fourteenth century were inspired by a remark Aristotle made in *De interpretatione* 1 16^a3–8:[6]

4 There is a modern edition of this work (although not a genuinely critical edition) in Scott [1977]. An English translation is available in Scott [1966]. The last chapter, *Soph.* 8, has been newly edited and translated in Hughes [1982]. Buridan's references to the *Sophismata* in TS are not consistent: they are future and past, frequently within a short compass—see *e. g.* TS 3.7.16 (past) and 3.7.23 (future).

5 Faral [1949] 496 lists several lost works of Buridan that probably dealt with logic: *De syllogismis*, *De relationibus*, and works pertaining to metaphysics as well as logic.

6 I have not translated Aristotle's original Greek, but rather the Latin version given by

> Therefore, the things that are in speech are the marks (*notae*) of the passions that are in the soul, and the things that are written are [the marks] of those that are in speech. And just as letters are not the same for all people, so the utterances are not the same. But the first things of which these utterances are marks are passions of the soul, the same for all people, and the things of which the latter are likenesses (*similitudines*)—real things—are also the same [for all people].

There are three distinct levels of language: *Written*, *Spoken*, and *Mental*. Each is a fully developed language in its own right, with vocabulary, syntax, formation-rules, and the like. These languages are hierarchically ordered; elements of Written Language (inscriptions) are the "marks" of Spoken Language, elements of Spoken Language (utterances) are the "marks" of Mental Language. The elements of likenesses *concepts*, which are mental particulars, "acts of the soul" (QM 5.9 fol. 33rb). The relationship of one language to another is not holistic, but piecemeal. A particular inscription will be related to a particular utterance, a particular utterance to a particular concept. Buridan calls the relations obtaining among these languages *signification*. A written word immediately signifies a spoken expression; a spoken word immediately signifies a concept of concepts of the mind; the concepts of the mind signify those things of which they are they natural likenesses. The written and the spoken term have an "ultimate" signification, namely what is conceived by the concept (TS 3.2.8, *Soph.* 1 Theorem 2). Clearly, a change in the signification of a term of one level will change the (ultimate) signification of the lower-level terms signifying it. The relation among the various levels, then, is rather like *encoding*.[7] Immediate signification is conventional, but the signification of concepts is natural, and indeed universal; concepts are "the same for all."

3.2 Nominalist Semantics and Equiformity

Logic, for Buridan, will include Aristotelian psychology as a component (QM 6.12 fol. 41vb): the constituent elements of Mental are concepts, and Buridan holds substantive doctrines about (*i*) the causal theory of concept acquisition, and (*ii*) the activity of the mind in combining concepts. Logic, of course, is not simply psychology, for logic includes a normative dimension.

The central theme dominating Buridan's logic, and indeed his ap-

Boethius in Minio-Paluello [1965]; Boethius conflates certain distinctions present in the original text: see Kretzmann [1974].

[7] This suggestive analogy is made in Normore [1976] 13.

proach to philosophy in general, is nominalism. Mediæval nominalism took many different forms, but at root is the denial of universal or metaphysically shared entities.[8] This has several forms in Buridan: he will deny the existence of universals; he will minimize the number of categories; he will reject various 'abstract entities,' such as propositions. This last is important, and will be discussed in more detail in Section 5.5; what is relevant to our purposes here is Buridan's attempt to create a nominalist semantics, a theory of logic and language that is based on particular inscription, utterances, and thoughts. The crucial notion in this enterprise is *equiformity*.[9]

Buridan rejects any abstract notion of 'proposition' to serve as truth-bearer and fundamental constituent of semantic analysis, and replaces it with a theory of logic and language designed to apply to the individual inscription.[10] Of course, all inscriptions are particular individuals, discrete and different from one another. Buridan is thus faced with a familiar dilemma: the laws of logic are general, and govern classes or types or sets of inscriptions, not individual inscription-tokens; 'class,' 'type,' and 'set' are abstract entities. Now whether mediæval nominalism ought to countenance such entities is not clear: they are not obviously the sort of metaphysically shared item it proscribes. But they do most of the same work, and they are abstract, and suspiciously so; mediæval nominalists should look on them with a jaundiced eye. But how is logic possible without recourse to such abstract notions?

There are many familiar attempts to escape the horns of this dilemma. Quine, for example, endorses sets, but argues that as abstract entities go they are pretty well-behaved (so well-behaved that he insists on calling himself a nominalist. Mediæval nominalism might take the same route, although then it must account for the ontological status of such abstract entities. Others have tried to give reductivist accounts of 'set' or 'class' or 'type,' suggesting that they are no more than reducible equivalence classes based on resemblance or similarity. This is Buridan's approach.

But according to the latter, is not 'resemblance' equally an abstract entity, a universal of the sort proscribed? Buridan avoids reifying

[8] Note that this formulation does not necessarily involve denying the existence of objects not in space and time, or immaterial objects—God is an example; nor does it require the denial of possible or future objects. What it does entail is that any such objects must be particulars.

[9] This useful term was coined by Hughes [1982] 5; we'll use it to refer to expressions that are sufficiently similar in relevant respects, although distinct by other criteria.

[10] In the rest of Section 3.2 I shall, for simplicity's sake, discuss only inscriptions; the same points can and should be made for utterances and individual acts of thought.

'resemblance' by interpreting it pragmatically. Each inscription-token is unique. Logical laws are stated for inscriptions that are similar (*similes*), that is, sufficiently resemble one another in the relevant respects. But there is no saying what respects are the relevant ones, or which degrees of resemblance are sufficient; these factors depend on the context, on our interests and aim. Inscriptions treated as the same in a given context are called "equiform," but there is no such thing as equiformity *tout court*, and so Buridan's nominalism is not compromised. Logical principles are therefore restricted to equiformity-classes of inscriptions. In some cases we may be forced to be very restrictive in our choice of equiformity-class:

The sentence printed below this one is false

The sentence printed below this one is false

$$2 + 2 = 17$$

The first two inscriptions cannot be equiform with respect to truth-value, for the first is false and the second true (QM 5.1 fol. 26vb). Equally, when discussing the Liar, for example, we have to carefully distinguish individual inscription-tokens (*Soph.* 8: Hughes [1982] 8.4.3, 13.5–6, 15.8.2).

Buridan seems to have kept an open mind on how arbitrary an equiformity-class may be. There may be an eventual limit, imposed by the causal theory of concept-formation and the requirement that concepts be natural likenesses of the things of which they are concepts, but Buridan does not say so explicitly . For most cases a handy practical criterion will serve to demarcate equiformity-classes; Buridan frequently uses the rule that terms supposit and appellate in equiform sentences just as in the original sentence (or part of a sentence: this will be important for the theory of consequences): see TC 3.7.41.

Buridan's pragmatic interpretation allows him to deal with classes of *possible* inscriptions. In general the laws of logic are not stated for actual sequences of inscriptions but rather specify what sequences of inscriptions would be acceptable, if formed.[11] This is, of course, an aspect of the normative element of logic, but for nominalists the point is deeper: logical laws cover not only actual inscriptions but possible inscriptions as well. There is no other way to describe basic principles of logic. For instance, any sentence can have a contradictory, but in actual fact such a contradictory may never have been inscribed; how then can we say that a non-actual inscription must be true if its actual contradictory is false? Buridan sensibly states his principles for actual and possible inscriptions, and, in some cases (such as the

[11] See TC 1.3.8 and TS 1.8.72, in which Buridan states how his logical laws apply to the Mental correlate of inscriptions, and the discussion of consequences in Section 7.2 below.

distinction of the possible and they possibly-true), they must be carefully distinguished, shedding new light on philosophically unexplored terrain.

A final point. The term 'equiform' might be read as 'having the same (or similar) logical form,' which is narrower than the usage I have given it. To be sure, Buridan does sometimes use it in this way, but then only as a special case of the more general sense: the relevant respects of similarity in question are the series of syncategorematic terms. Unless specifically noted, I shall follow Buridan in his practice of taking 'equiform' to mean any similarity-class of inscriptions, where the respects in which the inscriptions are similar are defined contextually.

3.3. Mental as an Ideal Language

Mental Language is of the utmost importance for Buridan's logic. For while Mental is a natural language, in a way in which Spoken or Written are merely conventional, we should not understand this the way today "natural languages" such as English or French contrast with symbolic language in rigor; rather, Mental is a natural language perspicuous in rigor. The basic claim, common to Buridan and many other fourteenth-century logicians, is that *Mental is a canonical language*, an ideal or logically perfect language.[12] This claim involved five theses:

[1] Mental is a universal language.
[2] Mental is adequate in expressive power.
[3] Mental is disambiguated.
[4] Mental is nonredundant.
[5] Mental sentences display their logical form.

Each thesis calls for further comment.

Ad [1]. The universality of Mental is a matter of its structure, not its content. That is, we do not necessarily all have the same stock of concepts; I may completely lack the concept of lion, which you possess, due to our different past interaction with the world. But the structure is the same for all, meaning (roughly) that we all have similar mental abilities: we can all combine simple concepts into complex concepts, for example. Moreover if two people each have the same term of Mental, then their concepts differ only numerically: your concept of a lion and mine may have been acquired through the experience of different lions, but the concepts are equiform, that is, for most practical purposes they may be regarded as the same.

[12] This point was first made with respect to Ockham, not Buridan, in Trentman [1970], but it holds generally for fourteenth-century logicians. Trentman does not consider [1]–[5], but they seem the necessary requirements for a language to be counted as 'ideal.'

The universality of Mental will provide the foundation for logic to be a full-fledged (Aristotelian) science.

Ad [2]. The adequacy of a language is a matter of its resources; a language is expressively adequate if it has the resources to express whatever can be expressed. But Mental is literally the language of though: we think in Mental. If we were telepaths we would speak to one another in Mental; angels, who are telepaths, do so.[13] Therefore Mental is expressively adequate, for whatever is thought must be expressed in Mental.

This approach will have certain difficulties: it is not immediately obvious how to handle non-declarative uses of language, performatives, and certain indexical terms. But the general aim of the project is clear even if not all of the details are; we shall leave them to one side.

Before we discuss [3] and [4] we need to clarify the role of translation-rules. A translation-rule correlates expressions of ordinary language with their perspicuous canonical representations. The best-known modern example of a translation-rule is Russell's rule about definite descriptions, where a sentence such as:

The present King of France is bald

is correctly "translated" in first-order logic as:

$$(\exists x)(Kx \wedge Bx \wedge (\forall y)(Ky \wedge By \rightarrow (x = y)))$$

For Buridan translation-rules are principles saying which elements of Mental an inscription or utterance corresponds to. We may call this correspondence *subordination*.[14] An ambiguous utterance or inscription is ambiguous because it is subordinated to more than one concept or complex on concepts in Mental; utterances or inscriptions are synonymous because they are subordinated to the same concept of complex of concepts in Mental. The utterance or inscription 'bank' is ambiguous because it could signify the concept of the side of a river, or it could signify the concept of an institution dealing in money. The utterances 'Tully' and 'Cicero' are synonymous, because they are subordinated to the same concept, the concept of a particular individual. Buridan takes ambiguity and synonymy to be features only of Spoken and Written. The exact nature of the translation-rules he proposes

[13] This point is made by Joan Gibson [1976].

[14] "Subordination" is a technical term used by Ockham; as used here it expresses (*i*) which concept(s) of Mental are immediately signified by an utterance; (*ii*) which concept(s) of Mental are immediately signified by the utterance that an inscription immediately signifies.

ins extremely important, for it is a key element of his nominalist approach. We shall discuss some principles when we discuss the distinction between absolute and appellative terms in Section 4.2. If the proper explanation of ambiguity and of synonymy is found in the translation-rules, then clearly Mental cannot have ambiguous or synonymous terms, for there is nothing further to which concepts of Mental are subordinated.

Ad [3]. The terms of Mental are concepts, which have a natural likeness to their objects. An ambiguous term in Mental, then, might be a concept that truly applied to two distinct kinds or groups of things. We may then have a broader concept than we originally thought, but not an ambiguous concept. More likely, though, we have a disjunctive concept, a concept having internal logical structure. Thus no term in Mental is ambiguous.

What of amphiboly, that is, ambiguous sentences, in which no single term is ambiguous, such as "Flying planes can be dangerous"? Buridan does not want to allow amphiboly in Mental, and equally does not want to compromise the principle that the translation-rules correlate terms with concepts. He avoids this problem gracefully by a general rule that the subjects of Mental sentences always stand for what they signify.[15] He justifies this rule by his realism about Mental as the "language of thought": to have a concept is a natural likeness; to have a concept that does not stand for that of which it is the natural likeness would be for a person to not think about what he is thinking about—which is impossible.

Ad [4]. The nonredundancy of Mental is a matter of its containing no synonyms. Suppose that 'vixen' and 'female fox' are in fact synonymous; they then differ only by their internal make-up, which is a matter of orthography for inscriptions and phonology for utterances. The analogy for Mental, then, would be to have two concepts which differ only in their "internal make-up." What could be analogous to orthography or phonology for concepts? Nothing at all, if we consider only simple concepts.[16]

[15] That is, they are always in personal and never in material supposition; these terms will be explained in Section 6.2. For this point, see for example *Summulae de dialectica* 7.3.45, where Buridan writes, "Nevertheless, it should be known that there is only material supposition in the significative utterance, as it seems to me, for the reason that no term in a Mental sentence supposits materially, but always [supposits] personally, for we do not use Mental terms conventionally as we use Spoken or Written [terms]: no Mental expression has several significations or acceptations; the passions of the soul are the same for all, as are those things of which they are the likenesses, as is said in *De int.* 1 [16^a3–8]." The text translated here is edited by Ebbesen in Pinborg [1976] 155.

[16] This answer needs to be made more precise, since I might acquire the concept of a

But if we allow complex concepts, it seems as though Mental must have synonyms, since the concepts corresponding to 'vixen' and to 'female fox' differ in their logical structure: the former appears simple, while the latter seems to combine the two concepts corresponding to 'female' and to 'fox.' Will not logical structure play the role in Mental played by orthography in Written and phonetics in Spoken?

Buridan's answer is that it does not, at least not so as to produce redundancy. The concept corresponding to 'female fox' does have a complex logical structure, but its signification differs from that of the concept corresponding to 'vixen.' For the signification of complex expressions Buridan endorses the *Additive Principle* (TS 2.3.10, *Soph.* 2 Theorem 5, QM 4.14 fol. 23vrb):

> The signification of a complex expression is the sum of the signification of its (categorematic) parts.

Hence the concept corresponding to 'female fox' signifies all females, foxes or not, and all foxes, female or not. This is quite reasonable if we think of signification as everything a term brings to mind—perhaps the original meaning of 'signification.' Of course, 'female fox' will stand only for vixens; that is not signification, but supposition, discussed in Section 6.1.

Ad [5]. A sentence of Mental perspicuously displays its logical form. To see how this is so, let us return to a problem postponed in the discussion of [4], the problem of complex expressions with the same logical components, such as "Socrates is taller than Plato" and "Plato is taller than Socrates." In both expressions we have the same components: a concepts corresponding to Socrates, a concept corresponding to Plato, and a concept corresponding to 'is taller than.' What distinguishes them? The utterances are distinguished by the temporal order in which the phonemes are uttered, and hence by the medium of time; the inscriptions are distinguished by the spatial sequence of the letters, and hence by the medium of space.

Is there such a discriminatory medium for Mental? Buridan thinks there is no need for one: such a medium would distinguish "Fred or Barney" from "Barney or Fred," and this is too fine-grained for Mental, for the latter are genuine synonyms. The point of such examples is that certain

lion by using Simba as a paradigmatic instance while you use Leo as a paradigm. But that is a difference dependent upon you and me having different concepts, whereas the question we are faced with is whether a single person can have two such concepts internally different. The claim that a single individual cannot form two different concepts by choosing two different paradigmatic instances—which he perhaps does not realize are of the same kind—is a substantive thesis, endorsed by Buridan, in the philosophy of mind, and relies on a theory of concept-acquisition.

logical operations are sensitive to *order* while others are not. Logical operations in Mental are thus like functions, taking concepts into expressions, some of which are sensitive to order (*i. e.* nonsymmetric) and others not. These 'functions' are themselves concepts: Buridan calls them *complexive* concepts (TS 2.3.12–13, 2.4.2–3), and they act to combine simple or complex concepts into complex concepts. They correspond to logical operators, that is, syncategoremata, discussed in Section 4.1. The complexive concepts which are the most important correspond to the two forms of the copula, to 'is' and 'is not,' which combine concepts to produce a Mental sentence.[17]

The complexive Mental concept corresponding to the affirmative copula will be nonsymmetric: that is what allows Buridan to construct different theoretical roles for the subject and predicate. The concept corresponding to 'is taller than' will be asymmetric, so TALLER (Socrates, Plato) implies NOT-TALLER (Plato, Socrates). The complexive concept corresponding to conjunction is symmetric: AND (Fred, Barney) = AND (Barney, Fred).

Thus expressions of Mental are individuated by their semantic properties alone; Mental therefore behaves like the semantics of Spoken or Written. The logical form of a Mental sentence, then, is displayed in its essential composition. Not that this need not be taken as a literal composition: the thought of a complex need not be a complex of thoughts. But the thought must have an internal logical articulation as we have described it.

Mental is thus a canonical language in virtue of [1]–[5]. Hence it is the primary concern of the logician. The surface structure of Mental is identical with its logical structure, and reflects the logical form of Spoken or Written.

4. The Properties of Terms

4.1 Syncategorematic Terms

To have a theory of logical form, we have to distinguish logical from non-logical constants; this is roughly the distinction between *syncategorematic* and *categorematic* terms. We take such a difference as primitive: logical constants are listed separately and they appear in the syntactical

[17] Buridan explicitly states these principles in *Summulae de dialectica* 1.3.2: "A Mental sentence consists in the complexion of concepts, hence presupposing simple concepts in the mind and superadding a complexive concept by which the intellect affirms or denies one of the concepts of the remainder... that complexive concept is called the copula of the Mental sentence. And so it is clear how the subject and the predicate of a Spoken sentence immediately signify the Mental subject and predicate, while this copula 'is' signifies an affirmative complexive concept and the copula 'is not' signifies a negative complexive concept." This text is edited in Pinborg [1976] 87.

rules in special ways. Buridan draws the distinction in terms of signification. Purely syncategorematic terms lack an ultimate signification, but this does not mean they lack all signification; every word which can be put into a sentence is imposed for some signification (TS 2.3.7). Syncategoremata have only an immediate signification, and have only an ultimate signification in combination with categorematic terms (TS 2.3.6).

What do syncategoremata immediately signify? Purely syncategorematic terms signify simple concepts which are complexive: the semantic functors described at the end of Section 3.3 (TS 2.3.11–143). Since the ultimate signification of a sentence; "Some men are sexists" and "No men are sexists" signify the same, as indeed do "God is God," "God is not God' and the term 'God.' The logical form of a sentence can be described as the exact series of syncategoremata (TC 1.7.3–4).

Some syncategorematic terms operate on terms to produce terms; others operate on sentences to produce sentences (TS 2.4.4). Whichever we examine, at the heart of all logical constants is the notion of their *scope*, that is, which terms in a sentence they affect. Buridan usually gives his scope-rules syntactically, and often simply explains the action of a syncategorematic term in a sentence by describing its effect of terms which come before or after the syncategorematic term. Such relations will correspond to order-relations in Mental, as we have seen.[18]

There are two forms of the copula, each of which corresponds to a complexive concept in Mental: 'is' and 'is not' (TS 2.3.12). Buridan flirts with the suggestion that the Mental copula is tenseless (TS 3.4.8), but he does not explore it in any detail. The copula produces sentences from terms or expressions, according to very complicated rules about what can act as a subject or predicate; a partial list of such rules is given in TS 2.6.1.

A negative particle (such as *non*) may act in two ways (TS 2.2.7). It may be a sentential-functor taking sentences to sentences; Buridan calls such a negation "negating" and it acts upon the copula. Or it may be a term-functor taking terms to terms; Buridan calls such a negation "infinitizing," and it acts upon the individual term, producing what is called an "infinite" term (TS 3.7.35). Buridan's logic will therefore distinguish sentences not only by their quantity but also by the character of the negation occurring within them, their quality.

Signs of quantity, that is, quantifiers, are more complex. Buridan

[18] I have translated Buridan's examples so that these rules apply directly to the translated version; there are some awkward results, such as "Some B A not is" rendering the uncommon idiom for negatives described in TC 1.8.70, or the rules describing confused supposition; I beg the reader's indulgence.

argues that such signs of quantity, no matter where they occur in the sentence, not only affect the term immediately following them but also act as "conditions of the whole sentence" (TS 2.2.15), a conclusion requiring careful argument (TS 2.28–14). Buridan characterizes such signs of quantity as either affirmative or negative, depending on whether they involve a negation, and as either distributive (that is, "universal") or as particular (that is, "existential"). There are several kinds of quantifiers, too; there is at least one for each category (TS 3.7.4), and each has a corresponding identificatory relative-term (TS 4.2.3, TC 3.7.19–2). For example, a universal affirmative sign in the category of Quality is "however," and it's corresponding identificatory relative-term is "such," as in the sentence "however Socrates is, such is Plato," *i. e.* every quality which Socrates has Plato also has (Rule RT in TS 4.8.1, or TS 3.7.27). Moreover some signs distribute parts of an integral whole; others a universal whole.

Conjunction and disjunction are both term-forming functors applied to terms and sentence-forming functors applied to sentences (TS 2.3.13) obeying the usual logical rules, except for certain uses such as forming 'conjunctive terms,' *e. g.* the term 'Peter and Paul' in "Peter and Paul lifted a table" (TS 3.2.3). Conjunctive terms are discussed in TS 2.6.64–67 and 2.6.77. Note that the conjunction of terms, their 'collective' sense, is distinct from term-combination as in *e. g.* adjective-noun expressions.

Terms such as 'if' and illative particles such as 'therefore' and 'hence' are sentential functors, producing consequences, just as the copula produces categorical sentences (TS 2.3.13, TC 1.3.2).

In addition to such typical syncategoremata, there are also exceptives (terms such as 'but' or 'except'); delimitives ('only,' 'at most,' 'at least'); and many others, causing Buridan to exclaim in the first treatise of the *Summulae de dialectica* that syncategoremata "are the source of virtually all the confusions which plague logic." Such syncategoremata are often mixed; in TS 2.4.4 Buridan suggests analyzing 'only' as it appears in "Only a man is running" as "A man is running and nothing other than a man is running," so that the concept corresponding to 'only' includes an affirmative copula and a negative distribution.

Buridan uses the analysis of syncategoremata to say what pertains to the logical form of a given sentence (TS 1.7.3): the copula, negations, signs of quantity, the relative-terms, the number of constituent elements, connectives, and most of all, the order in which these occur. Examples of each are given in TC 1.7.4.

Some terms are neither purely syncategorematic nor purely categorematic but are "mixed" or "mediate," such as 'somewhere,' which involves the purely syncategorematic term 'some' and implies the categorematic term 'place,' for it restricts the quantification to places (TS 2.3.1); such terms are present only in Spoken or Written, being analyzed into their components in Mental.

4.2. Absolute and Appellative Terms

Purely categorematic terms have ultimate signification and do not imply any syncategoremata (TS 2.3.1). Some categorematic terms have simple concepts corresponding to them, and others complex concepts (TS 2.4.5). The grammatical form of an inscription or utterance is not in general a good guide to the complexity or simplicity of the corresponding concept; since words signify by convention we can treat the word 'A' as equivalent to the sentence "Some man is running" (TS 2.4.5), and then 'A' immediately signifies a complex concept. How can we tell the difference?

The key distinction here is between simple (or incomplex) concepts and complex concepts. Buridan suggests that we can distinguish terms which correspond to a simple concept, which he calls *absolute* terms, from those which do not, by the theory of definition.

Definition, as it occurs in Spoken and Written, is the analogue of complexity in Mental; 'vixen' is definable as 'female fox,' and if a person possesses the concepts 'female' and 'fox' then he can form the complex concept 'female fox' to which the inscription or utterance 'vixen ' is subordinated. Thus the composition of concepts in Mental is reflected by the process of definition.

Again, if some Mental terms are literally composed of others then we impose a hierarchy on Mental terms: the primitive terms are the incomplex concepts in Mental, which we call absolute; others are produced through logical composition with the syncategoremata. Buridan argues that there must be such simple concepts:[19] "If anyone were to say that complex concepts exist, then they are composed of simples, for there can be no regress to infinity in the resolution of concepts." In QSP 1.4 Buridan merely argues for the existence of such simple or incomplex concepts; in TS 2.4.5 he explicitly says that 'man,' 'whiteness,' and 'white' correspond to such simple concepts.[20] Equally, purely syncategorematic terms correspond to simple, though complexive, concepts (TS 2.4.3).

[19] The argument is alluded to in QM 7.21 fol. 54vb, but the best presentation is given in QM 1.4 fol. 5ra.

[20] Obviously, the relevant form of simplicity in question is something like logical simplicity; the concept 'man' is not simple in regard to containing distinguishable physical parts (legs and arms, for example). Roughly, we may regard all terms appearing on each category-tree as *prima facie* candidates for simple concepts. Exactly what makes

There were two competing requirements on definitions in mediæval philosophy: (*i*) the definiens was to be synonymous with the definiendum; (*ii*) the definiens was to express the real nature or essence of the definiendum. Definitions satisfying (*i*) were called *nominal*, expressing the *quid nominis*, because they did not specify the nature of the definiendum and only gave information about how the term is applied (and hence are about the "name"; those satisfying (*ii*) were called *real* or *quidditative*.[21] With this technical machinery in place, we can begin sorting out absolute and non-absolute terms. Let us carefully set out Buridan's exact claims:

(1) A term corresponds to a complex concept if and only if the term has a nominal definition.

In TS 2.4.1 Buridan says that terms that correspond to complex concepts have nominal definitions; in QM 4.14 fol. 23va and QSP 1.4 fol. 5rb he says that terms with nominal definitions correspond to complex concepts. Together these yield the equivalence stated in (1), which is explicitly endorsed in Rule Sup-12 (TS 2.6.1). From (1) we may easily derive the next thesis (2.4.1):

(2) A term correspond to an incomplex concept if and only if the term has no nominal definition.

The motivation for (1)–(2) is obvious; if a term is subordinated to a complex concept, then by definition it is synonymous with the expression stating how the relevant concepts are combined. This is why Buridan suggests that (*i*) indefinable substantial terms correspond to simple concepts (QM 4.14 fol. 23va and QSP 1.4 fol. 5vb); (*ii*) purely syncategorematic terms correspond to simple complexive concepts (TS 2.4.3). Thus we may view all non-absolute terms as mere abbreviates for their nominal definitions. Mental, for obvious reasons, need contain only absolute terms and purely syncategorematic terms; complex concepts may be logically constructed, by complexive syncategoremata, from simple concepts.

The thesis complementary to (1)–(2) would be that a term corresponds to a simple concept if and only if the term has a real definition. But here we must introduce another distinction among categorematic terms:

such a simple concept "simple" is a very difficult question; we shall have something to say about this matter below.

[21] For example, QM 7.5 fol. 44va: "Some definitions are simply quidditative, which precisely indicate what [a thing] is, such that they do not indicate that of which it is or that from which it is... There are other definitions expressing the *quid nominis*; indeed, often some name involves (*implicat*) exceedingly many diverse concepts of diverse things, and a definition expressing the *quid nominis* ought to designate those diverse concepts explicitly. Such definitions are fitting for substantial as well as accidental terms."

some terms are *appellative*, and others are not.[22] Appellative terms are more complicated than absolute terms. We shall investigate their nature carefully.

Buridan characterizes at least some appellative terms by the *Remainder Principle*:[23]

> If a term signifies something it doesn't stand for, the term is appellative.

By 'stand for' Buridan means what a term refers to or supposits for (discussed in Section 6). Appellative terms falling under the Remainder Principle have no real definition (QM 7.5 fol. 44va); presumably they have a nominal definition and so by (1) correspond to complex concepts. Buridan lists several examples of appellative terms: (*i*) every term in an oblique case;[24] (*ii*) nondenoting terms, which may be *impossibilia* such as 'round square' or *figmenta* such as 'centaur' (TS 1.4.7); (*iii*) concrete terms in categories other than Substance;[25] (*iv*) transcendental terms convertible with 'being,' such as 'thing,' 'one,' and the like (QM 4.5 fol. 15vb); (*v*) the term 'potency' (QM 9.6 fol. 59ra); (*vi*) most combinations of terms, so that complex subjects and predicates are connotative. What is more, terms in oblique cases combined with a substantive connote the relation between the subject and what the term would stand for in the nominative case. In this case, or when an attributive adjective is combined with a substantive, the term appellates the adjacence of the associated property with the subject.

What, exactly, is appellation? Buridan explains it thus (TS 1.4.1, 5.1.1, 5.2.5; *Soph.* 4 Remark 2):

> A term appellates that which it connotes as in some way adjacent or non-adjacent to that for which it stands or is apt to stand.

'Connotation' is a semantic relation, a kind of indirect or oblique form of signification.[26] The concrete accidental term 'white' stands for white things,

[22] The two distinctions absolute/non-absolute and non-appellative/appellative are often identified, but this is a substantive semantic thesis and is open to question: in TS 1.4.8 Buridan says that 'white' is appellative, and in TS 2.4.5 he explicitly says that 'white' corresponds to a simple concept. Therefore some appellative terms are absolute.

[23] See TS 1.4.1, 5.1.1, 5.2.5; *Soph.* 1 Theorem 6. Modern scholars often state the Remainder Principle as a biconditional, but Buridan *never* states it in this way—nor should he, since he countenances 'non-external' appellation (discussed below).

[24] Rule App-6 in TS 5.4.1 and the discussion in 5.4.4–5.4.7; such terms, like attributive adjectives, also appellate in combination with a term in the nominative case (TS 1.4.4-5).

[25] Nonsubstantial abstract terms are more difficult; they need not have appellation, and are categorized by reduction (QM 4.6 fol. 17va).

[26] Buridan's usage is not regular; sometimes he characterizes terms as connotative, par-

connotes whiteness, and appellates whiteness insofar as it inheres in those things (TS 1.4.8). Equally the term 'wealthy' stands for a man, connotes his riches, and appellates those riches as adjacent to him as a possessor. 'Adjacence' is Buridan's general term for the metaphysical relation between two items, *e.g.* inherence; non-adjacence its opposite. Thus 'blind' appellates vision privatively, as non-adjacent to its subject.

In most cases, a term connotes something other than that for which it stands, and so is appellative by the Remainder Principle. but there are exceptions. In QM 7.4 fol. 44ra Buridan says that the appellative term 'creative' connotes a power which is not really distinct from the possessor of the power; it has no 'external' connotation. Buridan mentions 'extraneous' and 'distinct' (*alienae*) connotation in QM 4.1 fol. 13ra, and in TS 1.4.6 Buridan says that 'rational' in the combination 'rational animal' connotes no accident, for there is no appellation of a distinct (*alienae*) disposition: here it is a constitutive differentia.[27] These issues immediately embroil us in metaphysics, for Buridan holds that such modes of adjacence are all Aristotle's categories really amount to. That is, the primary metaphysical types of adjacence simply *are* the Aristotelian categories, which are therefore not classification of (different kinds of) beings. Corresponding to these types of adjacence will be various modes of predication, each involving a different inherence-relation. Here is how Buridan puts the matter in QM 4.6 fol. 17va:

> There are ten categories or generalissima because concrete accidental terms are connotative...the categories should be distinguished by the distinct modes of predicating (*modos praedicandi*) [something] of primary substances—[that is], of singular terms contained under the genus *quid* or *aliquid*...

The same point is stated more forcefully in *Soph.* 4 Remark 3:

> Thirdly, it should be remarked that the different modes of predication, such as *in quale*, *in quantum*, *in quando*, *in ubi*, how one thing is related to another, and so forth are taken from (*proveniunt*) the different modes of adjacence of the things appellated to

ticularly in his commentaries on Aristotle, and sometimes he calls such indirect signification appellation, particularly in his independent logical works. This looseness is not vague; his intention is usually clear in context.

[27] There is a problem here: we might take 'rational animal' to be the real definition of 'man,' and hence to correspond to a simple concept. But in QM 7.21 fol. 54vb Buridan says that definitions by genus and differentia are nominal, not real. There are two reasons for rejecting this claim: (*i*) if definition by genus and differentia is not 'real,' then we have no example of real definitions, and indeed may lose our intuitive grasp on this notion altogether; (*ii*) 'man' cannot be synonymous with 'rational animal' by the Additive Principle.

> the things for which the term supposit. The different categories are taken *(sumuntur)* from these different modes of predication.

Buridan discusses such modes of adjacence in QM 5.8; he states the key problem, Bradley's regress, in fol. 31rb-va, and in fol. 31vb asserts that there is a "mode of relation" he calls an 'inseparable disposition,' which is the inherence of an accident in a subject. These dispositions are inseparable; to destroy them is to destroy the inherence, and conversely; and "they are accidents which are inseparably related to their subjects in this manner."[28] These 'dispositions' are qualities of qualities which are inseparable, but they are not a new kind of entity; they are unusual entities of an old kind, namely qualities.

The semantic counterpart of these ontological issues is now clear. a term like 'white' (*i*) *stands for* (supposits for) an individual person; (*ii*) *connotes* the quality whiteness; and (*iii*) "appellates the quality insofar as it is adjacent to what it stands for," that is, *appellates* the special disposition of inherence (TS 1.4.8), which itself is a quality (indeed a quality of a quality), inseparable from the white thing itself without destroying the white thing. Of course, "to signify the added disposition is not to signify that the disposition is added" (QM 5.7 fol. 31ra). A term is therefore appellative if it is apt to satisfy (*i*)–(*iii*).[29] Obviously, we may treat the semantic relation of 'appellation' as a kind of naming, denoting the inseparable disposition which is the appropriate form of adjacence. For 'wealthy' we may treat the mode of adjacence as various mental qualities of the people involved which constitute the recognition of ownership, for example.

The Mental correlate of an appellative term is a primitive complexive concept: roughly, it is the functor 'thing-having-x' which is applied to the absolute concepts of abstract qualities, *e. g.* 'thing-having-whiteness.' The primitiveness of the functor means that there need be no nominal definition of the term in question, though of course there may well be. The functor must be complexive well, since Buridan allows accidents like whiteness to exist without inhering in a particular subject,[30] and the inseparable

[28] See Normore [1984] who discusses this text and the metaphysical problems in detail.

[29] The qualification 'apt to' is necessary, because (*i*)–(*iii*) may fail in actual practice: the term may fail to refer; what is connoted may not exist; what is connoted may fail to inhere, *i. e.* this disposition be destroyed (QM 4.9 fol. 19ra; TS 5.2.6). This last case is theologically crucial, since it describes the Eucharist, in which the qualities of the bread remain without inhering in a subject. In each of these cases the truth-value of the sentence containing the term is affected.

[30] The semantic version of this principle is that 'φ-ness' is not synonymous with 'what it is to be φ,' which Buridan defends in QM 4.6 fol. 30vb.

disposition must be involved.

The theory of definition was our initial guide to whether a term was absolute or appellative. We can now see that the usefulness of the theory of definition was in that definition provides a generally reliable guide to whether a term in Mental is complex or not. But in certain cases the theory of definition is not sufficient, and we had to investigate deeper metaphysical issues. The net result is that *appellative terms are those with structure in Mental*, not merely those which have definition; such structure can be introduced by the primitive complexive functor described in the preceding paragraph.

4.3 Intentional Verbs

One of the key uses of the doctrine of appellation is to analyze the behavior of terms in sentential contexts with *intentional verbs*.[31] His remarks equally apply to the participles and nouns derived from them. Such verbs (and the terms derived from them—henceforth I shall omit this clause) differ from other verbs in that the "verbal action" each specifies "goes over" to their object not directly, into the things for which the terms supposit, but indirectly, by means of "certain mediating concepts indicated by those terms" (TS 1.6.12-14, TC 3.7.3). In particular, such verbs cause the terms with which they are construed to *appellare suas rationes*, that is, to appellate the concept or *ratio* by which the terms were imposed to signify (TS 3.8.25, TS 5.3.1 Rules App-1 and App-2, TC 3.7.5).

For Buridan, we are only mediately in touch with the things the concepts are about, by means of mediating concepts: perhaps a single concept, perhaps a Mental sentence. Such concepts give the *ratio significandi* to an inscription or an utterance; knowledge, in this sense, is not direct but requires an instrument (QM 12.8 fol. 70vb). As Buridan says in TS 3.7.10, the logical analysis of a sentence such as:

> Socrates knows *A*

is given by:

> Socrates knows *A* according to the *ratio* or concept by which the term 'A' is applied, that is, according to the concept-of-*A*

Buridan is therefore a "descriptionalist": there is no pure immediate knowing; all knowledge is mediated by some concept(s) specifying a description

[31] By "intentional verbs" I mean verbs that are (*i*) cognitive or epistemic, such as 'know,' 'understand,' 'believe,' and the like; (*ii*) verbs of desire, such as 'want,' 'intend,' 'hope,' and the like; (*iii*) promissory-verbs, such as 'owe' or 'promise' and the like. The fullest list, though Buridan acknowledges its incompleteness, is found in TC 3.4.7; their characteristics are discussed in *Soph.* 4 Sophisms 7–15, TS 3.8.24-31 and 5.3.1-8, TC 1.6.12-16 and .73-10, QM 4.8 fol. 19ra and 4.14 fol. 23va, QSP 2.12 fol. 38va.

under which we are in toiuch with an object.

Intentional verbs behave in an odd way: when a term follows the verb[32] we have an opaque context: Coriscus may not know the one approaching is his father and he knows his father; the syllogism is prevented by the apellation of a *ratio*, and we can conclude only that Coriscus does not know his father as the concept 'the-one-approaching' applies to him, which is perfectly acceptable. Buridan describes this appellation as similar to material supposition (TS 3.8.30, TC 3.7.6), for substitutivity is prevented. On the other hand, when the term precedes the verb, it is said to appellate all its *rationes* indifferently (Rule App-5 in TS 5.3.1 and the discussion; TC 3.7.7): in this case substuitutivity is preserved and we have a transparent reading:[33] the sentence "The one approaching is someone Coriscus does not know" is false, for the *ratio* 'the-father-of-Coriscus' equally applies to the one approaching.

Buridan's analysis permits the inference *a parte priori* to *a parte post* for some *ratio*, which we shall call the Entailment Principle. The converse entailment *a parte post* to *a parte priori*, generally fails, as the nature of opacity suggests. but in certain cases the latter inference does hold, and in particular for the verb 'know' (*scire*); we shall call this the Converse-Entailment Principle (*Soph.* 4 Remark 8, TS 3.8.27). The objectual version allows us to infer from "Socrates knows A" the sentence "There is an A Socrates knows," and the sentential version (Sophism 13) allows us to infer from "Socrates knows A to be φ" the sentence "There is an A Socrates knows to be φ." Buridan's key argument for the Converse-Entailment Principle is that we should otherwise have to deny that we have knowledge of items in the world.[34]

The Converse-Entailment Principle runs into two difficulties: counterintuitive substitution-instances, and the lack of existential import (*i. e.* when no A exists the sentence should be false).

Buridan takes up the first difficulty in *Soph.* 4 Sophism 14: Socrates,

[32] This is Buridan's grammatical way of drawing scope distinctions: a term appears *a parte post* and so in the scope of the verb, or *a parte priori* and so outside the scope of the verb.

[33] Strictly speaking it is incorrect to call the 'opaque' and 'transparent,' for they are not alternative ways of reading one and the same sentence but rather Buridan's way of regimenting the difference between the logical form of two different sentences; I shall use these terms as convenient abbreviates for distinguishing the two classes of sentences, which should not occasion any confusion.

[34] Note that Buridan is careful to state the Converse-Entailment Principle only for *scire*: it clearly fails for most intentional verbs, which allow for intentional inexistence.

who has been studying astronomy, has been imprisoned and cannot see the sky. We are permitted in this case to pass from "Socrates knows that some stars are above the horizon" (by his astronomical studies) to "There are some stars Socrates knows to be above the horizon." Which stars? Those which are in fact above the horizon, which in the posited case is the constellation Aries. But surely this seems false, for Socrates cannot see the sky.

Buridan's reply is to insist on the different reading *a parte priori* and *a parte post*. The constellation Aries is indeed what Socrates knows, but he knows it only under the complex *ratio* 'some-stars-above-the-horizon,' according to the Entailment Principle. This *ratio* will of course latch onto some actual stars, though Socrates does not know which. The Converse-Entailment Principle allows us to infer "[There are] some stars [which] Socrates knows to be above the horizon" and, since sustitutivity works *a parte priori*, we may infer from the fact that the constellation Aries above the horizon "The stars of Aries Socrates knows to be above the horizon." But the Entailment Principle licenses us to pass back only to "Socrates knows stars (under some *ratio*) to be above the horizon," and the *ratio* in question is 'some-stars-or-other.' This, Buridan holds, is not counterintuitive at all but the natural view of the matter.

But this answer might seem to be a cheat.[35] For "it surely trades on the peculiar characteristic [of *scire*] in that what you know must be so." Yet this is exactly what Buridan has been emphasizing all along, and is the very reason why the Converse-Entailment Principle holds only for *scire*.

The second difficulty mentioned above was that it seems I can know that thunder is a sound in the clouds even in the absence of any thunder, but by the Converse-Entailment Principle then "Any thunder I know to be a sound in the clouds" should be true, and in the posited case the subject-term is empty; affirmative sentences with empty subject-terms are automatically false (TC 2.3.3, QM 4.14 fol. 23va).[36] Buridan's answer involves his theories of ampliation and natural supposition, which we shall discuss below in Section 6.5.

[35] Which is exactly what Geach [1972] 134 calls it.

[36] by the Square of Opposition, negative sentences with empty subject-terms are automatically true. Hence "The present King of France is bald" is false, and "Pegasus is not a winged horse" is true.

5. Sentences

5.1 Sentences as Assertions

Sentences express assertions: an inscription or utterance is a sentence only through subordination to a Mental sentence, which is an act of thinking. Inscription which are not being read, utterances which are not heard (as a tape recorder playing in the wilderness), are sentences in only a derivative fashion. To construe sentences assertions, as Buridan does, has three important consequences. First, only a Mental sentence is true or false; inscriptions and utterances have truth-value only derivatively, as we call a furry winter coat 'warm' (TS 1.3.3-11). Strictly, only the act of thinking which is the Mental sentence is the act of assertion. Second, no part of an assertion may itself be an assertion, as Frege also maintained; hence no part of a sentence is itself a sentence (Rule Sup-6 in TS 2.6.1, Hughes [1982] 13.4.1.1.1). However, a sentence may have constituent parts which are equiform to or supposit for sentences.[37] Third, the reason for the existential import of universal affirmatives is clear: a sentence like "All swans are white" is not to be understood as the disguised conditional $(\forall x)(Sx \rightarrow Wx)$, from which $(\exists x)(Sx \wedge Wx)$ does not follow, but rather as something like "Consider the swans: Each is white" or "About the swans: They are all white," where 'Consider the swans' or 'About the swans' captures the assertive force of a sentence. The existential claim "Some swans are white" is then "Consider the swans: some are white" which in fact does seem to follow from the universal sentence.[38] This is why Buridan takes quantification to be a *condition* of the whole sentence, as discussed in Section 4.1.

Buridan says that every sentence is either *categorical* or *hypothetical* (TC 1.3.2), which is an exclusive and exhaustive division.[39] These should

[37] Buridan will speak loosely about the 'sentences' which make up a consequence, but this is a permissible looseness, not a deviation from his doctrine. It is therefore incorrect, strictly speaking, to represent Buridan's logic by means of the propositional calculus; it is equally incorrect to metalinguistically quote sentential letters. But as Buridan often speaks loosely, relying on his general rule that terms have the same semantic role in equiform sentences (TC 3.7.41), I shall occasionally use such quasi-technical devices in my exposition.

[38] This method for interpreting universal and existential sentences so that existential import is validated I take from Belnap [1973]; it works because 'each' presupposes the existential import of the subject-term—see TS 3.7.19 and the discussion of anaphoric reference in Section 6.4.

[39] Buridan admits a class of sentences which he first calls "quasi-hypothetical," but immediately says that such sentences are in fact hypothetical, and they are so-called

be regarded as sentence-frames;[40] the expressions which may be substituted in each frame may be logically complex. The categorical sentence-frame functionally defines its constituents as subject and predicate (TS 1.3.8); its general form is therefore:

(subject)-[copula]-(predicate)

There seem to be three types of hypothetical sentences: conjunctive sentences, disjunctive sentences, and consequences The last functionally defines its constituents as antecedent and consequent; its general form is therefore:

(antecedent)-[illation]-(consequent)

where the illation is usually signaled by 'therefore' or its equivalent, or by 'if' preposed to the antecedent or its equivalent; Buridan favors the former method. Disjunctive or conjunctive sentences are sentences which are disjunctions or conjunctions or parts equiform to sentences; Buridan does not investigate their general form; neither shall we, since their behavior is familiar.

5.2 Categorical Sentences

The general form of the categorical sentence discussed above, namely subject-copula-predicate, is the actual form of the Mental sentence; in Spoken or Written not all of these components will be apparent (TC 1.8.72). For example, existential statements have misleading surface grammar; the Spoken or Written expression "S is" is represented in Mental as "S is a being" or "S are beings" (TS 1.3.8 and TC 1.8.84). Equally, subjects paired with intransitive verbs are resolved into the copula with a participle, so "S runs" becomes "S is running"; and the like. Buridan tentatively suggests that the copula in Mental is timeless (TS 3.4.8), but does not explore the implications. We have remarked above the complexive nature of the copula. Strictly speaking there are two forms of the copula: 'is' and 'is not,' which correspond to distinct functions in Mental (TS 2.3.12).

It is hard to describe the nature of the subject and the predicate, which Buridan discusses in TS II-2 and II-6. A complex series of rules specify some restrictions on which expressions can serve as subject or predicate. In general, any noun phrase can be subject or predicate; this includes (*i*) any substantive nominative term, whether discrete or common, and any adjective taken substantively (Rules Sup-1 and Sup-5); (*ii*) all referring expressions (Rule Sup-2); (*iii*) infinitive verbs (Rules Sup-4 and Sup-7); (*iv*) grammatical combinations of nouns and adjectives, or, more

only because they are very similar to certain categorical sentences (TC 3.7.20)

[40] Buridan actually suggest such frames in several places: TC 1.8.62, 2.6.34, 3.4.59, for example.

generally, nouns and oblique terms Rule Sup-8); (*v*) certain combinations of categorematic and syncategorematic terms (Rules Sup-9 and Sup-10). Conversely, taken *per se* the following cannot serve as subject or predicate: (*i*) pure syncategoremata (Rule Sup-2); (*ii*) oblique terms not combined with a substantive (Rule Sup-3); (*iii*) finite verbs (Rules Sup-4 and Sup-7); (*iv*) sentences (Rule Sup-6).

Categorical sentences may be distinguished with respect to quantity, quality, or mode. Regarding quantity, first, signs of quantity are properly not parts of the subject or predicate, but are "conditions of the whole sentence" (TS 2.2.15), the same way a negating negation (though not an infinitizing negation) is a condition of the copula. Second, the quantity of a sentence is determined by the syncategorematic terms present in the sentence. Distributive and particular signs of quantity are the usual syncategorematic terms which specify the quantity of a sentence, but other syncategoremata may also affect the quantity—negations, exceptives, and the like (see Section 4.1). Finally, the quantity of a sentence is, in general, given by the semantic generality of the subject of the sentence, and not by the items actually denoted; "All unicorns have horns" is distributive, and so has more quantity than "Some men are sexists," despite the fact that 'unicorn' is an empty term and 'man' is not.

Modern logicians recognized differences of quantity and of mode: sentences are universal or particular, assertoric or modal. Differences of quality—whether a sentences is affirmative or negative—are not usually taken into account. Yet Buridan takes two forms of the copula, 'is' and 'is not,' as primitive; hence there are two fundamental types of sentences, affirmative and negative. Is this defensible? The answer turns on the significance attached to the distinction between predicate negation and sentence negation, which cannot be clearly drawn in modern logic: there is no difference between belonging to the extension of the complement of a predicate and not belonging to the extension of the predicate. Buridan, however, insists on the difference: the truth of "Socrates is non-blue" suggests that Socrates has a property, being non-blue, or at least presupposes that he exists.[41] Thus for Buridan the truth-conditions for sentences differing in quality may be quite different. Fore example, since negatives are denials which do not presuppose the existence of their subjects, negatives with empty subject-terms are automatically true: of course unicorns fail to have a given property; they fail to have every property, since they do not exist.

[41] This may be the key intuition behind the mediæval view that quantity is irrelevant to ontological commitment, but quality is not. See Section 6.9 below.

Conversely, affirmative sentences with empty subject-terms must be false, even if the predicate is infinite.

Categorical sentences may also be distinguished with respect to their mode: they are either assertoric or modal (TC 2.1.3). Buridan distinguishes them in the following way: although every sentence is either possible, necessary, or contingent, only those sentences explicitly including a "mode" are counted as modal; the terms 'possible,' 'necessary,' 'contingent,' 'true,' 'false,' 'known,' and so forth are *modes*. Buridan thus treats 'modal logic' as much wider than the logic of possibility and as "that whole found in the sentence aside from the mode and the copula and the negations and the quantifiers, or other determinations the mode or the copula" (TC 2.3.3).[42] Examples are "That a man runs is *composite*, the latter *divided*: a modal sentence is composite when the mode is put as the subject and the dictum is predicated, or conversely (TC 2.2.2); a modal sentence is divided when part of the dictum is put as the subject and the other part as the predicate, and the mode is taken as a determination of the copula (TC 2.2.5).

Divided modal sentences ampliate their subjects and can be treated easily as a case of sentences with a special ampliative copula (see Section 6.8). Composite modal sentences, on the other hand, are a peculiar type of assertoric categorical: we may understand

It is possible that p

as equivalent to

A possible [sentence] is that-p

where the analyzed sentence has an ordinary copula and the predicate is 'that-p' (TC 2.7.1-5). Such sentences are indefinite and may thus be quantified:

Some possible [sentence] is that-p

Every possible [sentence] is that-p

To understand this we need to know what it means to call something a 'possible' or a 'possible [sentence].' Buridan implicitly endorses the following analysis:

> "Some possible [sentence] is that-p" is true if and only if for a sentence p^* equiform to p, how p^* (if formed) signifies to be can be the case.

There are two points to note. First, the clause 'a sentence p^* equiform to p' must be included because the predicate of the original sentence is 'that-p,'

[42] Strictly speaking this definition is too loose, for the dictum of "It is possible that a man is running is "that a man is running," and so includes an occurrence of the copula occurrences of logical particles. It is less clear how to explain 'non-primary' in a non-trivial fashion; this is precisely Frege's distinction between 'force' and 'assertion.'

which is not itself a sentence.[43] Indeed, no part of a sentence is a sentence. Second, the clause 'if formed' is not within the scope of 'can be the case': that is precisely Buridan's distinction between the possibly-true and the possible.[44] The analysis given above is true of a possible sentence; roughly a sentence is 'possible' when it describes a possible situation, and 'possibly-true' when it is true of and in a possible situation. Thus we may also give the analysis:

> "Some possibly-true [sentence] is that-p" is true if and only if for a sentence p^* equiform to p, how p^* signifies to be can be the case, if formed.

A sentence is possibly-true only if it can be a part of the possible-situation it describes. The standard example Buridan uses to distinguish the possible and the possibly-true is the sentence "no sentence is negative": it describes a possible situation, for it is surely possible that all utterance have been affirmative - but the sentence itself can never be part of such a situation, since it is not affirmative in form.

5.3 Hypothetical Sentences

In addition to categorical sentences there are hypothetical sentences, that is, sentences whose main connective is not the copula. There are three types of hypothetical sentences: conjunctive sentences, disjunctive sentences, and consequences (QM 6.10 fol. 40rb). The truth-conditions for the first two are straightforward and given in terms of the truth-conditions of sentences equiform to their constituent parts. Consequences, however, are more complex.

In particular, we may specify truth-conditions for each kind of hypothetical sentence. Hypothetical disjunctive sentences are true just in case one of the sentences equiform to its disjuncts is true; hypothetical conjunctive sentences are true just in case each of the sentences equiform to its conjuncts is true.[45] Standard laws of the propositional calculus are appropriately represented in Buridan, *e. g.* DeMorgan's Laws (TS 3.7.37).

The truth-conditions for consequences are more complicated, for they are not simply truth-functional: their truth or falsity depends on the

[43] In Latin this may be either a subordinate clause introduced by *quod* or an accusative-infinitive construction, neither of which is grammatically a sentence.

[44] See Prior [1976] for an excellent pioneering discussion of the distinction between the possible and the possibly-true.

[45] See QM 6.10 fol. 40rb: "It should be known that a conjunctive [sentence] requires for its truth that any of its categoricals are true, and it suffices for its falsity that one is false. Conversely for disjunctives: their falsity requires that each part is false and it suffices for their truth that either part is true."

semantic analysis of the sentence equiform to the protasis and apodosis (discussed in Section 7.2).

The truth of hypothetical sentences, then, depends on our understanding the truth of categorical sentences: it is not a simply recursive dependence, in which the truth-value of the complex is truth-functionally determined by recursive rules. The acceptability of a consequence depends on the modal connection of the truth-values of the antecedent and consequent: it must be impossible for things to be as the antecedent says with things not being as the consequent says. Therefore, despite the fact that not all hypothetical sentences can have their truth-value specified recursively and truth-fuctionally, the analysis of the truth of categorical sentences is the key ingredient in determining the truth or acceptability of a hypothetical sentence. After considering some basic principles of sentential logic, we shall take up the problem of determining the truth-value of categorical sentences (Section 5.5).

Obviously, hypothetical sentences are similar to the contemporary logician's 'molecular formulae.' But there are important differences: the parts of hypothetical sentences are not sentences, though parts of well-formed formulae may be formulae; equally, the truth-value of a hypothetical sentence is not a simple recursive matter—indeed, consequences are properly called acceptable or not, not true or false. These differences stem from a more basic difference, namely Buridan's construal of sentences assertions, discussed in Section 5.1. Conjunctive and disjunctive hypothetical sentences may be straightforwardly treated as assertions: a conjunctive assertion is closely related to a conjunction of assertions; a disjunctive assertion is closely related to a disjunction of assertions. (See TS 2.6.59, in which Buridan describes this as the "natural condition" of disjunction.) In these cases, although the main connective is not the copula, it is closely related to it so no problems need arise.[46] But, as is notorious among modern logicians, the notion of 'conditional assertion' is not so amenable to direct treatment. Here it seems we may be forced to distinguish between the predicative force of copulation and assertive force. And indeed, that is what Buridan does, tough he takes a different route that Frege in classifying consequences as acceptable rather than as possessing truth-value. This is a deep point about Buridan's system which we shall cover thoroughly in Section 7.

[46] There may be as many conjunctions or disjunctions in a conjunctive or disjunctive sentence as one likes; they may even be related by DeMorgan's Laws. Because of associativity, we may treat them as pairwise sentence-functors without loss of generality.

5.4 Principles of Sentential Logic

The core of sentential logic has several elements: (*i*) opposition in all its forms, such as contradiction, contrareity, and subcontrareity; (*ii*) rules of inference or entailment, including equipollence and conversion; (*iii*) principles of well-formedness, both as criteria for the acceptability of sentences and rules for compounding sentences. The theory of consequences is crucial for (*ii*), and will be discussed in detail in Section 7. We have generally canvassed (*iii*) in the preceding sections. The present section will explore (*i*).

In TC 1.8.1 Buridan sets forth a general principle:

> For any contradiction, one of the contradictories is true and the other false, and it is impossible that both are true together or false together; again, every sentence is true or false, and it is impossible for the same sentence to be true and false at the same time.

Several principles governing opposition are endorsed here: bivalence, *i. e.* "every sentence is true or false"; the Law of Non-Contradiction, *i. e.* "it is impossible for the same sentence to be true and false at the same time"; and what we may call the 'Law of Contradictories,' *i. e.* contradictories must have opposite truth-value. (See also TC 1.17 for bivalence and the Law of Non-Contradiction.) Buridan never discusses bivalence at any length; like most mediæval logicians, he simply assumes its truth.[47] On the other hand, the Law of Non-Contradiction and the Law of Contradictories are carefully investigated in QM 4.11–15.

Buridan begins his discussion in QM 4.11 by asking whether contradiction is the greatest form of opposition, and argues first that, strictly speaking, sentences alone are contradictories, such as "Socrates is running" and "Socrates is not running." However, there are also contraries; sometimes terms themselves, or their referents, are called contraries; most properly, though, only sentences can be contraries. There are two kinds of contraries: (*i*) sentences which are contraries due to their terms, such as "Socrates is white" and "Socrates is black"; (*ii*) sentences which are contraries due to their logical form, such as "Every man is running" and "No man is running" (fol. 21ra). Technically, we can distinguish two additional principles: the 'Law of Contraries,' *i. e.* such a pair of sentences can both be false but cannot both be true (discussed by Buridan in TS 2.6.39), and

[47] Most mediæval logicians considered bivalence to be part of the very definition of 'sentence,' so the question did not arise. However, reflections on the problem of future contingents stimulated discussion of bivalence, leading some (*e. g.* Petrus de Rivo) to give it up. Buridan, however, in his discussion of future contingents in QM 6.5, maintains bivalence.

the 'Law of Subcontraries,' *i. e.* such a pair of sentences can both be true but cannot both be false.

In QM 4.13 Buridan asks "whether the sentence 'it is impossible that the same be present (*inesse*) and not present to the same at once, with respect to the same, and so forth for other conditions' is the first complex principle" (fol. 22ra), that is, the Law of Non-Contradiction. His answer is complex. Sentences may be classified according to quality, quantity, mode, time, and the like; Buridan argues that there is no general way to state truth-conditions applicable to all these different types of sentence.[48] For example, it is possible that the same be possibly present and possibly not present to the same at once. In particular, there will be no general definition of 'contradictory,' 'contrary,' or 'subcontrary,' but particular definitions for each type of sentence. Hence the formulations given above are only *schematic*: there is a form of the Law of Non-Contradiction for each type of sentence; "all of the aforementioned principles are indemonstrable, although some are more evident and simple [than others]" (fol. 22va).[49] Equally, there will be particular forms of the Laws of Contradictories, Contraries, and Subcontraries.

The traditional Square of Opposition expresses the logical relations among a limited class of sentences: the contraries "All S is P" (A-form) and "No S is P" (E-form) entail as subalternates, respectively, the subcontraries "Some S is P" (I-form) and "Some S is not P" (O-form); the 'diagonal' pairs are contradictories, *i. e.* A-form and O-form on the one hand, E-form and I-form on the other hand. If we focus on their abstract logical properties, though, we can see that the sentences on display in the Square of Opposition are not very general: they are present-time assertoric categorical sentences, the terms of which are logically simple, where the predicate-terms are common nouns and have an implicit particular quantification,[50] indefinite sentences are treated as implicitly quantified (TS 2.2.12), and singular sentences added. Buridan's rules, however, are stated generally, so the Square of Opposition represents only a limited amount of theory.

[48] Buridan's arguments for this thesis will be discussed in Section 5.5 and Section 6.9.

[49] Buridan devotes QM 4.15 to exploring whether the Law of Non-Contradiction holds for tensed sentences, a difficult case; they will be discussed in Section 6.8.

[50] We should also note here that common terms appearing in sentences are all quantified,, explicitly or implicitly: an unquantified term may be read as involving distributive or particular quantification, depending on the requirements of the sentence, context, and good sense: "Every S is P" is subordinated in Mental to either "Every S is every P" or "Every S is some P" (typically the latter). Indefinite sentences are quadruply ambiguous.

5.5 Truth and Sentential Signification

What are the truth-conditions for categorical sentences? Buridan investigates this question in *Soph.* 1–2, TC 1.102, and QM 6.8. He begins by citing the standard definition of truth: " A (categorical) sentence is true if howsoever it signifies so it is (*qualitercumque significat ita est*)." To understand the definition we need the notion of the signification of a sentence. Buridan considers and rejects four possible answers.

First, let us suppose that the definition refers to the immediate signification of all of its terms. But then we have the unfortunate conclusion that every Spoken sentence is true, because its immediate significate is the Mental sentence, which does exist (*Soph.* 1 Sophism 1); indeed, an inscription or utterance is only a sentence in virtue of the corresponding Mental sentence. The definition cannot mean the immediate signification of the terms.

Second,[51] suppose the definition concerns the ultimate signification, of the terms in the sentence. Since Syncategoremata by definition lack ultimate signification, they are excluded. Yet this will have two unfortunate results. First, a sentence and its contradictory may differ only in their syncategoremata, as do "Socrates is a sexist" and "Socrates is not a sexist"; they ultimately signify the same. Hence the proposed definition of truth will not assign opposite truth-values to a sentence and its contradictory, which is absurd. Second, by the Additive Principle, the ultimate signification of a sentence such as "A man is an ass" is the sum of the ultimate significations of its categorematic parts, namely men and asses. But there surely are men, and there surely are asses; does this mean that the sentence "A man is an ass" is true?

Third, several philosophers though the significate of a sentence was an abstract entity called a *complexe significabile*, rather similar to the modern notion of a proposition. Buridan rejects this view for a variety of reasons: (*i*) even proponents of the *complexe significabile*, such as Gregory of Rimini and Adam Wodeham, admitted that it was not to be found in any of the Aristotelian categories; Buridan draws from this the conclusion that it is nothing, and it is impossible to see how a sentence is true or false in virtue of what is nothing at all. Yet (*ii*) suppose it were not nothing, but something; then it either exists or fails to exist, and presumably in the former case the sentence which signifies it is true and in the latter case the sentence is false. But then one is maintaining that a false sentence literally has no signification, which seems absurd: such sentences are false, not meaningless.

[51] For the next series of arguments see QPA 1.5 and QM 6.8.

Finally, (*iii*) suppose an existent *complexe significabile* is the significate of true or of false sentences in some fashion. But suppose things changed in the world. The abstract entity which is the *complexe significabile* is unchanged, not being part of the world. But then a sentence is true or false not in virtue of the world at all, which seems obviously ridiculous. Hence the *complexe significabile* is not sufficient to account for truth-value.

Fourth, suppose that the significate of a sentence is the true or the false, in the Fregean manner. But then the proposed definition is both circular and uninformative.

The problem with all of these suggestions, Buridan thinks, is that they fail to notice that sentences are far more than the terms which compose them; they are the way in which we talk about the world. Buridan modifies the proposed definition by adding a single clause:

> A categorical sentences is true if and only if howsoever it signifies so it is, in the thing(s) signified.

To understand what is 'in' the thing(s) signified we must move beyond the simple signification of terms: we have introduced the notion of reference, which for Buridan is part of the theory of supposition. Hence we need to understand the nature of supposition before we can account for the truth or falsity of sentences.

6. The Theory of Supposition

6.1 Supposition and the Theory of Reference

Supposition is a semantic relation, holding between term(s) and things(s). The relation of signification, however, is also a relation of term(s) and thing(s). Yet it is one matter to assign certain terms to certain things, so that a language may be set up in the first place; this is the contribution of signification. It is quite another matter to actually use that language to talk about things; this is explained by supposition, which accounts for the referential use of (significative) terms. Hence there are two major differences between supposition and signification: first, terms have signification wherever they are found, inside or outside a sentence, but it is only in a sentence that we use terms referentially, that is, actually talk about things and say something about them. Therefore:

(1) A term has supposition only in a sentential context.

Second, we do not always use terms to talk about what those terms signify; we use them in other ways as well. therefore:

(2) The kind of supposition a term has depends on its sentential context.

These principles govern the theory of supposition, and provide a context for the various divisions of supposition.

The theory of supposition should not be assimilated to formal logic, but to the philosophy of logic; it is the mediæval theory of reference. The claim that supposition, or any relation R, is in fact a reference-relation is justified if the R-relation is sufficiently similar to our uses of 'reference.' There are two main uses in contemporary philosophy of language. The first, associated with Davidson, takes the R-relation to figure in a compositional theory of how sentences acquire their meaning and truth-value, especially if in this theory the R-values are the referents. As we shall see in Section 6.8, the theory of supposition does provide an account of the truth-value of sentences, although not their meaning. Hence this first way is inconclusive.

The second way, with Quine, is definitive. On this approach we take the variables of quantification to be the paradigm case of the sort of term having an R-value. Now this approach is usually rejected out of hand because there are no variables of quantification in supposition theory: terms are bound, not variables. Yet such rejection is facile; there may be no overt variables of quantification, but we can look at the mediæval treatment of variable-binding operators in the language. In particular, consider the case of anaphoric pronouns whose antecedents are quantified terms, such as 'he' in "Some man has a daughter and he loves her." This is a case of 'relative supposition' (discussed in detail in Section 6.4) in which the supposition of 'he' is determined by the supposition of 'Some man.' Equally there are various indexing devices available; in the example, gender disambiguates the particular antecedent to each pronoun. Such antecedents can be multiplied and indexed at will in supposition theory. Hence supposition has the key characteristics of a reference-relation, and may be treated as such.

Supposition theory is a theory of reference. It is a unified theory, which has as its goal to specify what a term is used to talk about in a given sentence.[52] The various divisions of supposition illustrate the ways in which a term may supposit for something. Buridan's division of supposition can be put in outline form as follows:

[52] The claim that supposition theory is in fact a unified theory, and that it should be construed as I suggest in Section 6, is a matter of controversy. Some of the arguments are taken up in the following sections. My interpretation has the virtue of making supposition theory a reasonable philosophical enterprise, as well as fitting in well with other branches of mediæval logic, such as fixing coreferentiality by the Doctrine of Distribution in the theory of the syllogism. The distinction of referential and attributive uses of the particular sign given below is, it seems to me, well-confirmed by the remarks made by Buridan in his discussion of determinate and non-distribute confused supposition. but the reader should not that the interpretation I offer is a matter of dispute.

- SUPPOSITION
  - Improper
  - Proper
    - Material
    - Personal
      - Discrete
      - Common
        - Relative
        - Absolute
          - Natural
          - Accidental
            - Determinate
            - Confused
              - Distributive
              - Non-Distributive

Of course, we must specify the scope of Buridan's theory; this is the point of the first division, the distinction between proper and improper supposition. A term as improper supposition when it is used rhetorically in a sentence and not literally. The theory of supposition is developed only for the literal uses of terms, as indeed are modern theories of reference, and both are equally far from explaining the meaning of 'literal' or understanding non-literal uses of language.[53]

Personal and material supposition illustrate the *kind* of thing a term may refer to in a sentence, namely its ultimate significate or itself, whether as inscription, utterance, or concept. the various divisions of personal supposition specify *how many* of its ultimate significates a term may stand for: exactly one (discrete); at least one (determinate); several (non-distributive confused); all present instances (distributive); all past, present, and future instances (natural). The status of a term, as defined in TS 6.1.1, requires this interpretation. The theory of supposition, of course, does not say exactly which things a term stands for; like any semantic theory, that is left to the facts of the matter to decide—terms may fail to refer, the reference will change with changes in the world, and so on. But it does specify the semantic function a term may have in different sentential context. This point is important: supposition theory will explain the semantic role of terms in a

[53] This is surprising, since Buridan holds that the speculative sciences each study a single Mental term, to which other terms are related as individual men in an army are related to their leader (QM 4.3 fol. 14ra and QSP 1.2 fol. 3rb–va): such attributions to a primary element are semantic but not exactly 'literal,' and we would expect a theoretical account of such uses.

sentential context—evaluating the truth of the sentence is another matter. Hence it is a mistake to suppose that supposition theory will say exactly which things a term in fact supposits for. Rather, supposition theory will specify what things a term semantically supposits for, and then it is a separate question whether the supposition is successful. Failing to appreciate this point can lead only to confusion.

6.2 Personal and Material Supposition

The general motive for distinguishing material and personal supposition is clear: a sign may be used to refer to things (its ultimate significates), or it may be used to refer to other signs. In the former case a term has *personal supposition*; in the latter, *material supposition*.[54] For example, in the sentence "Every man is running" the term 'man' has personal supposition; we are asserting something about individual men, who are the ultimate significates of the term 'man.' But in the sentences "Man has three letters," "Man has one syllable," "Man is a concept," the term 'Man' is used to talk about not individual men, but rather an inscription, an utterance, and a concept respectively, each of which is itself a sign. Note that the subject-terms of all these sentences are the same.[55] Buridan defines personal supposition as follows (TS 3.2.1):

> Supposition is called 'personal' when a term supposits for its ultimate significate(s).

Thus the term 'man' has personal supposition in the sentence "A man is running". The divisions of personal supposition, discussed in Sections 6.3-7, describes which of its ultimate significates a term may supposit for in particular. This same question is also at the heart of the theory of ampliation and restriction, discussed in Section 6.8, which analyzes how the referential domain of a term in personal supposition may be either widened or narrowed respectively. WE can generalize Buridan's account in TS 1.2.9 to arrive at the following characterization of personal supposition:

> A term t has personal supposition in a sentence if and only if either (i) some sentence of the form "This is t" is true, or (ii) some clause of the form '. . . and that is t' can be added to an existential sentence or a sentence presupposing an existential sentence to produce a true sentence. The demonstrative or relative pronoun and the copula of (i) and (ii) should be taken in the appropriate number, tense, and

[54] This is too loose: the term 'utterance' personally supposits for signs. But it may serve as a handy initial characterization.

[55] Don't worry about whether the examples of material supposition need quotation-marks; that is one of the jobs performed by the theory of material supposition.

mode.

The motivation for adding (*i*) in TS 1.2.9 is to circumvent worries about whether we can point to God, who is not open to direct inspection (in this life), which might seem to be required for ostension. In fact (*ii*) is more general: it will serve to sidestep worries about "pointing to" past, future, or possible existents as well, which are also not open to direct inspection. Thus in the sentence "Some man will lecture tomorrow" the term 'man' has personal supposition, since some sentence of the form "This is a man" is true.

A term is in material supposition, on the other hand, if it does not supposit for what it ultimately signifies (TS 3.2.1):

> But supposition is called 'material' when an utterance supposits for itself or something similar to itself or for its immediate significate, which is the concept according to which it is imposed to signify, as the term 'man' in the sentence 'Man is a species.'

Logicians preceding Buridan posited another kind of supposition, simple supposition, in which a term was taken to stand for a universal or common nature. William of Ockham had retained simple supposition as a category, but his nominalist scruples forced him to say that in simple supposition a term supposits for the concept it immediately signifies, which, in the case of common terms, simply is the universal. It was left to Buridan to see this extra division of supposition as a relic of realism (TS 3.2.4-5): since a concept is just as much a sign as an utterance or an inscription, albeit naturally and not conventionally significative, Buridan drew the conclusion that supposition for a concept as the immediate significate was just another form of material supposition (TS 3.2.6).

Buridan argues that material supposition is proper only to Spoken or Written, and that there is no material supposition in Mental: "no term in a Mental sentence supposits materially, but always personally" (*Summulae de dialectica* 7.3.4). Hence Buridan is not even faintly interested in divisions of material supposition: such a series of divisions would only be of limited interest to the logician, whose concern is with Mental and not with Spoken or Written. Mental therefore has a complete apparatus to accomplish all of the functions of material supposition. For example, the sentence "Man is a species" is properly represented in Mental in the following way (*Summulae de dialectica* 7.3.4):

> I say that the Mental sentence corresponding to the sentence "Man is a species" (taken as it is true) is not a sentence in which the specific concept of man is put as the subject, but rather is a sentence in which the concept by which the specific concept of man is conceived

> is put for the specific concept of man, from which it is clear that the aforementioned paralogism is a fallacy *in dictione* by the change of supposition.

The "aforementioned paralogism" is the argument "Man is a species, and Socrates is (a) man; therefore, Socrates is a species." It fails because there is a change in supposition, perspicuously seen in Mental: the sentence "Man is a species" is subordinated to the sentence "The concept by which the specific concept of man is conceived is a species," which is true: genera and species are concepts of concepts, not individual general concepts themselves.

Material supposition acts like quotation in many ways, serving to distinguish the use of a term from its mention, but the differences should not be overlooked; material supposition is much wider than our use-mention distinction. The similarities are obvious: in sentence such as "Man has three letters" or "Man is a monosyllable" the term 'man' has material supposition; we should render these by using the quote-functor, giving " 'Man' has three letters" or " 'Man' is a monosyllable." However, there are differences even in such cases. First, applications of a quote-functor produces a *new* term, one which names the personal or material supposition. Indeed, Buridan was familiar with the device of naming expressions, but he never proposes it as an account of material supposition.[56] Equally quotation does not seem to require a sentential context the way material supposition does. Second, the quote-functor may be iterated,[57] but material supposition cannot; there is no mediæval analogue of the sentence " ' 'Socrates' ' names 'Socrates' names Socrates, who was Greek."[58] Material supposition could be no more than a first-order fragment of quotation theory. Third, the substitution classes differ; a term can materially supposit for what is only similar to it, so that accusative-infinitive phrases supposit for sentences, and changes in case and gender are permitted. In the composite modal sentence "It is possible for Socrates to be a bishop: the expression 'for Socrates to be a bishop' materially supposits for the sentence "Socrates is a bishop" (or a similar sentence).

Material supposition, then, is more inclusive than the distinction of use and mention, even when we restrict ourselves to the material supposi-

[56] Sometimes Buridan uses the mediæval French word 'li' or 'ly' prefixed to a term: this however was not a mediæval quote-functor but rather a metalinguistic comment indicating that the following term has material supposition.

[57] Provided that no syntactic ambiguity arises, as would be the case in ' 'p' '$\wedge$' 'q' ': but a scope convention will clarify such cases.

[58] Unless such iteration can be accomplished through anaphoric reference: see the discussion of relative supposition in the next section.

tion of utterances or inscription. Allowing material supposition of concepts merely points up the relevant differences.

When is a term in personal supposition or in material supposition? Unlike Ockham, Buridan does not try to give precise rules, but rather trusts to good sense and good logic (TS 3.2.15-22), usually taking context to decide. Since there is no material supposition in Mental, Buridan is not at fault for not providing precise rules; the vagaries of Spoken or Written are met individually.

6.3. Discrete and Common Supposition

We now properly begin the divisions of personal supposition, which were drawn in an effort to clarify which of its significates a term is used to refer to. By 'discrete term' Buridan means a singular referring expression—that is, an expression which is predicable of only a single item. Hence it is obvious when using a discrete term that one is talking about the very thing the term refers to: there are no other choices. Hence personal supposition is divided into *discrete* and *common* (TS 3.3.1). As examples of discrete terms Buridan offers us 'Socrates' and 'this man.' Presumably all proper names are discrete, despite the fact that two people may have the same name; the 'name' is not really common in this case. Demonstratives combined with common terms are also discrete. Buridan never says so, but pure demonstratives should also be discrete terms. Any singular referring expression is, as a matter semantics, a discrete term: if the Scholastics had articles, they would surely have treated definite descriptions as discrete terms.[59]

However, not every term which refers only to one thing is a discrete term; in a world with only one man, the common term 'man' has only one referent, but it is nonetheless common, since it is predicable of many. Only those expressions which can be predicated of only one referent are discrete terms as a matter of semantics, although other terms may as a matter of fact e used to talk about only one referent. Criteria for distinguishing the kinds of common supposition a term can have will depend on the relation of the sentence in which the common term appears to sentences with discrete terms. Common terms, of course, will be general referring expressions.

[59] In general the Scholastics did distinguish descriptive knowledge from direct knowledge, rather like Russell's "knowledge by description" and "knowledge by acquaintance." In QM 7.20 fol. 54rb–va Buridan explores how names actually correspond to descriptions (conjunctions of properties) in the absence of the thing named. See also Perreiah [1972].

6.4. Absolute and Relative Supposition

As instances of *relative supposition* Buridan examines[60] what is now called anaphoric reference: a 'relative-term' is not a term in the category of relation, but a reference indicator, something "relative to a thing said before or recalled" (TS 4.1.3). Anaphoric terms include what Buridan calls *identificatory relative-terms* (TS 4.2.1): the personal, reflexive, and possessive pronouns (TS 4.3–4, TS 4.6–7): relative pronouns (TS 4.5); terms paired with categorical quantifiers, as 'such' is paired with the universal affirmative sign 'however' of the category of Quality (TS 4.8); they also include differentiating relative-terms, such as 'another,' 'other,' 'different,' and like (TS 4.9).

There are two rules governing identificatory relative-terms terms: (*i*) the relative-term supposits only for those things for which its antecedent is verified (Rule RT-1 in TS 4.3.1); (*ii*) the supposition of the relative-term is the same as the supposition of its antecedent (Rule RT-2 in TS 4.4.1).[61]

The motivation for Rule RT-1 is apparent in "Some men are sexist and they are disgusting": the relative-term 'they' supposits not simply for men, nor simply for some men, but for those men who are sexists. 'They' are not simply men, for we are not claiming that men are disgusting; nor are they simply 'Some men,' for in that case the sentence could be true if there are non-sexist disgusting men, which is certainly not what we mean. Equally, in the sentence "Some men are not sexists and they are liberated" the relative-term 'they' supposits for those men who are not sexists; if there are none, the sentence is false. Therefore such cases of anaphoric reference act as restricted quantifiers, referring to those items the antecedent supposits for which the predicate also supposits for. This explains why Buridan treats issues of pronomial reference together with identificatory relative-terms paired with the categorial quantifiers.[62] Now Buridan treats

[60] Buridan considers the division of supposition into absolute/relative as a division of personal common supposition because (*i*) the terms 'this' may be either a demonstrative or a relative pronoun, but when it fails to have discrete supposition it is used as a relative pronoun and so has relative supposition (TS 4.5.1); (*ii*) the distinguishing features of relative-terms show up only in personal supposition, since any expression, conventionally significant or not, may be in material supposition.

[61] This is very loose of Buridan: strictly, the relative-term supposits for what its antecedent supposits for—in "Man is a noun and it has three letters" the relative-term 'it' does not have material supposition, but personally supposits for the term 'man' *in material supposition*. Perhaps this provides a way of iterating material supposition.

[62] This reveals a deep connection between the behavior of quantifiers, the subject and predicate of a sentence, and attributive combination: taken referentially, an attributive combination like 'white man' acts to restrict the supposition of 'man' to those who are

all such cases on a par, and so 'pronouns of laziness' will also have a quantificational aspect: in "Socrates is disputing and he is ugly" the relative-term 'he' does not simply refer to Socrates, but to Socrates as disputing—and if Socrates is not disputing, then 'he is ugly' will be false due to its empty subject-term (TS 1.2.9). There are two reasons which motivate Buridan here: (*i*) recall that no part of a sentence is a sentence, and so "Socrates is disputing and he is ugly" is a conjunctive sentence making a single assertion; it is no wonder that the semantics for 'he is ugly' depend on the semantics for 'Socrates is disputing.' What is more, (*ii*) if Socrates is not disputing then it is in fact questionable what 'Socrates is disputing' refers to, and hence what 'he is ugly' refers to; it is a philosophical question whether such a sentence refers to Socrates and makes a false claim abut him or simply fails to refer, which Buridan points up by asking for the reference of 'he.' While the first alternative is customary today, Buridan opts for the second alternative as more fruitful philosophically.

Reflexives must be in the same sentence as their antecedent term and are construed with them (Rule RT-4 in TS 4.6.1): from "Every man amuses himself" we correctly infer "Socrates amuses Socrates, and Plato amuses Plato, and so on." term like 'such' and 'so-much' supposit for what their antecedents do, where the antecedent is governed by the appropriate kind of quantifier (Rule RT-6 in TS 4.8.1).

Differentiating relative-terms are also paired with antecedent terms but serve to distinguish the terms which follow them; the rules governing them are complex (Rule RT-7 in TS 4.9.1). For example, "Socrates is other than Plato" should be analyzed as "Socrates is a being and Plato is a being and Socrates is not Plato," by Rule RT-7(b). Strictly speaking this is not anaphoric reference, but the logical analysis of sentences containing certain terms.

6.5 Natural and Accidental Supposition

Natural supposition is a form of ampliation: the present tense of the copula does not accurately indicate the temporal range of the terms occurring in the sentence. Just as a term such as 'man' signifies all its significates, men, for all times, so too in certain sentential contexts it supposits for all of its significates, not merely present existents.[63] Such supposition is paradig-

white (Rule Res-2 in TS 6.3.1), which are the supposits of the sentence "[Some] men are white." Perhaps this is why Buridan tries to minimize the difference between the *complexio indistans* (attributive combination) and the *complexio distans* (copulation of subject and predicate) in QM 4.14 fol. 23va and QM 6.6 fol. 37vb. This subject is touched on in Nuchelmans [1973] 244–246.

[63] This may be why such supposition was called 'natural,' namely the supposition coin-

matically common, for the term is used to refer to everything which it may stand for. Buridan defines it as follows (TS 3.4.1):

> Supposition is called 'natural' when a term indifferently supposits for everything for which it can supposit, past and future as well as present; this is the sort of supposition we use in demonstrative science.

This definition is repeated in QNE 4.6 501:[64]

> According to the older logicians, the common supposition of a term is twofold: natural and accidental… it is [natural] when it supposits indifferently for all of its supposits, whether past or future; demonstrative science uses this supposition.

We may say, then, that a term has natural supposition if it supposits for everything (past, present, future) which it signifies. As Buridan indicates, he is reviving an old usage, for the logicians of the thirteenth century had discussed natural supposition.

Buridan cites four reasons why we should admit natural supposition: (*i*) epistemic verbs require it (TS 3.4.3); (*ii*) the disjunctive subject-terms of sentences such as "What was or is or will be *A* is running" have it (TS 3.4.4); (*iii*) natural supposition is produced by terms such as 'perpetually,' 'eternally,' and the like (TS 3.4.5); (*iv*) demonstrative science uses it (TS 3.4.6–7 and QNE 6.6 501–502). Buridan argues that we can and do construct concepts indifferent to time (TS 3.4.8–12 and QNE 6.6 501): our concepts do not have any temporal reference, for it is possible to conceive a thing "without understanding along with it a determinate time" (QNE 6.6 501). Indeed, this is the only way our concepts can signify things of all times.

Natural supposition leaves us with a problem: cases like (*i*)–(*iii*) quite obviously require omnitemporal reference, but in discussing (*iv*) Buridan cites sentences which do not have any clear indication of their natural supposition, such as "Thunder is a sound in the clouds." Buridan says that such an utterance or inscription is properly subordinated to the Mental sentence "Any thunder, whenever it was or is or will be, is or was or will be a sounds in the clouds" (TS 3.4.7); in particular he calls the former sentence an abbreviation (*ad breviloquim*) of the latter.[65] the temporal quantifier

cides with the complete signification of the term, to its full or 'natural' extent.

[64] The formulation is close enough to be identified.

[65] In QNE 6.6 501 he says "This sentence 'Man is an animal' or 'Every man is an animal' is about all [men] through natural supposition: of whatever and whenever it is true to say 'man' then it is true to say 'animal,' and so 'Thunder is a sound in the clouds' is true, by referring singulars to singulars." I take this to be equivalent to the reconstruction proposed above, and Scott [1965] to be in error in suggesting

'whenever' has 'it was or is or will be' in its scope, and restricts the supposition of 'thunder' to times when thunder exists, for each case of which it either was or is or will be a sound in the clouds (matching the times to the tense of the copula). Buridan specifically rejects the conditional reading of such sentences (502).

Hence whether a sentence such as "thunder is a sound in the clouds" has natural supposition will depend on our understanding of the sentence, that is, what the sentence corresponds to in Mental: the same Spoken or Written sentence (or equiform sentences) may be ignored, doubted, opined, and demonstratively known; to hold otherwise would be to make all knowledge strictly a matter of semantics. Sentences of natural science are thus to be understood as follows:

> Socrates knows "Thunder is a sound in the clouds" (aparticular utterance or inscription).
>
> Socrates knows that any thunder, whenever it was or is or will be, was or is or will be a sound in the clouds.

The latter expresses the Mental sentence to which the utterance or inscription is subordinated. The Converse-Entailment Principle discussed in Section 4.3 then licenses:

> Thunder (whenever it was or is or will be) Socrates knows to have been or to be or to be going to be a sound in the clouds.

Therefore for any instance of thunder Socrates knows it at least under the *ratio* 'sound-in-the-clouds' (matching tenses appropriately). This removes the difficulty about empty subject-terms, for the subject of this last sentence is nonempty; indeed, it is ampliated to all and only those times at which there is thunder.[66]

Supposition is accidental, on the other hand, when the supposition of the term is not indifferent to time (TS 3.4.1), that is, refers to items existing only at some determinate time or other (QNE 6.6 501). The remaining divisions of supposition apply only to accidental supposition, for the restriction to a given time excludes some excludes some items from the (putative) reference-class of the term.

that such sentences are disguised conditional statements. They are not, but rather complex categoricals, as Buridan says.

[66] There is a further problem here: such a sentenced may have truth-at-all-times, by that might not seem enough to capture the necessity involved in knowledge and science: do we not need to require that every possible instance of thunder is equally a sound in the clouds? Buridan does not mention this. On his behalf we might suggest either (*i*) natural supposition includes ampliation to possibles; (*ii*) there are no never-actual possibles—but each suggestion obviously has further problems.

6.6 Determinate and Confused Supposition

A term is said to have *determinate supposition* in a sentence when a sufficient condition for the truth of the sentence is that it be true for some determinate singular falling under the common term (TS 3.5.1), where 'some' here means for at least one such singular (TS 3.5.3). Buridan gives two conditions which must be met for a term to have determinate supposition in a given sentence: rather, they are conditions which apply in virtue of inferential connections which obtain between the original sentence and related sentences:

(1) From any given singular falling under the common term the sentence with the common term follows, all else remaining unchanged (TS 3.5.5).

(2) All of the singulars can be inferred disjunctively in a disjunctive sentence (TS 3.5.6).

Roughly, a common term will have determinate supposition if it is in the scope of a particular quantifier which is not in the scope of another logical sign. Thus the term 'man' in "Some man is not a sexist" has determinate supposition, because (*i*) "Socrates is not a sexist; therefore, some man is not a sexist" is an acceptable consequence; (*ii*) "Some man is not a sexist; therefore, Socrates is not a sexist or Plato is not a sexist or Aristotle is not a sexist *et sic de singulis*" is an acceptable consequence. The conditions for determinate supposition, then, roughly correspond to existential generalization and instantiation.

A term in determinate supposition is used to talk about at least one of the things it signifies. To what, then, does it refer? The answer is clear; 'some man' in such sentences refers to some man. Geach has noted a problem here:[67] if 'supposition' is a relation of reference, then what does 'some man' in the sentence "Some man is φ" refer to when the sentence is false? If 'some man' refers to some man, then we may ask *which* man. But the sentence is false, and so it cannot refer to those men who are in fact φ, for there are none. Perhaps it refers to every man, being false in each particular case? But then there would be no difference between 'some' and 'every,' since 'every man' refers to every man.[68] Thus the very idea that a

[67] In Geach [1962] 6–7 the problem is discussed at some length; I can treat it only briefly here.

[68] It is only fair to point out that existential quantification may be in no better shape: the variables bound by a quantifier range over the entire domain, so that universal and existential quantifiers equally 'refer' to the entire domain. The mediæval and modern cases only appear different; the modern case seems simpler because the work of fixing the reference of a term is embodied in the interpretation-function, after which

term can refer to only part of its extension is incoherent.[69]

Geach himself (unwittingly) offers a solution ([1962] 7):

> Suppose Smith says, as it happens truly: "Some man has been on top of Mount Everest." If we now ask Smith "Which man?" we may mean "Which man has been on top of Mount Everest?" or "Which man were you, Smith, referring to?" Either question is in order…

Without realizing it, Geach is pointing out that 'some' can have a *referential* or an *attributive* use.[70] In its referential use, 'some' picks out a particular man or men, so that we may immediately assess the sentence's truth-value. In its attributive use, though, 'some' commits us to the sentence's truth-value, so that we may determine the reference (extension) of the term. The referential use roughly corresponds to 'some (*i. e.* this or these)' and the attributive use to 'some (*i. e.* some-or-other).'

To return to Buridan: a term used referentially has determinate supposition, and a term used attributively has non-distributive confused supposition. The particular sign of quantity 'some' normally has determinate supposition, that is , is used referentially; the inference-rules given above guarantee this. In TS 3.5.1 Buridan says that a term has determinate supposition when it is true *for some determinate supposit*, that is, referentially. The very point of the first rule is to insist on the referential reading. Further support is found in the uncommon idiom for negatives, which is equivalent to branching particular quantification, each taken referentially though not necessarily for the same item(s). But 'some' in certain sentential contexts can, in combination with other logical terms, have non-distributive confused supposition, that is, be used attributively. Such uses will be discussed in the next section. The rules of supposition are primary; it is a mistake to think that a term always has the same reference, no matter what the sentential context. That is one of the central points of supposition theory, noted in Section 6.1.

Hence 'some man' in the false sentence "Some man is φ" refers to some man, *i. e.* to some particular man. The sentence is false because that

determining truth-value is simple. As we shall see, this is but to embrace one possible solution. Mediæval logic will take another path.

[69] This is Geach's central claim, and he argues against the Doctrine of Distribution by means of such puzzles. But Geach is more abusive than conclusive; his objections, as I shall argue, are ill-founded.

[70] This distinction was first noted in Donnellan [1966], who applied it only to definite descriptions; he was able to do so in virtue of the fact that definite descriptions are disguised existential quantification - which have an attributive and a referential use.

particular man is not φ. Even if other men are φ, the sentence will be false, for they are not being referred to. Modern logicians are accustomed to read 'some' attributively, and so to take the φ-ness of other men to suffice for the truth of the sentence. Yet nothing forces us to accept the attributive reading; the referential reading is certainly a possible interpretation, and in fact seems to accord with ordinary language and linguistic intuitions far better than the attributive reading.

It might be objected that on the referential reading there is no difference between discrete and determinate supposition. But this is an artifact of our examples; we have taken 'some' to stand for a single individual. In such a case there is no difference. But 'some' may also stand for a determinate number of individuals: I may assert "Some men are bald," referring to six friends of mine whom I know or believe to be bald. Taken referentially, if any is bald, the sentence is true; if not, not. The semantic understanding of determinate supposition should now be evident. A term has determinate supposition if it is taken referentially for at least one determinate individual it signifies. Thus what Geach took to be the great sin of mediæval logic turns out to be its great virtue. There is nothing incoherent in the notion that a term may stand only for some of its significates on a particular occasion of its use in a sentence: Geach's worries stem from not taking seriously the pragmatic dimension of semantics, and the contribution of the logical grammar of a given sentence to determining the reference of a term.

A final point. The same term may appear in different sentences, and in each case have determinate supposition. The theory of supposition has nothing to say about when such occurrences are coreferential. Nor should it; supposition theory specifies the reference of a term in a given sentential context. Coreferentiality involves more than a single sentence;[71] additional semantic principles are needed for the wider context of several sentences. Clearly such principles are vital for the theory of inference: they are provided in the general theory of the syllogism. In particular, the *dictum de omni et nullo*, discussed in Section 8.2, fixes coreferentiality.

A common term has *confused supposition* in a sentences if it is not sufficient for the truth of that sentence that it be true for a singular term falling under the common term (TS 3.5.1). This is equivalent to saying that conditions (1)–(2) for determinate supposition are not met (TS 3.5.7).

[71] Strictly speaking, this is not true; the same term may appear several times with a sentence. Coreferentiality in such cases is established (or not) anaphorically, by relative supposition.

6.7 Distributive and Non-Distributive Supposition

There are two forms of confused supposition; Buridan gives a condition for each, and then a series of syntactical rules. A term in *distributive confused supposition* is used to talk about any or all of the things it signifies; a term in *non-distributive confused supposition* (sometimes called "merely confused supposition") is used to talk about several of the things it signifies indifferently.

A common term has distributive confused supposition in a sentence when any of the singulars falling under the common term can be inferred individually, or all inferred conjunctively in a conjunctive sentence (TS 3.6.1). Hence the term 'man' in "Every man is running" has distributive confused supposition, because (*i*) "Every man is running; therefore, Socrates is running" is an acceptable consequence; (*ii*) "Every man is running; therefore, Socrates is running and Plato is running and Aristotle is running *et sic de singulis*" is an acceptable consequence. Distributive confused supposition, then, is similar to universal quantification. The semantic relations involved in distributive confused supposition are clear: reference is made to everything (presently existing) which the term signifies; it is "distributed" over each individual.

Buridan gives five rules for when a common term in a sentence has distributive confused supposition; they syntactically indicate, by means of scope conventions governing negation and quantification (and comparison), when a common term has distributive confused supposition. First, a universal affirmative sign distributes the common term it governs (Rule DC-1 in TS 3.7.1); Buridan has a long analysis of what counts as such a universal affirmative sign (TS 3.7.2–3.7.33). Second, negations affect distribution: an infinitizing negation distributes terms in its scope (Rule DC-3 in TS 3.7.42), while a negating negation distributes every term in its scope which would otherwise not be distributed (Rule DC-2 in TS 3.7.34)– and terms which imply negative syncategoremata have similar effects (Rule NDC-5 in TS 3.7.50). Third, comparative contexts produce distribution, that is, the use of a comparison, and adjective of degree or a superlative (Rule DC-4 in TS 3.7.45).

On the other hand, a common term in a sentence has non-distributive confused supposition when neither 9i) any of the singulars individually follow, nor (*ii*) do the singulars follow disjunctively in a disjunctive sentence, although sometimes a sentence with a disjunctive extreme follows (TS 3.6.1). the term 'animal' in "Every man is an animal" is in non-distributive confused supposition, because (*i*) "Every man is this animal" does not follow; (*ii*) "Every man is this animal or every man is that animal or..." does not follow—although in this case, "Every man is this animal or that animal or…" does follow.

Semantically, a term has non-distributive confused supposition when it is used attributively of its extension. This explains why a sentence containing such a term does not entail any of the singulars individually or in a disjunctive sentence—this characterizes referential uses—but why a sentence with a disjunctive extreme does follow. The attributive use of a term will apply to a definite number of individuals in its extension (depending on the facts of the matter), but it does so indifferently, applying to each for exactly the same reasons. Such sentences presuppose their truth to determine the (actual) reference of the term, *i. e.* the subclass of the signification which actually have the property in question. In this sense, non-distributive confused supposition is close to our use of existential quantification.

Thus distributive confused supposition will correspond to universal quantification, determinate supposition to existential quantification used attributively, and discrete supposition to naming or denotation. It is clear that this scheme is complete: there is no room left for other sorts of reference. Mediæval semantic theory, therefore, codifies reference-relations by their use and by their extension.

The syntactic specification of non-distributive confused supposition is more motley than distributive confused supposition; two rules deal with the effect of quantification, and two with special terms. First, a universal affirmative sign causes non-distributive confused supposition in a term not in its scope which is construed with a term in its scope (Rule NDC-1 in TS 3.8.1 and 3.8.4), and a common term has non-distributive confused supposition where it is in the scope of two universal signs, either of which would distribute the rest were the other not present (Rule NDC-2 in TS 3.8.13). third, temporal and locative locutions "often" produce non-distributive confusion (Rule NDC-3 in TS 3.8.19), as do terms involving knowing, owing, and desiring (Rule NDC-4 in TS 3.8.24), where the latter have certain special characteristics.

Buridan's syntactic rules accomplish more than merely telling us which kind of confused supposition a term has; they permit the extension of supposition-theory beyond the narrow realm of simple categorical sentences, to deal with complex cases of multiple quantification such as "Every woman gives some food to every cat." Nevertheless, if we restrict ourselves to sentences in which the subject and predicate are logically simple, then we can briefly summarize the kinds of supposition possessed by the subject-term and the predicate-term in the Square of Opposition:

(1) In universal affirmatives such as "All S is P" [A-form], S has dis-

tributive confused supposition and P has non-distributive confused supposition.

(2) In universal negatives such as "No S is P" [E-form], S and P each have distributive confused supposition.

(3) In particular affirmatives such as "Some S is P" [I-form], S and p each have determinate supposition.

(4) In particular negatives such as "Some S is not P" [O-form], S has determinate supposition and P has distributive confused supposition.

If we augment the Square of Opposition by adding sentences with singular subject-terms, then in singular affirmatives and negatives the predicate ahs the same supposition as the corresponding particular form, and the subject has discrete supposition. Do not overlook the fact that Buridan's theory is much more general than (1)–(4) suggest.

Once the supposition of a term has been determined, the sentence may be inferentially related to other sentences. The most interesting such sentences are those in which terms which do not have discrete supposition are replaced by terms which do have discrete supposition; depending on the inferential direction, the relations are relations of *ascent* or *descent*.[72] For example, the I-form sentence "Some dog is healthy" entails "Some dog is this healthy thing or that healthy thing or..." where 'some dog' can be replaced by the reference-class in question, since it has a referential use.

Modern mediæval scholarship finds a fatal flaw in the theory of supposition at this point, known as the "Problem of the O-form." The difficulty is as follows. Suppose that only Socrates and Plato exist, each of whom is Greek, and consider the (false) O-form sentence "Some man is not Greek." If we descend under the subject-term, then (assuming that the reference is to Socrates and Plato) we get "Socrates is not Greek or Plato is not Greek." Each of the disjuncts is false, and so the sentence is false. However, if we descend under the predicate-term, we get "some man is not this Greek and some man is not that Greek." If 'this Greek' and 'that Greek' refer to Socrates and Plato respectively, then the first conjunction is true, taking 'some man' to be Plato, and the second conjunct is true, taking 'some man' to be Socrates; both conjuncts are true, and so the conjunction is true, though the original sentence is false.

The interpretation of supposition-theory offered above suggests a

[72] Buridan uses these terms only rarely, as in TS 2.6.75–76 and 3.8.22–23, but does frequently talk about 'descending' from a general sentence to a string of less general sentences. Despite the fact that it was a standard part of supposition-theory, though, Buridan does not give any systematic or theoretical account of the doctrine.

solution. In the latter case in which we descend first under the predicate-term, a crucial step in the argument was that 'some man' in each conjunct be taken not referentially, but attributively. Thus each occurrence of 'some man' in "some man is not this Greek and some man is not that Greek" should be read as 'some man or other (indifferently).' Yet this is to assign non-distributive confused supposition to the subject-term; in the example, the subject-term was said to have determinate supposition. To put the point a different way: if the subject-term has determinate supposition then an equivalent descent can *only* be made taking the subject-term thus corresponds to a 'priority of analysis' rule, and the so-called 'problem' of the O-form is resolved.

6.8 Ampliation: Time and Modality

Natural supposition, as we have seen, ampliates a term to stand for all of the items it signifies, whether past, present, or future (Rule Amp-5 in TS 6.2.1). Accidental personal supposition, though, may be concerned with the actual or the possible, with the present or past or future. Buridan, like most mediæval logicians, takes tensed sentences to be complete: they are not to be replaced by tenseless sentences with a time-index, or expanded to a canonical form; their truth-value can be assessed immediately. Now personal supposition as described above strictly applies to present-time assertoric discourse; the extension theory of *ampliation* (TS 6.1.1): a term is said to be ampliated if the scope of its reference is widened, made more "ample" (TC 1.6.4). Naturally, this suggests that ampliation is possible only for terms which are able to be predicated of many, that is, terms which are semantically general; singular referring expressions cannot be ampliated and remain singular. This is what Buridan maintains: in *Soph.* 5 Sophism 3 he asserts that discrete terms cannot be ampliated. Hence the theory of ampliation will deal only with common terms.

Buridan suggests in TS 3.4.8–10 that the copula of a Mental sentence is timeless, but whether this is so will not make any appreciable difference to his theory; he still has to explain sentences about times other than the present. Buridan admits two forms of temporal ampliation: (*i*) the copula itself is modified, either adverbially or by tensing the verb; (*ii*) certain terms in themselves imply a difference of time. In either case a term is ampliated from its original status (TS 6.2.1), which is the present supposition it possesses.

Let us begin with the case in which the copula is tensed; by TC 1.6.3 we know that a term retains its supposition for present items, and so Buridan offers the following paradigms for tensed sentences:

- "S was P": What was or is *S* was *P* [rule Amp-1 in TS 6.2.1;

TC 1.6.4].

- "S will-be P": What is or will-be *S* will-be *P* [Rule Amp-2 in TS 6.2.1; TC 1.6.4].

The supposition of the subject is thus ampliated to include past or future existents as well as present existents, analyzed as a hypothetical sentence with a disjunctive subject. This sentence is not equivalent to a disjunction of two sentences.[73] These paradigms have the peculiar consequence that "An old man will be a young man" may be true: this sentence is equivalent to "What is or will be an old man will be a young man," and this sentence may be true of a bouncing baby boy, who will eventually be an old man, will (first) be a young man (*Soph.* 4 Sophism 4). On the other hand, Buridan's analysis avoids nasty ontological puzzles; there is no way to move from the sentences "There was a statue" and "That statue no longer exists" to the conclusion "there is a statue which does not exist." However, a version of the "what is..." locution can temporally fix the extension of the subject-term: the subject-term of "What was *S* was/is/will-be *P*" refers only to present existents, of "What will-be *S* was/is/will-be *P*" refers only to future existents.

The predicate-term of sentences in which the copula is tensed is carried along to supposit for items at the time of the verb, provided that the term itself does not imply a different time (TC 1.6.1 and 1.6.20). Thus in a sentence of the form "*S* will be *P*" the subject-term is ampliated to supposit for what is or will be *S*, while the predicate-term supposits for

[73] Buridan argues for this point in TC 2.4.1–7. The nonequivalence mentioned is established for tensed sentences and for modal sentences. Buridan's argument proceeds as follows: consider the sentence (S*) "What is *B* can not-be *A* or what can be *B* can not-be *A*"—an equivalence Buridan will deny. He argues that when there are no actual *B*, the first disjunct of (S*), 'What is *B* can not-be *A*,' will be true, since it is a negative sentence with an empty subject-term. (Equally "Unicorns do not have wings" is true for the same reason.) Since one of the disjuncts of (S*) is true, (S*) is true. Buridan now claims that (S), however, may be false while (S*) is true, clearly showing the nonequivalence of (S) and (S*). He suggests that (S) would be false for the case of a possible (but non-actual) *B* that must be *A*, that is, cannot not-be *A*. Since (S) ampliates its subject to stand for possibles, (S) would be false. An example will help. Suppose that God destroys the human race, and consider the sentence (S) "A man can not-be rational." The proposed equivalent (S*) "What is a man can not-be rational or what can be a man can not-be rational" will be true, since its first disjunct is a negative sentence with an empty subject-term. But it is part of the nature of man to be rational; it is part of the human essence. Thus any possible man must be rational, that is cannot not-be rational, and so (S) is false while (S*) is true. Buridan opts for understanding (S) by the equivalence (S**) "What is or can be *B* can not-be *A*," whose truth-conditions are the same as those for (S).

what will be *P*.

In the second case individual terms produce temporal ampliation; the most obvious instance is of tensed participles, such as 'going-to-dispute' (*disputurus*, TS 5.2.10–12); when such a term appears in predicate position it takes us to a time relative to the time of the verb: in "Aristotle was going-to-dispute" the predicate-term 'going-to-dispute' supposits for the future relative to the time of the verb, which is some point in Aristotle's life (TS 5.2.11, 6.2.7; TC 1.6.20). this is quite like the systematization of tense-logic given by Reichenbach [1947]; there is no reason these relations cannot be iterated.

Other terms also produce temporal ampliation; we can divide them into two categories (Rule NCD-3 in TS 3.8.19): (*i*) terms implying in themselves a difference of time, such as 'dead'; (*ii*) terms determining, relatively or absolutely, a different time, such as 'tomorrow' or 'afterwards' (relative) or a dating of the sentence as '12 July 1358' or 'eternally' (absolute).

Buridan is a temporal divisibilist. He holds that all intervals are divisible into intervals, and intervals only, infinitely; in particular, Buridan rejects an ontology which includes instants or moments of time. Two arguments for temporal divisibilism are offered in *Soph.* 7: first, there is no single instant at which a spoken sentence exists, since the subject-term is uttered before the rest of the sentence; but it is part and parcel of Buridan's nominalism that a sentence, whether utterance or inscription, has a truth-value only when it exists; hence truth-value is relative to a given temporal interval (Sophism 1). Second, time is divided into past, present, and future; but the past and the future admittedly have duration. To suppose that the present has no duration would introduce an asymmetry into the account of time. Hence the present is not an instant, but an interval (Sophism 4). Therefore tensed sentences are evaluated for truth-value with respect to a given divisible interval. The interval with respect to which a given sentence is evaluated is determined by context, and may be as long or as short as you like; events within a given interval are treated as co-present. In order to preserve bivalence, Buridan relies on the asymmetry between affirmative and negative sentences:

> An affirmative sentence is true with respect to a given interval if and only if it is true in some subinterval.
>
> A negative sentence is true with respect to a given interval if and only if it is true in every subinterval.

Hence (*i*) "Socrates is asleep" and "Socrates is awake" can both be true with respect to the same temporal interval, and (*ii*) the consequence "Socrates is asleep; therefore, Socrates is non-awake" is acceptable for the same interval.[74] But (*i*) and (*ii*) are compatible; Buridan rejects the consequence "Socrates is non-awake; therefore, Socrates is not awake," which would deny bivalence: it is illegitimate to pass from an affirmative sentence with an infinite predicate to a negative sentence with a finite predicate.

Of course, the subject and predicate of tensed sentences may be quantified; this will not affect the analysis unless the quantifier is temporal, *e. g.* "whenever" or "sometime," in which case it will affect the supposition in an obvious way.

If there is no temporal ampliation, that is, in a sentence with a present-time verb and a non-ampliative predicate such as "S is P," Buridan allows the consequence "S is P; therefore, S is or exists" (QM 2.4). More generally, Buridan explores the logic of ampliated sentences in TC 3, in Theorem III-9 through Theorem III-12, where he argues (TS 3.4.72) that their logic wholly derives from the logic of non-ampliated sentences.

Ampliation is relevant only to divided modal sentences, since composite modals may be treated as ordinary quantification across sentences; divided modals behave in much the same way as tensed sentences. In fact, they are so similar it is not clear why Buridan doesn't give a uniform treatment of both. For example, he could distinguish composite and divided temporal sentences, such as "It will be the case that Smith is the mayor" and "Smith will be the mayor," treating such sentences like composite and divided modals. The copula of present-time assertoric discourse seems doubly indexical, for presentiality and actuality; I do not know why Buridan did not exploit this apparent parallel.

Divided modals in which 'possibly' or 'necessarily' adverbially modify the copula, or where the verb is 'can' or 'must,' ampliate the subject to supposit for possibles (TC 2.4.1):

- "S can be P": What is or can be S can be P [Rule Amp-3 in TS 6.2.1].
- "S must be P": What is or can be S must be P [Rule Amp-4 in TS 6.2.1, Theorem II-2 in TC 2.6.8].

In both cases the subject-term is ampliated to stand for possibles; this has the interesting corollary that when we know the truth of a modal sentence we know an infinite, the possibles (QSP 1.11). In fact, there is a close link between time and modality: Buridan says that possibility is "directed toward the future" but note that he does *not* say that every possibility in actualized at some time; he gives the example "Many things are possible which neither were nor are nor will be" (TC 1.6.9), and in stating rule Amp-

[74] Consequences, and hypothetical sentences generally, are assessed for truth-value with each of their constituent parts relativized to the same temporal interval.

4 (TS 6.2.1) he suggests that a term may supposit for things which never exist, the "merely possible."

What are the ontological commitments or temporal and modal discourse? Contemporary philosophers, following Quine, take the idiom of ontological commitment to be determined by quantity alone: existentially-quantifed sentences are the bearers of ontological commitment. Buridan, however, takes the idiom of ontological commitment to be determined by time, quality, and mode: present-time affirmative assertoric sentences are the bearers of ontological commitment. This alternative view has several important consequences. First, quantity is irrelevant; universal as well as existential present-time affirmative assertoric sentences carry ontological commitment—the so-called "existential import" of universal sentences. Thus "All unicorns have horns" commits one to the existence of unicorns as surely as "Some unicorns have horns" does. Second, negative sentences do not carry any ontological commitment; their truth-value is determined by the truth-value of their contradictory affirmative. Thus "Some unicorns do not have horns" does not carry any commitment to the existence of unicorns.[75] Third, Buridan insists that time and mode are relevant to ontological commitment. Time is not normally taken into account by contemporary logicians; Quine, for example, takes "eternal sentences" to be the logical form of sentences in his canonical language. Buridan, though, allows tensed sentences in Mental, and so his insistence that time is relevant to ontological commitment is a substantive disagreement with the modern tradition. Thus "Some man will be born in exactly ten years" does *not* carry any ontological commitment for Buridan. Equally, mode is relevant; "Some man can be standing in the doorway" does not carry any ontological commitment for Buridan.

Let us explore this difference further. Modal logicians today usually reduce *de re* to *de dicto* modality, and cash the latter out in terms of what actually happens in peculiar conditions: modal sentences are reduced to ordinary assertoric sentences with the ontology expanded to include possible worlds. Thus "Socrates can be bald" is canonically represented as "There is a possible world in which Socrates is bald."[76] Mediæval logicians,

[75] Quality is not completely irrelevant for contemporary philosophers; there is no ontological commitment if a negation precedes the existentially-quantified sentence on the modern view. But "Some unicorns do not have horns" would be represented as $(\exists x)(Ux \wedge \neg Hx)$, which does commit one to the existence of unicorns.

[76] It makes no difference whether the reference to possible worlds is included in the sentence (*e. g.* as an operator) or is part of the semantic apparatus, included in the interpretation function assigning truth-values to sentences; the net result is the same—

on the other hand, reduce *de dicto* to *de re* modals, or countenance both sorts. In particular, Buridan refuses to reduce modal discourse to any other form: rather than being a disguised form of assertoric discourse under peculiar conditions, modal discourse is a primitive form of discourse in normal conditions. Similarly, tensed discourse is not reducible to present-time or tenseless discourse with an expanded ontology including past and future existents; it is a primitive form of discourse in normal conditions. We might capture the difference by saying that mediæval logicians sever quantification (and hence supposition or reference from ontological commitment. The truth of modal or tensed sentences does not require us to expand our ontology. Buridan is a nominalist: he does not admit any abstract entities. But it is not at all clear that he must likewise refuse to quantify across past or future items, or possible items, so long as they are not abstract. Indeed, since supposition is a theory of reference, it seems clear that can can talk about such items; what is important is that we not take them to exist. And that is the moral of Buridan's alternative conception of ontological commitment.

6.9 Truth-Conditions

Armed with the theory of supposition, we can return to the truth-conditions of categorical sentences. First, Buridan takes the proposed definition of truth "howsoever a sentence signifies to be, so it is, in the thing(s) signified"—to apply directly to a present-time affirmative assertoric sentence. But there are other kinds of sentence. Past-time sentences are true if howsoever the sentence signifies to have been, so it was, in the thing(s) signified; future-time sentences are true if howsoever the sentence signifies to be going to be, so it will be, in the thing(s) signified; and modal sentences are true if *e. g.* howsoever the sentence signifies to be able to be, so can be, in the thing(s) signified (TC 1.1.7–11, QM 6.8). As remarked at the end of Section 5.5, the application of this definition requires the theory of supposition, explaining what is meant by the clause "in the thing(s) signified." If we confine ourselves to the simple sentences of the traditional Square of Opposition, then we may state their correspondence truth-conditions[77] as follows:

(1) If an A-form sentence such as "Every S is P" is true, then everything the subject supposits for the predicate supposits for.

(2) If an I-form sentence such as "Some S is P" is true, then something the subject supposits for the predicate supposits for.

an assertoric sentence true in an expanded ontology.

[77] I borrow this name from Hughes [1982] 22.

(3) The truth-value of the E-form (*e. g.* "No S is P") and O-form (*e. g.* "Some S is not P") sentences is determined by (1) and (2).

Now (1) and (2) are rather similar to giving truth-conditions in terms of set inclusion among the extensions of the terms. But note that they are stated as necessary conditions for truth, not as sufficient conditions. That is because of problems with Liar-sentences (TC 1.5.5–1.5.7; *Soph.* 2 Theorem 12 and *Soph.* 8 Sophisms 7 and 11 especially).

Recall that Buridan takes sentences to be assertions, and hence the sentential form requires certain contextual prerequisites be met for the sentence to count as a sentence. An affirmative assertion (sentence) indicates that its terms supposit for the same, according to the requirements of the given sentence; a negative sentences indicates the opposite (TC 1.5.1–2). But this is not all, for Buridan also holds what Hughes has felicitously named the *Principle of Truth-Entailment* ([1982]) 110): a sentence 'p' and a sentence of the form 'A exists,' where 'A' names p, together entail a sentence of the form 'A is true.' (Naturally 'p' as it occurs in the latter sentences is in material supposition and only equiform to the original sentence.) This principle is quite intuitive if we remember that sentences are assertions: an utterance which actually counts as an assertion contextually presupposes that the assertion is true. Buridan sometimes expresses this loosely, by saying that the truth of a sentence is nothing other than that very sentence itself (QM 2.1 fol. 8vb). He is thinking of Mental sentences, of course, but the point is that 'truth' is eliminable: we do not need a truth-predicate.

What are the causes of the truth of falsity of a sentence? Buridan discusses this question in TC 1.2 and QM 6.8. We have seen (in Section 5.5) how Buridan rejects various suggestions about what a sentence signifies, especially the *complexe significabile*. But what then is the cause of the truth or falsity of a sentence? Some fact or event? But then it seems as though we have to admit negative facts, or perhaps future facts, and the like. Modern logicians sidestep some of these difficulties by taking negation as a sentential operator, so that negated sentences are molecular; their truth-value is then derived from the truth-values of atomic sentences. Buridan however, admits as basic two forms of the copula ('is' and 'is not'), and so cannot treat the truth-value of negative sentences as derivative. Rather, he takes a more radical line: there is *no* cause of the truth of a negative, just as there is no cause of the falsity of an affirmative. The causes of the truth of an affirmative are equally causes of the falsity of the corresponding negative: the way things are. And that is the end of the story. Further support for this view can be found in the fact that Buridan states correspondence truth-conditions only for A-form and I-form sentences, since there is no cause of the truth of the E-form and O-form sentences.

Buridan also has several remarks about the "number" of causes of truth, which are as one would expect: the sentence "All men are sexists" has more causes of its truth that "Some men are sexists." These causes of the truth of the sentences are related by set-inclusion, it should be noted. Such principles are crucial for proving equipollence and conversions, as we shall see in Section 7.3.

7. Consequences

7.1 Conditionals, Inferences, and Consequences

Buridan's theory of *consequences* covers material treated by modern logic under the separate headings of a theory of conditionals and the rules of inference. These are not distinguished by the logicians of the fourteenth century;[78] this need not be an error—we need to recall the philosophical motivations for drawing the distinction initially.

Consequences in modern logic are distinguished as conditionals and rules of inference; they are not merely equivalent: to show that it is possible to pass from one to the other a Deduction Theorem is needed, which is not a trivial matter. It is not obtainable in incomplete systems. Each is specified syntactically, but conditionality is represented by a sign in the language (as primitive or a defined abbreviation) appearing in formulae, whose behavior is given by axioms, while rules of inference are neither stated in the language nor in the syntactic recursive grammar, but metalinguistically govern the production of a formula from other formulae. Inference rules are tied to deducibility and provability, and hence to validity by the notion of syntactic consequence; conditionals are loosely tied to truth and interpretation.

Buridan, as noted, does not distinguish an object-language from a metalanguage, so it would be difficult for him to arrive at precisely our distinction between conditions and rules of inference.[79] Yet aside from the incompleteness of logical systems, a worry it would be anachronistic to

[78] Things were not always so: the twelfth-century philosopher and logician Peter Abelard clearly distinguishes arguments and conditionals, and even argues for a Principle of Conditionalization and Deconditionalization—a mediæval "deduction theorem." But Abelard's work seems to have been completely lost to the fourteenth century, for unknown reasons. The unpublished studies of Christopher Martin on Abelard and Boethius on conditionals are invaluable.

[79] The problem is exacerbated if Spoken language is taken as primary, for the distinction is usually not drawn verbally; if drawn at all, it typically relies on non-verbal cues. Buridan mentions such cues in TS 2.6.79 when distinguishing composite and divided senses.

ascribe to Buridan, there are two standard philosophical reasons given for distinguishing conditionals and rules of inference. The first is classically expressed by Lewis Carroll in "What the Tortoise Said to Achilles," and takes the form of an infinite regress. Briefly, the Tortoise asserts "p" and "If p then q" but rejects Achilles' claim that he must perforce assert q: that only follows, maintains the Tortoise, if he asserts "If p and if p then q, then q." Yet even if he accepts this principle, he continues, he need not assert q, for that does not follow unless he also asserts "If if p and if p then q, then q," and so on *ad infinitum.* But this argument will not serve its purpose. First, if correct, it shows that any system containing a Deduction Theorem includes this infinite regress, though it be carefully disguised in an axiom-schema or a schematic rule of inference. Second, the argument suggests the seeds of its own destruction: do we not need justification for the rule of inference, and will not this justification stand in need of justification? The distinction does not prevent the regress.

The second argument for distinguishing conditionals from rules of inference is based on philosophical analysis of ordinary language: it captures in a formal way the difference between assertions which do or do not require a commitment to the truth of the first statement. One is so committed when using the inferential form, one is not so committed when using the conditional form. Perhaps these considerations about acceptance and commitment are what motivate the infinite regress given by the Tortoise—who, after all, is dialectically arguing with Achilles, and so seeks persuasion.

The second argument is on the right track, but then there is no overwhelming reason to distinguish the cases as being of different kinds rather than as species of a single genus, the mediæval notion of consequence. Hence we are not forced to distinguish them.[80] Consequences are similar to inference-rules in three principal ways: first, we have a consequence only when it is impossible that the antecedent obtain with the consequent failing to obtain; otherwise we do not have a consequence at all. Conditionals are identified syntactically, though, and are no less conditional for being false in an interpretation. Consequences thus seem more similar to inference-rules, for there is no inference when the first formula obtains and the second does not. Second, consequences are not specified syntactically, but are defined by the relations obtaining between what the antecedent and consequent assert

[80] While the theory of consequences explores some principles about commitment, the theory of *obligationes* explores in greater detail which inferential connections obtain when the sense of a term or a sentence is altered, all else remaining the same—this is the form known as *institutio*. See Spade and Stump [1982] for a discussion of *obligationes*.

(to speak loosely). Third, consequences are called acceptable, not 'true' or 'false.'

Consequences are similar to conditionals in two principal ways: first, the sentences which appear as antecedent and consequent are *used*, not mentioned, despite the fact that technically they are not sentences (no part of a sentence is a sentence). Second, the definition of consequence is remarkably similar to Lewis-Hacking's notion of strict implication, in the modal characteristics of the definition.

7.2 The Definition and Division of Consequences

In TC 1.3.4–12 Buridan considers the proper definition of "consequence," eventually proposing the following (see especially TC 1.3.11):

> A sentence of the form "if p then q" or "p; therefore, q" or an equivalent form is a *consequence* if and only if a sentence p^* equiform to p and a sentence q^* equiform to q are so related that it is impossible that both (i) it is the case as p^* signifies to be, and (ii) it is not the case as q^* signifies it to be, provided that they are put forth together, and *mutatis mutandis* for each class of sentences.

This definition calls for some comment. First, the constituent parts of a sentence are not themselves sentences, which is why we must specify sentences equiform to the parts of a grammatically consequential sentence, *i. e.* the protasis and apodosis of the sentence; Buridan carefully calls these the 'first part' or the 'second part' of a grammatically consequential sentence.[81] When we have a consequence, we may call these the antecedent and consequent, but not otherwise. Second, the clause 'provided they are put forth together' is meant to rule out cases where either part is not asserted (TC 1.3.8). Third, the final clause '*mutatis mutandis* for each class of sentences' is meant to remind us that the tense and mood of the verbs in (i) and (ii) have to be taken seriously, since Buridan denies a general formula for truth and gives criteria for each class of sentence: assertoric or modal, past-time or present-time or future-time, affirmative or negative, and so forth. Finally, note that the definition is semantic, because it defines 'consequence' in terms of the relation holding between what p^* and q^* assert or say is the case.[82]

[81] Buridan will occasionally call a grammatically consequential sentence a "consequence," even if it fails to satisfy his complex conditions, but this is no more than a mere abbreviation, a harmless way of talking eliminable upon request. I shall also indulge in this looseness when there is no danger of being misunderstood.

[82] Buridan is careful to argue that the supposition and appellation of terms in the protasis or apodosis is the same as their supposition in the equiform sentence p^* and q^*

Buridan almost never calls a consequent 'true' and 'false' in this treatise. Rather, he calls them *acceptable (bona)* or not, though technically an unacceptable consequence is not a consequence (this is an instance of the laxity noted above). A consequence is acceptable if it satisfies the strict definition for consequences as given above. In that respect consequences are indeed like inferences, which are valid of invalid, not true or false.[83]

Buridan's division of consequences is found in TC 1.4.9, as follows:

CONSEQUENCES
- Formal
- Material
  - Simple
  - *ut nunc*

The first division is between *formal* and *material* consequences. A consequence is formal if and only if it satisfies the *Uniform Substitution Principle* (TC 1.4.2–3): it is acceptable for any uniform substitution for any of its categorematic terms; otherwise the consequence is material. Formal consequences are in this respect like tautologies, and so like strict implication: they remain acceptable (true) for any uniform substitution of categorematic terms (non-logical constants).

Material consequences fail this test, but may yet be necessary; such material consequences are called *simple.* As an example Buridan points out that the sentence "A man runs; therefore, an animal runs" is not formal, for a substitution-instance is "A horse walks; therefore, wood walks" (TC 1.4.3). The genus-species relation between 'man' and 'animal' need not be preserved under substitution. On the other hand, it is clearly a necessary consequence.[84] Material consequences come in two forms: simple and *ut nunc*. A simple material consequence is a consequence which is not formal but satisfies the strict definition of consequence given above. They are acceptable only through reduction to a formal consequence, namely "by the addition of some necessary sentence or sentences which, when assumed with the antecedent, render the consequence formal" (TC 1.4.4). In the example above the necessary sentence is "All men are animals," so that "A man

(TS 3.7.41). But note that the definition is not stated by talking about the truth of p^* and q^*; this is due to complications which arise from taking Liar-sentences into account: what a sentence signifies to be is the case is a necessary, but not a sufficient, condition for the truth of the sentence.

[83] In a few places Buridan says "*consequentia valet*," which I have translated as 'valid.'

[84] In the spirit of Carnap's "meaning-postulates" we could modify our substitution-rules to preserve relations among terms, as was done by the later Scholastics, but Buridan's refusal to do so indicates an admirable attempt to sever logic from metaphysics.

runs and all men are animals; therefore, an animal runs" is a formal consequence. Simple material consequences are thus treated as enthymematic (TC 1.4.5–6).

The final kind, *ut nunc* consequences, are strictly speaking not necessarily consequences: they are acceptable if we replace the 'impossible' in the definition of consequence with 'it is not the case.' An *ut nunc* consequence, then, is a sentence such that it is not the case that the antecedent obtains and the consequent fails to obtain. But here again we may take the tense of the verb in 'it is not the case' seriously, and so discuss consequences which hold as a matter of fact at other times—as Buridan says, *ut nunc*, *ut tunc*, or *ut nunc pro tunc*. Thus Buridan's theory of tense-logic will be confined to part of the theory of *ut nunc* consequences. Not that the *ut nunc* consequence may behave just like the material conditional; all that is required for it to be acceptable is the factual lack of the antecedent obtaining with the consequent failing to obtain.

Consequences are further examined in the first seven theorems in TC 1. Theorem I-1, Theorem I-5, and Theorem I-7 state the key characteristics of consequences: they can never lead from truth to falsity, nor from possibility to impossibility, nor from the necessary to the non-necessary; equally, the necessary follows from anything, and from the impossible (such as the conjunction of contradictories) anything follows. In Theorem I-3 the law of contraposition is stated for consequences, and in Theorem I-4 the law of transitivity. Together, all of these characteristics define the nature of an acceptable consequence.

7.3 Assertoric Consequences

The rest of the theorems in TC 1 are devoted to particular assertoric consequences. These are two forms: *equipollence*, that is, sentences which follow from each other as consequences, and *conversion*, in which the subject-term and the predicate-term of a sentence are the same, but their positions are reversed or the syncategoremata are altered (or both). In TC 1.8.47, Buridan says that all equipollences and conversions are contained in Theorem I-8 (given in 1.8.40 but as revised in 1.8.43):

> [Theorem I-8 (revised)] (*a*) Any two sentences of which neither can have some cause of its truth which is not a cause of the truth of the other sentence follow from the same sentences; (*b*) any two sentences of which one has, or can have, more causes of its truth that the other sentence, although every cause of the truth of the latter is a cause of the truth of the former, are so related that the sentence with more causes of its truth follows from the sentence with fewer, but (*c*) not conversely.

Though difficult to state precisely, Theorem I-8 is extremely important: sentences will be shown to be equipollent by having the same causes of their truth, and so by (*b*) mutually follow from each other. This result Buridan deemed sufficiently important to repeat it in a separate theorem, theorem I-9. Equally, one sentence will be shown to be the conversion of another by having the same or more causes of its truth than the original sentence, that is, by Theorem I-8(*a*) or (*b*). These two theorems are the basis of not only assertoric equipollences and conversions, but also of modal equipollences and conversions (TC 1.8.50), though these are explored later. These theorems are perfectly general; Buridan points out (TC 1.8.51 that the sentences "Of-any-man no ass is running" and "Of-no-man an ass is running" are equipollent. Hence his theory is applicable far beyond the simple sentences which appear in the Square of Opposition.

Buridan gives only some examples of equipollences in TC 1.8.51; they are the equivalences of signs of quantity and negations, so that *e. g.* 'Every ... is not —' is equipollent to 'No ... is —'. The equipollences are standard, but note that Buridan does not take them as definatory of the signs of quantity, as modern logicians define one quantifier by another; their equipollence must be established, which is a corollary of Buridan's semantic approach to quantification.

Conversions are treated at greater length, in Theorems I-10 through I-17. Buridan begins in Theorem I-10 with consequences by subalternation and conversions of universals into particulars; the theorem is stated for terms with distributive confused supposition, and generalized to terms with non-distributive confused supposition in Theorem I-II. The next pair of theorems, Theorem I-12 and Theorem I-13, deal with conversions of sentences containing ampliation or non-ampliating restrictions (such as the "what is..." locution).

Theorem I-14 is a turning-point in Buridan's discussion, specifying rules of simple and accidental conversion:[85]

> [*Theorem I-14*] (*a*) From any universal or particular affirmative assertoric sentence there follows a particular affirmative by conversion of the terms; (*b*) from any universal negative there follows a uni-

[85] Conversions are *simple* when the subject-term and predicate-term are reversed and the result is equipollent. E-form and I-form sentences convert simply, *e. g.* "No S is P" and "No P is S" are equipollent. Conversions are *accidental* when the subject-term and the predicate-term are reversed and the quantity is changed; the result is not equipollent. Thus an A-form is converted accidentally to an I-form, such as "Every S is P" to "Some P is S," and an E-form is converted accidentally to an O-form, such as "No S is P" to "Some P is not S."

versal and a particular negative; and (*c*) no sentence follows from a particular negative by its form alone.

Buridan discusses this theorem in TC 1.8.70–95. The extended analysis is required by cause of the generality of Theorem 1-14: it applies to sentences in which the logical constituents (subject-copula-predicate) may be implicit rather than explicit, such as the one-word sentence "*Ambulo*"; the copula of such sentences, whether explicit or implicit, may be present-time, past-time, or future-time; the predicate term r the subject-term, or both, may be ampliative; the 'what is...' locution may be used; the sentence may be existential, and so (apparently) lack a predicate-term; the rules of grammar may be violated in reversing the subject-term and predicate-term; the subject-term or the predicate-term may be logically complex, containing syncategoremata; they may contain or consist in terms in oblique cases; certain syncategoremata may require special analysis. Buridan takes each of these possibilities into account, and his discussion is a marvel of complex precision.

A singular sentence follows from another singular sentence by conversion of the terms, as Buridan notes in Theorem I-15, paying attention to ampliation and the form of the singular term.

In the next pair of theorems Buridan considers the results when the subject-term or predicate-term is changed "according to finite and infinite," *i. e.* the effects of term-negation (infinitizing negation) on conversions. This may be called 'conversion' in a broad sense, even though the terms themselves are altered (TC 1.8.101). Theorem I-16 and Theorem I-17 set forth such relations of conversion; it is often necessary to suppose that the varied term is non-empty, the assumption of the *constantia terminorum*.[86]

It is a travesty to present mediæval logic as though only sentences appearing on the Square of Opposition were investigated. Buridan's discussion of assertoric consequences is perfectly general, and adequate for his philosophy of logic and language.

[86] The technical names for these conversions are as follows. First, if the subject-term and predicate-term are not reversed, then (*i*) if the predicate-term is changed according to finite and infinite the result is called *obversion*; (*ii*) if the subject-term is changed according to finite and infinite the result is called *partial inversion*; (*ii*) if both subject-term and predicate-term are changed according to finite and infinite the result is called *full inversion*. Second, if the subject-term and the predicate-term are reversed, (*i*) the obversion of the resulting sentence is called *obverted conversion*; (*ii*) the partial inversion of the resulting sentence is called *partial contraposition*; (*iii*) the full inversion of the resulting sentence is called *full contraposition*. (Do not confuse the latter case with 'contraposition' as applied to consequences themselves.)

7.4 Divided Modal Consequences

In TC 2.2.5 Buridan discusses the equipollence of the various modes in combination with negations, and in Theorem II-1 he establishes such equipollences as a formal result. There are no surprises; 'necessary' is equipollent to 'not possibly not,' and so forth.[87] Yet there is an important methodological corollary of such equipollences which Buridan states in TC 2.6.7: theorems for divided modals need be stated only for the case in which the mode is affirmed.

Conversions of divided modals are treated in Theorem II-5 and Theorem II-6. The results are straightforward: affirmative divided modals *de possibili* convert both simply and accidental (to affirmative divided modals *de possibili*, of course); universal negative divided modals *de necessario* convert simply. No other conversions are explored.

Buridan does, however, explore 'mixing' theorems, that is, theorems about which consequences obtain between sentences of different modes. In Theorem II-3 the relation between assertoric sentences and divided modals *de necessario* is stated: the only consequence which obtains is that from a universal negative. In Theorem II-4 the relation between assertoric sentences and divided modals *de possibili* is stated the only consequence which obtains is that from an affirmative assertoric to a particular affirmative *de possibili*. Buridan specifically remarks the lack of consequential connection between divided universals *de possibili* and their corresponding assertorics (TC 2.7.32), and between divided particulars *de possibili* and their corresponding assertorics (TC 2.7.33).

In TC 2.6.33 Buridan defines the mode 'contingent': it is equipollent to 'possibly and possibly-not.' This mode may also figure in divided or composite modals, and Buridan explores its behavior in Theorem II-7, Theorem II-8, Theorem II-14, and Theorem II-19. His motivation for so doing is not clear; by equipollence, theorems about composite or divided *de possibili* sentences will settle questions about contingents as well. The formula is simple: replace the contingent sentence with a conjunction of *de possibili* sentences. Because this theoretical simplication is available, we shall not discuss Buridan's treatment of contingent sentences, either as consequences or as syllogistic.

[87] The equipollence of necessity and possibility requires divided modals *de necessario* to ampliate their subject-terms to stand for possibles, as Buridan points out in Theorem II-2; this in turn supports his remark in TC 2.6.22 that a *de possibili* divided modal follows from a *de necessario* divided modal, but not conversely.

7.5 Composite Modal Consequences

Buridan begins his discussion of composite modal consequences with Theorem II-9, which states that the following consequence-scheme is acceptable:

Some [dictum] is [mode]; therefore, every [dictum] is [mode].

Thus "Some [sentence] 'The sentence written on the wall is false' is possible; therefore, every [sentence equiform to] 'The sentence written on the wall is false' is possible" is an acceptable consequence. In particular the sentence which is the sole inscription on the wall 'The sentence written on the wall is false' is possible. This view does not land us in paradox; recall that to say a sentence is "possible" is to say that it describes a possible situation. The particular inscription on the wall does just that: for example, it describes the possible situation in which only '$2+2 = 17$' is written on the wall. Sentences need not describe their own actual situation to be possible. This is the point of Theorem II-9: if an inscription or utterance is possible, then it describes a possible situation, and all equiform inscriptions of utterances are equally possible since they may be taken to refer to that situation.[88] The inscription on the wall "The sentence written on the wall is false" is possible, but never possibly-true. With this point noted, Buridan's theorems about composite modal equipollences and conversions do not pose any special problem.

Conversions of composite modals are straightforward. If the dictum is the subject, then all composite modals convert simply with the exception of the universal affirmative composite modal, which is converted accidentally (Theorem II-10). If the mode is the subject, then all composite modals convert simply with the exception of the particular negative composite modal, which is not converted (Theorem II-11). Buridan also discusses "conversions with respect to the dictum," in which the composite modal sentence itself is not converted, but the dictum of the composite modal is converted. Such conversions with respect to the dictum are discussed in Theorem II-12, Theorem II-13, and Theorem Ii-14, and pose no special problems.

As with divided modals, Buridan also offers mixing theorems for composite modals in relation to assertoric sentences, and in particular, the dictum of the composite modal. We may simplify Theorem II-15 as follows:

> [*Theorem II-15 (revised)*] (*i*) From any composite affirmative modal *de vero* there follows its dictum, and conversely; (*ii*) from any composite modal *de necessario* there follows its dictum; (*iii*) from any

[88] Note that this principle requires strong accessibility among possible worlds: only in S5, in which every possible world is accessible from every other possible world, is a claim like this acceptable.

sentence there follows a composite modal *de possibili* of which it is the dictum.

These claims have a modern appearance: (*i*) is a version of the Tarski biconditional; (*ii*) is like the law "$Tp \rightarrow p$"; (*iii*) is like the law "$p \rightarrow Tp$." Equally, Theorem II-16 can be seen as expressing the distributive laws "$(\Box p \wedge (p \rightarrow q)) \rightarrow \Box q$" and "$(\Diamond p \wedge (p \rightarrow q)) \rightarrow \Diamond q$." Of course, Buridan expresses these theorems as consequences, which are not simply conditionals or rules of inference; nor is it strictly speaking correct to formalize his logic with the propositional calculus, since no part of a sentence (such as '$\Box p$') is a sentence (such as 'p'), but merely contains a part equiform to a sentence. These differences noted, Buridan's claims are still quite similar to the modern theses.

Finally, Buridan gives some mixing theorems which 'mix' composite modals and divided modals. Theorem II-17 says that a particular affirmative divided modal follows from an affirmative composite modal (with the dictum affirmed); Theorem II-18 says that a universal negative divided modal follows from a universal composite modal (with the dictum denied). These are the only consequential relations holding between composite and divided modals.

8. The Syllogism

8.1 The Definition of the Syllogism

When we turn to syllogistic, it is necessary to recall that Buridan is not working an artificial language and stating a symbolic calculus: he is working a fragment of a natural language, although a highly stylized fragment (scholastic Latin), and he possessed neither our modern array of metalinguistic equipment nor our interest in formal metalogic. It is easy to lose sight of these obvious facts, for Buridan's syllogistic has an astonishing degree of rigor.

In Buridan's hands syllogistic is a logical instrument of great flexibility and power, not the rigid and sterile doctrine it later became; it is directly based on his philosophical semantics. Buridan seems almost uninterested in the part of assertoric syllogistic which most people have traditionally identified as 'syllogistic,' quickly moving on to temporal syllogisms, oblique syllogisms, variation syllogisms, and modal syllogisms (discussed Section 8.4).

The initial definition of 'syllogism' in TC 3.2.1 is given only after a long description of what kind of consequence a syllogism is (TC 3.1.1–17). The syllogism is a formal consequence, and in that sense syllogistic is merely a branch of the theory of consequences; it is distinguished by having a conjunctive antecedent and a single consequent, made up with three terms. This paradigm is generalized when Buridan turns to the oblique syllogism, and allows more than three terms, provided that the terms connected in the conclusion are parts of the terms of the premisses (TC 3.6.2–3), a result required for variation syllogisms as well: more precisely, multiple-term syllogisms are permitted under strict assumptions about the relations of the terms.

We can be more exact by introducing some of Buridan's technical terminology:

(1) The *major sentence* is the first premiss in the conjunctive antecedent, and the *minor sentence* the second premiss in the conjunctive antecedent (TC 3.2.3).

(2) The *extremes* of a sentence are its subject and predicate; the *syllogistic extremes* are the extremes of the conclusion (TC 3.6.2).

(3) The *syllogistic middle* is a term or part of a term common to the premisses; the *major extreme* is the extreme in the major premiss which is not the syllogistic middle, and the *minor extreme* is the extreme in the minor premiss which is not the syllogistic middle. The major and minor extremes, or parts thereof, are connected in the conclusion (TC 3.2.3 and 3.6.2).

When the syllogistic extremes are the major and minor extremes, and the middle is the same term in both premisses, we have the additional paradigm of the syllogism; the deviations above are introduced to allows for development of the syllogistic.

We need three further technical notions to develop syllogistic. First, we may define a *syllogistic figure* as the ordering of the syllogistic middle to major and minor extreme in the premisses (TC 3.2.4); second, we call a conclusion *direct* or *indirect* if the major extreme is predicated of the minor extreme in the conclusion, or conversely; finally, we call a particular pair of premisses characterized only with respect to quantity and quality a *conjugation*.

From these definitions it is clear that there are exactly four figures (TC 3.2 4–8): (1) the first figure, in which the syllogistic middle is subject in the major sentence and predicate in the minor; (2) the second figure, in which the syllogistic middle is predicate in major and minor; (3) the third figure, in which the syllogistic middle is subject in major in minor; (4) the fourth figure, in which the syllogistic middle is predicate in the major sentence and subject in the minor. But if we transpose the premisses of the fourth figure we have the first figure, changing the conclusion from direct to indirect or conversely. Buridan correctly notes this point and does not bother to treat the fourth figure independently (TC 3.2.9). He is quite correct; given the way he has defined the syllogistic figures, they are not really distinct; it is only on other definitions of the syllogistic figures that there can be a genuine 'fourth figure' controversy.[89]

8.2 Syllogistic Semantic Principles

A syllogism is a formal consequence with a conjunctive antecedent and a single sentence as consequent, containing three or more terms. To understand why a syllogism satisfies the Uniform Substitution Principle is to understand the semantic framework of a syllogism, and this requires us to examine how terms are conjoined—not merely in a single sentence but across two sentences, the conjunctive antecedent. Buridan sets out this semantic framework in two rules for the acceptability of syllogisms, which require the notion of distribution.

Buridan's two rules are given as his version of the traditional *dictum de omni et nullo*, where one rule applies to syllogisms with affirmative conclusions and the other to syllogisms with negative conclusions. His statement is given in TC 3.4.5–7, but Buridan's distinctions along the lines of discrete and common terms are unnecessary. In their full generality they are as follows:

> [*Rule 1*] Any two terms which are called the same as a third term by reason of the same thing for which that third term supposits, not collectively, are correctly called the same as each other.
>
> *[Rule 2]* Any two terms, of which one is called the same as some third term of which the other is called not the same by reason of the same thing of which that third term supposits, are correctly called not the same as each other.

Two points must be noted: first, the 'not collectively' clause in Rule 1 excludes cases in which two terms are collectively or conjunctively called the same as a third term, as when "matter and form are said to be the same as one and the same composite, and the matter is not the same as the form (TC 3.4.3)."[90] Second, the careful use of the phrases 'called the same' and 'called not the same' indicate that Buridan is here talking in what we should call the formal mode: they characterize sentences which, if

[89] See the discussion of various definitions of the fourth figure and the related controversies in Rescher [1966] Chapter II.

[90] Two terms are used collectively if a sentence in which they are the conjoint subject does not entail the conjunctive sentence in which each conjunct has only one of the terms as the subject. See further Buridan's discussion of collective terms in TS 3.2.3.

formed, are respectively affirmative or negative in quality.[91] This is the key to Buridan's semantic characterization of the *dictum de omni et nullo*: his talk of supposition is entirely in the formal mode. Syllogistic is a branch of the theory of formal consequence, and so the test for the acceptability of a syllogism is whether it satisfies the Uniform Substitution Principle, but the principle applies only if certain relations among the supposition of the terms obtain. In a particular sentence the relations are made explicit by the nature of the syncategorematic terms present, and Rule 1 and Rule 2 state how the terms must be semantically related, *i. e.* in terms of their supposition.

The clause 'by reason of the same thing for which that third term supposits' is the central semantic contribution of the *dictum de omni et nullo*, because it makes explicit the coreferentiality required for the same term appearing in different sentences. Coreferentiality is the underpinning of the theory of inference. Now there are many ways of fixing reference; how can we tell if the clause 'by reason of the same thing for which that third term supposits' is satisfied? That is: when are different occurrences of a middle term coreferential? There are three cases. First, the third term in question may be a singular referring expression, that is, a discrete term, and here there is no trouble, for a discrete term can supposit only for a single thing, as a matter of semantics. A syllogism with a discrete term as syllogistic middle is called an 'expository syllogism' (Theorem III-4 in TC 3.4.23–25). Second, if the middle is a common term, then we may simply stipulate that the syllogistic extremes are called the same or not the same for the same thing(s) for which the middle term supposits; this stipulation must be visible in the sentences, since the syllogism is a formal consequence, and consists in adding an identificatory relative-term to the minor (Theorem III-6 in TC 3.4.29–30).

The third case also involves a common middle term ,but where there is no such stipulation of suppositional sameness. We do not simply want coextension here; we want coextension as a matter of the semantics, for a syllogism is a formal consequence—indeed, a matter of the semantics obvious from the syntactic form. The answer is given by the theory of distribution.

Buridan does not specifically address distribution while discussing the syllogism, but he does not need to: it is covered in his account of common personal supposition. A term is *distributed* if in a sentence it is taken

[91] We have seen Buridan make the same point in the material mode, for example when he says that an affirmative sentence "indicates" that its terms supposit for the same thing(s), as determined by the correspondence truth-conditions listed in Section 6.9.

to supposit for all it signifies, that is, if it is used to talk about or refer to everything it signifies. The most obvious case of distribution is where a term is joined with a distributive sign (a universal quantifier), not in the scope of a negation. Buridan's rules for distributive and non-distributive supposition, discussed above, explicitly state when a term is said to be distributed and when it is not. The theory of distribution, and hence of syllogistic, is therefore of widespread applicability, but if we confine ourselves to simple sentences as on the Square of Opposition, we may give some rules for distribution: universals distribute subjects, negatives distribute predicates, and no other terms are distributed. Hence in an A-form sentence the subject alone is distributed, in an E-form sentence the subject and the predicate are distributed, in an I-form neither subject nor predicate is distributed, and in an O-form sentence the predicate alone is distributed.[92]

The motivation for the theory of distribution is clear; we want to avoid the case in which one extreme is called the same or not the same as part of what the middle term supposits for, while the other extreme is called the same or not the same as part of what the middle term supposits for, while the other extreme is called the same or not the same as the other part of what the middle supposits for. In that case there is no connection between the extremes through the middle, and no inference will hold by the semantics alone. Distribution is a way of making sure that the foregoing case does not occur, by talking about everything the middle term supposits for. Indeed, it obviously follows from the theory of distribution that no syllogism made up with two negative premisses is acceptable (Theorem III-2 in TC 3.4.15), as Buridan notes. The principles governing the doctrine of distribution are given in Theorem III-7 and Theorem III-8 (TC 3.4.34–36).

8.3 Reduction and Proof-Procedure

Buridan follows tradition in taking the first four moods of the first figure to be evident of 'perfect': *Barbara*, *Celarent*, *Darii*, *Ferio*. Their acceptability is shown directly by the preceding principles; what I have called Buridan's "proof-procedure" for syllogisms is his way of reducing all other syllogisms to these.[93] In each of the various forms of assertoric and modal

[92] Classically, a syllogism is acceptable if and only if (*i*) the middle term is distributed exactly once; (*ii*) an extreme term is distributed at most once; (*iii*) if the conclusion is negative exactly one premiss is negative; (*iv*) if the conclusion is affirmative neither premiss is negative.

[93] The traditional name for each mood indicates the method of reduction in the following way: (*i*) the initial letter of the name indicated which of the four basic first-figure syllogisms it is reduced to; (*ii*) the first three vowels characterize the quantity and quality of each premiss (and all other vowels are ignored); (*iii*) the letter 's' following a

syllogistic Buridan takes up, he first points to the evidentness and perfection of those moods corresponding to the first-figure moods listed; no further justification is given.[94] Hence it is incorrect to think that Buridan (or any other mediæval logician) has a formal metatheory of deductive systems. Buridan offers two methods for showing the acceptability of a syllogism: the *Reductio*-Method and the Method of Reduction. Both are founded on Aristotle, and supplement the principles listed in the preceding section.

The*Reductio*-Method, traditionally required only to show the acceptability of *Baroco* and *Bocardo*, is stated as a general principle governing consequences with a conjunctive antecedent (Theorem III-3 in TC 3.4.17–18):

> A grammatically consequential sentence of the form "$p \wedge q$; therefore, r" is a (syllogistic) consequence if and only if for the sentence $\neg r$ contradictory to r (*i*) the sentence "$p \wedge r$; therefore, $\neg q$" is acceptable for the sentence $\neg q$ contradictory to q; (*ii*) the sentence "$q \wedge \neg r$; therefore $\neg p$" is acceptable for the sentence $\neg p$ contradictory to p.

Since the antecedent of such a sentence is itself equiform to a conjunctive sentence, the theorem holds by consequential contraposition (Theorem I-3), De Morgan's Laws, and consequential importation. The *Reductio*-Method consists in assuming that a syllogistic consequence fails to hold and showing that clauses (*i*) and (*ii*) are satisfied. We can easily illustrate this method by proving the acceptability of *Bocardo*: "Some M is not P, and all M is S; therefore, some S is not P." Taking the contradictory of the conclusion "All S is P" and the minor premiss we can syllogize in *Barbara* "All S is P, and all M is S; therefore, all M is P," and this conclusion contradicts the original major premiss; equally, taking the contradictory of the conclusion and the major premiss we can syllogize in *Baroco* "All S is P, and some M is not P; therefore, some M is not S," and this conclusion contradicts the original minor premiss. Hence *Bocardo* is acceptable.

vowel indicates that the premiss characterized by the vowel is to be converted simply, that is, characterized by the vowel is to be converted *per accidens*, that is, the terms transposed in the subalternate; (*v*) the letter 'm' indicates that the premisses are to be transposed; (*vi*) the letter 'c' indicates that the reduction is a *reductio ad absurdum*. For example, take *Camestres*, of the second figure. The vowels indicate that the form is "All P is M, and no S is M; therefore, no S is P." To reduce *Camestres* we apply (*iii*) to the minor and to the conclusion, and by (*v*) we transpose the premisses, giving us "No M is S, and all P is M; therefore, no P is S"—a syllogism in *Celarent*.

94 Buridan discusses the perfection of the assertoric syllogism in TC 3.4.56–57 (with an aside on ampliation and the 'what is. . . ' locution in 3.4.56), the oblique syllogism in TC 3.7.9, syllogisms including identificatory relative-terms in TC 3.7.23, and the divided modal syllogism in TC 4.2.4 and 4.2.28.

The other way to show the acceptability of a syllogistic consequence is the Method of Reduction, stated in Theorem III-4 (TC 3.4.10–22):

> If the sentence "s; therefore q" is a consequence, then a sentence of the form "$p \wedge q$; therefore, r" is a consequence if and only if a sentence of the form "$p \wedge s$; therefore, r" is a consequence.

This theorem holds by Theorem I-4; we may illustrate the Method of Reduction to show the acceptability of *Datisi*, which has the form "All M is P, and some M is S; therefore, some S is P." We know from the preceding sections which conversions hold, and in particular that we immediately infer an I-form from an I-form by transposition of the terms (obversion); thus the sentence "Some M is S; therefore, some S is M" is a consequence. Hence we may replace the minor of *Datisi* with "Some S is M" and syllogize in *Darii* "All M is P, and some S is M; therefore, some S is P," and since this is an acceptable consequence so is *Datisi*.

8.4 Assertoric Syllogistic

Buridan investigates four main forms of assertoric syllogistic: the traditional syllogistic; the temporal syllogism, in which the terms or copula are temporally ampliated; the oblique syllogism, in which the sentences contain oblique terms; and the variation syllogism, in which the middle term is finite in one premiss and infinite in the other.

Buridan states which syllogisms in particular are acceptable in various theorems. His theorems are a model of rigor.[95] It will be useful to investigate certain general points systematically. First, let's review Buridan's procedure for the traditional syllogism—a distinctly non-traditional approach.

In TC 3.4.37 Buridan notes that the semantic principles for the syllogism discussed above specify the traditional syllogistic, and in 3.4.38 he approaches the problem in a combinatorial way. A conjugation is a pair of premisses; if we simply list them as permutations of quantity and quality, there are sixteen possibilities (numbered for further references):

(1) [AA] affirmative universal and affirmative universal
(2) [AI] affirmative universal and affirmative particular
(3) [AE] affirmative universal and negative universal
(4) [AO] affirmative universal and negative particular
(5) [EA] negative universal and affirmative universal
(6) [EI] negative universal and affirmative particular
(7) [EE] negative universal and negative universal

[95] For the sake of convenience, examples of each kind of syllogism Buridan finds acceptable are given in the notes to each theorem. Some difficulties are also noted.

(8) [EO] negative universal and negative particular
(9) [IA] affirmative particular and affirmative universal
(10) [II] affirmative particular and affirmative particular
(11) [IE] affirmative particular and negative universal
(12) [IO] affirmative particular and negative particular
(13) [OA] negative particular and affirmative universal
(14) [OI] negative particular and affirmative particular
(15) [OE] negative particular and negative universal
(16) [OO] negative particular and negative particular

The principles regarding distribution allow us to reject some conjugations immediately, no matter what the figure, since they have an undistributed middle. By Theorem III-2 we reject all made up of a pair of negatives, namely (7)–(8) and (15)–(16); since the middle is not distributed at all in the case of two particular affirmatives, we reject (10) as well (TC 3.4.39–40). Now we must move to particular figures.

In the first figure, (9) and (13)–(14) have an undistributed middle, and so they are to be rejected (3.4.41–42). Buridan accepts the remaining eight as "useful" (*utile*).[96] However, there are some surprises. (1) can conclude directly, in which case we have 1-AAA (*Barbara*), or indirectly, in which case we have I-AAI (*Baralipton*); (5) may also conclude directly or indirectly, and we have respectively 1-EAE (*Celarent*) and 1-EAE (*Celantes*); (2) may also conclude directly or indirectly, and we have respectively 1-AII (*Darii*) or 1-AII (*Dabitis*). Then we have (3), which can conclude only indirectly as 1-AEO (*Fapesmo*), and (11), which can conclude only indirectly as 1-IEO (*Frisesomorum*). But then the surprises start.

We need to understand what Buridan calls the "uncommon idiom for negatives." The common idioms for negation, of course, are where the negation precedes either the copula or the predicate. In the "uncommon idiom" the predicate-term precedes the negation, so we have a sentence such as "Some A B not is (*Quaedam A B non est*)." Now Buridan first mentions this "uncommon idiom" in TC 1.8.70, but he is nowhere very explicit about its logical behavior. From his remarks, though, we may say that the "uncommon idiom" for negatives is equivalent to a sentence in which the subject and predicate are both particularly quantified. That is, a sentence such as "Some A B not is" is to be read as though it were "Some A is not some B," where the quantification is branching: this sentence is true

[96] 'Useful' is a predicate of a figured conjugations, indicating that a conclusion can be added to produce a syllogistic consequence; a figured conjugation is in fact what Aristotle called a "syllogism" (TC 3.4.48). When a conclusion is so added the resulting syllogism is called a *mood*.

if and only if there is some A, say A_i, which is not the same as some B, say B_j. This holds no matter what the rest of the A and the B are like. The reason Buridan does not simply write this as I have suggested, as "Some A is not some B," is that this violates his rules for distribution and scope.

Now take (6): when it concludes directly, we have 1-EIO (*Ferio*). But it may also conclude indirectly, so we have 1-EI with a conclusion not in the common idiom for negatives, as "No M is P, and some S is P; therefore, some P S not is [*i. e.* some P is not some S]." This is not normally part of the traditional syllogistic, but Buridan is perfectly correct to say that it is an acceptable syllogism. Similarly, (4) and (12) conclude directly, though not in the common idiom for negatives, so we have respectively 1-AO "All M is P, and some S is not P; therefore, some S P not is [*i. e.* some S is not some P]," and 1-IO "Some M is P, and some S is not M; therefore, some S P not is [*i. e.* some S is not some P]."

In the second figure, we reject (1)–(2) and (9), because the middle is not distributed (TC 3.4.49); there are eight remaining useful moods. They are as follows (not that the moods which conclude both directly and indirectly are not given different traditional names): (5) concludes directly and indirectly, so we have 2-EAE (*Cesare*); (3) concludes directly and indirectly, so we have 2-AEE (*Camestres*); (6) concludes directly only as 2-EIO (*Festino*); (4) concludes directly only as 2-AOO *Baroco*). These are all of the acceptable syllogisms recognized by traditional syllogistic, but Buridan finds another four acceptable syllogisms. There is (11), which concludes only indirectly as 2-IEO; this is the same as *Festino* with the premisses transposed (making the conclusion indirect), and so Buridan names this "*Robaco*." Buridan also finds (12) acceptable, as 2-IO with the conclusion not in the common idiom for negatives, and therefore having the form "Some P is M, and some S is not M; therefore, some S P not is [*i. e.* some S is not some P]." Finally, (14) is acceptable, as 2-OI with the conclusion not in the common idiom for negatives, and therefore having the form "Some P is not M, and some S is M; therefore, some S P not is [*i. e.* some S is not some P]."

The third figure requires us to reject (12) and (14), since the middle is not distributed (TC 3.4.51); the remaining nine moods are acceptable, and they are all in the common idiom for negatives. The traditionally accepted moods are as follows: (13) concludes indirectly as 3-OAO (*Bocardo*); (1) concludes both directly and indirectly as 3-AAI (*Darapti*, no separate names); (9) concludes both directly and indirectly as 3-IAI (*Disamis*, no separate names); (2) concludes both directly and indirectly as 3-AII (*Datisi*, no separate names); (5) concludes directly as 3-EAO (*Felapton*); and (6) concludes directly as 3-EIO (*Ferison*). Buridan adds three acceptable syllogism, each of which has the premisses of a traditional mood transposed: (4) concludes indirectly as 3-AOO, which Buridan names "*Carbodo*," since the premisses are transposed from *Bocardo*; 93) concludes indirectly as 3-AEO, which Buridan names "*Lapfeton*," since the premisses are transposed from *Felapton*; and (11) concludes indirectly as 3-IEO, which Buridan names "*Rifeson*," since the premisses are transposed from *Ferison*.

Buridan is correct when he embraces his non-traditional moods: they are in fact acceptable. Thus even the most traditional part of assertoric syllogistic turns out to have surprises when Buridan turns his attention to it. Buridan then considers which moods are acceptable if the terms or copula is temporally ampliated: the temporal syllogism, discussed in TC 3.5.54–72. We may summarize his results briefly, since they are the basis for Buridan's tense-logic.

If the middle term is ampliative (we may regard an ampliative copula in the minor as an ampliative middle), then if no other term is ampliative the following moods alone are acceptable: (*i*) in the first figure, both *Celarent* and *Celantes*; (*ii*) in the second figure, both *Cesare* and *Camestres*, each concluding directly and indirectly; (*iii*) in the third figure, all of the standard moods are acceptable, namely *Bocardo*, *Ferison*, *Felapton*, *Carbodo*, *Rifeson*, *Lapfeton*, and *Darapti* and *Disamis* concluding directly and indirectly. On the other hand, if there is an ampliative major extreme (we may regard an ampliative copula in the major as an ampliative major extreme), then if no other term is ampliative the following moods alone are acceptable: (*i*) in the first figure, *Darii*, *Ferio*, *Baralipton*, *Celantes*, *Dabitis*, *Fapesmo*, *Frisesomorum*; (*ii*) in the second figure, *Festino*, *Baroco*, *Tifesno*, *Robaco*; in the third figure, all of the standard moods are acceptable, namely *Bocardo*, *Ferison*, *Felapton*, *Carbodo*, *Rifeson*, *Lapfeton*, and *Darapti* and *Disamis* concluding directly and indirectly.

These two accounts may be put together to obtain a complete account of temporal syllogistic (TC 3.4.70–72), in the obvious way: if the major extreme and middle term ampliate to the same time, the traditional syllogistic applies; if they ampliate to different times, then the acceptable syllogisms are those each account finds acceptable (namely the acceptable third-figure syllogisms). All cases reduce to these two.

The oblique syllogism is Buridan's next subject, which he treats in Theorem III-13 through theorem III-17 (TC 3.6.1–3.7.30). These theorems largely regulate the behavior of oblique term in distributive contexts, which is at the heart of syllogistic consequence. Buridan treats the oblique syllogism in a way unlike all other syllogistic forms he investigates: his procedure maintains the high level of rigor, but the theorems are rather more like recipes—they indicate how to build an acceptable syllogism from a sentence with certain characteristics. For example, if the oblique term appears at the very beginning of the sentence, then if we treat it as the subject and the rest of the sentence as the predicate all of the standard moods apply. We shall not consider the oblique syllogism in any detail here.

The last form of assertoric syllogism Buridan considers is the variation syllogism, that is, a syllogism in which the middle term is finite in one premiss and infinite in the other[97] (*i. e.* the term is "varied" from one to the other), in Theorem III-18 and Theorem III-19 (TC 3.7.31–45). We may summarize his claims: in every figure, there is an acceptable pair of variation syllogisms for the conjugations (1)–(2), (7)–(10), and (15)–(16); in the first figure, there is also an acceptable pair of variation syllogisms for the conjugations (3)–(4), respectively 1-AE and 1-AO, and for the conjugations (11)–(12), respectively 1-IE and 1-IO; there are no additional variation syllogisms in the second figure; in the third figure, there is an acceptable pair of variation syllogisms for the conjugations (3)–(6), respectively 3-AE, 3-AO, 3-EA, 3-EI, and the conjugations (11)–(14), respectively 3-IE, 3-IO, 3-OA, 3-OI. These are the only acceptable variation syllogisms.[98]

8.5 Composite Modal Syllogistic

Buridan investigates three forms of modal syllogistic: syllogisms whose premisses are purely composite modals; syllogisms whose premisses are purely divided modals; and syllogism whose premisses and conclusions are mixed—either a mix of composite and divided modals, or of assertoric and modal sentences.

Composite modals are the easiest case. If we recall from above that in a composite modal the mode is subject and the dictum predicate, or conversely, and that Buridan allows such sentences to be quantified, then composite modal logic is simply a branch of the standard assertoric syllogistic. For example, "No possible [sentence] is that snow is white, and some utterance is that snow is white; therefore, some utterance is not a possible [sentence]" is a straightforward syllogism in *Festino*.

But there is a more interesting aspect to composite modal syllogistic, namely if we consider what we may call "syllogisms with respect to the

[97] Such syllogisms have four terms rather than three, but Buridan's extended definitions of syllogistic extremes and middle permit this case.

[98] Note that some of these variation syllogisms have only conclusions that are not in the common idiom for negatives, namely 1-AA, 1-IE, 1-IO, 1-OO, 2-II, 2-OO, 3-II, 3-IE, 3-10, 3-O1.

dictum," on analogy with Buridan's conversions with respect to the dictum, discussed in TC 2.7.18–24. By Theorem IV-1 (TC 4.1.10–12) Buridan asserts we may prefix the indefinite mode 'necessary' or 'true' to the premisses and conclusion of a standard assertoric syllogism and get an acceptable syllogism, which is never the case for the modes 'possible' or 'false.' For example, if we have an assertoric syllogism in *Darapti* of the form "All M is P, and all M is S' therefore, some S is P" we may form the acceptable composite modal syllogism (with respect to the dictum) "It is necessary all M be P, and it is necessarily all M be S; therefore, it is necessary all S be P." Given such an acceptable syllogism, by equipollence we can convert the premiss or conclusion: since 'It is necessary that p' and 'It is impossible that not-p' are equipollent, we may equally express the above syllogism as (for example) "It is impossible some M be not P, and it is necessary all M be S; therefore, it is impossible no S be P" (Theorem IV-2 in TC 4.1.13–14). Obviously, the acceptability of the composite modal syllogism depends on the mode in question; Buridan makes a brief negative venture into epistemic-doxastic logic by pointing out that no such syllogism is acceptable if the mode is known,' 'opined,' 'doubted,' or the like (Theorem IV-3 in TC 4.1.15–16).

8.6 Divided Modal Syllogistic

Divided modal syllogistic is far more interesting, and Buridan devotes TC 4.3.1–74 to the subject. The first series of theorems, Theorem IV-4 though Theorem IV-6, concentrate on syllogisms whose premisses are *de necessario* or *de possibili*; the second series of theorems, Theorem IV-7 through Theorem IV-9, concentrate on which syllogisms are acceptable if the restrictive 'what is' locution is added to the subject of one of the premisses (which effectively restricts the referential domain of the subject to present or actual existent). The remaining theorems, Theorem IV-10 through Theorem IV-20, are mixing theorems, stating which mixtures of assertoric and modal premisses produce valid syllogisms.

The pure divided modal syllogistic, given in Theorems IV-4 through IV-6, is in many ways the most interesting; from the modern point of view we may see Buridan as settling matters about the iteration of modal operators. Take an example: the following divided modal syllogism is acceptable (corresponding to (2) with a direct conclusion):

All M is possibly P.
Some S is necessarily M.

Therefore: Some S is possibly P.

The way to validate the syllogistic conclusion is as follows. From the premisses we can clearly infer that some S is necessarily something which is possibly P; we may abbreviate this by writing "Some S is necessarily possibly P." Here we have the modes iterated. Buridan's claim about the syllogistic form, then, may be seen as a claim about equivalences among such iterations; in the given example, Buridan is holding that 'necessarily possibly' entails 'possibly'—as it certainly does. Since some S is necessarily possibly P, then, that means that some S is (simply) possibly P, which is the divided modal syllogism.

We may extend this point: if we take part of Buridan's divided modal syllogistic as specifying how iterated modal terms are to be treated—what single modal term corresponds to an iterated pair of modal terms—then which divided modal syllogism Buridan finds acceptable, and what iteration-reductions they embrace, will tell us (roughly) what system of modal logic Buridan is using.[99] It seems to be S5: 'necessarily necessarily' is replaced by 'necessarily'; 'possibly possibly' is replaced by 'possibly'; and 'possibly necessarily' is replaced[100] by 'necessarily.' The validity of this claim rests on our interpretation of certain modal syllogism Buridan accepts; since Buridan states only the general theorem, it is not always obvious what form he is endorsing. I have noted the cases requiring strong modal assumptions in the notes to the translation with an asterisk.

This characterization, of course, simply takes S5 and other modal systems to be specified syntactically. We may semantically characterize S5 in the well-known way of taking the accessibility-relation among a system of possible worlds to be an equivalence relation (reflexive, symmetric, and transitive). The quantifiers in divided modal premisses, then, would quantify across possible worlds, the modal terms state which worlds are accessible, and Buridan's replacement of iterated modalities reflects the fact that the accessibility-relation is an equivalence relation. On this construal, it is easy to see the point of introducing the 'what is' locution and the several mixing theorems: these are devices which index some of the claims made to the actual world.

Buridan simply specifies the modal character of the premisses in a figured conjugated, and asserts that the conclusion will have a certain modal character. In my notes to each theorem I have tried to list the instances of the theorem. However, while Buridan's approach to assertoric

[99] The correspondence is not exact, since the various modal systems are usually distinguished by two-way iterations, not the one-way iterations we have derived from modal syllogistic. Later mediæval logicians, such as Strode, will argue over two-way iterations explicitly, but Buridan does not.

[100] It is so replaced according to Theorem IV-6, applied to (9) in the third figure with a direct conclusion.

syllogistic is relatively unproblematic, there are certain problems with his modal syllogistic. First, it is sometimes difficult to give the modal character of the conclusion inexactly the general form Buridan describes. In order for his claims about the modal character of the conclusion to hold in each case, it is sometimes necessary to use a tortuous combination of negations, where the conclusion is much more readily expressed by using the equipollent modal term. Again, sometimes even this device does not work; in the case of Theorem IV-13(b) for mixed premisses in the third figure for (5) and (6), the conclusions derivable cannot be fit into Buridan's specifications.

But there is amore serious problem with Theorem IV-5, for modal premisses in the second figure in (3) and (6): no conclusion at all seems to be entailed by the premisses. The troublesome pair is as follows: (*i*) "All *P* is possibly *M*, and no *S* is necessarily *M*"; (*ii*) "No *P* is necessarily *M*, and some *S* is possibly not *M*." Try as I may I cannot find any conclusion which follows acceptably from (*i*) or (*ii*). Perhaps my interpretation of his strictures is incorrect; perhaps he has simply made an error. If the latter, then the wonder is that there are not more errors, given the abstract level at which theorems are stated and proved. For all that, Buridan's modal syllogistic is an astonishingly rigorous and precise construction, worthy of our admiration. It shows the mediæval logical mind at its best.