© Peter King, *The Philosophical Review* 96 (1987), 146–149

REVIEW

William Heytesbury: On Maxima and Minima. Chapter 5 of "Rules for Solving Sophismata," with an anonymous fourteenth-century discussion. Translated, with an Introduction and Study, by JOHN LONGEWAY. Synthese Historical Library, Vol. 26. D. Reidel: Dordrecht, The Netherlands, 1984. Pp. x, 201, index of sophisms, index of names, bibliography. $29.50

William Heytesbury was one of the guiding lights of the fourteenth-century Merton School of Oxford Calculators, a group of philosophers who brought sophisticated mathematical techniques to bear on physics. The fifth chapter of his *Regulae solvendi sophismata* deals with maxima and minima, that is, with assigning limits to the active and passive powers of physical agents. The investigation of such problems is interesting for several historical and philosophical reasons. First, it contributed directly to the notion of intensive qualitative variation, a key move in the development of modern physics. Second, it involves issues central to mathematics and the philosophy of mathematics: the existence of limits—or, more generally, partitions of linear continua; measurement; the rates of change of functions; and the nature of continuity. Finally, such investigations were conducted using the resources of later mediæval logic and semantics, a fact we have only lately begun to realize and appreciate. Philosophers interested in physics, in logic, in mathematics, as well as those interested in mediæval philosophy, should be interested in this book.

The sort of puzzle Heytesbury takes up can be illustrated through a simple example (p. 2 and p. 79). If Socrates has only finite strength, then his power is limited, and it is natural to identify the limit with the greatest weight Socrates can in fact lift. But Socrates cannot lift a weight with a power of resistance equal to his capacity (since he must exceed the resistance to lift such a weight), and by the same token he can lift any lesser weight. From this is follows that there is no greatest weight Socrates can lift: for any given weight, either Socrates cannot lift it, or he can lift a greater weight, namely one between the given weight and the weight with a power of resistance equal to his capacity. Yet how can Socrates's strength be finite and yet lack a limit? Through the examination of such puzzles (called 'sophisms'), Heytesbury proposes a classification of the kinds of capacities, rules for when a limit may be assigned to a capacity, and whether an assigned limit is within the scope of a capacity or beyond it ('internal' or 'external' limits).

The first section of Longeway's book is given over to a translation of Chapter 5 of Heytesbury's *Regulae solvendi sophismata*, based primarily on the Locatellus 1494 incunabulum corrected against a few manuscripts. Given that there is a critical edition of the full *Regulae* in progress, it might have made more sense to wait for its appearance before offering a translation, especially since Longeway himself is preparing the edition of Chapter 5 (p. ix). As it stands, his translation will have to be corrected once the critical edition is available. (He asserts that only minor changes will be required.) Yet having any translation at all, even though not based on a critical edition, is a welcome contribution to the growing library of mediæval philosophy available in English. The translation, with respect to the incunabulum, seems quite reliable. It is not easily read; the fault is not Longeway's, or even Heytesbury's, but the difficulty of the material and the abstract level at which the discussion takes place. Longeway's manuscript deviations are generally quite sensible, and only in a few minor places would I quarrel with his judgement—for example, in §6.8 he follows the manuscripts with *talis* when the *aliqua* of Locatellus would entail less generality, in keeping with the rest of Heytesbury's reply (p. 31).

The second, and most extensive, section of the book is given over to an edition and translation of an anonymous treatise, written sometime in the latter half of the fourteenth century, which discusses Heytesbury at some length. It explains his rules more fully, offers different analsyses of assigning limits, and treats several new sophisms. Longeway's edition is based on the two known manuscripts of the work (there are no incunabula). There are only a few passing references to this treatise in the secondary literature, so Longeway's edition and translation are a genuine contribution to our understanding of the period.

The last section is devoted to Longeway's study of the theory presented in both treatises. It is thorough and careful, discussing many matters in detail, and will help guide the novice through the twists and turns in the material. I cannot discuss all of the issues he raises, but I would like to briefly address one of his central themes: that Heytesbury, and others of the period, should be viewed as advancing theories in what we now call elementary point-set topology (p. 137), even coming close to Dedekind's notion of a 'cut' defining the real number field (pp. 169–161). This seems to me to be a mathematically naïve way to approach such mediæval treatises, even if we accept the anachronism involved in talking of sets. Grant that we start with a dense set on which a total ordering is defined (with no minimal or maximal element), there are two different routes to the study of continuity. Longeway endorses the route which accepts a second-order continuity axiom stating that every interval has a limit. At least, his explanations of Heytesbury's discussion of the kinds of capacities in question seem to presuppose such an axiom. But there is another route to the study of continuity. Any number of primitive relations R might be introduced such that, if we take t as our index set (the totally-ordered dense set we began with) and x as the quality which is indexed (such as heat, as Heytesbury often does), then we might introduce the first-order axiom:

$$(\exists R)(\forall t)(R(x;t) \text{ defines an interval} \Rightarrow \text{ there is a limit})$$

The two routes are not equivalent. The latter neither entails nor is entailed by archimedean principles, that is, that definable processes terminate at a final stage, whereas the former route does entail such principles. The mathematical formalization of the second route seems much closer to the mediævals' actual practise: a sophism will pose a case in which a physical quantity is said to vary with regard to an index set, such as heat varying over time, and then the exact relation between the physical and the index variables is examined to see whether it defines the kind of limit which exists. This route has the further advantage of clearly showing the intersection of physical theory, logic, and mathematics characteristic of the Oxford School. But little of the concrete details of Longeway's study will be affected by this high-level difference of opinion, and much of what he has to say is quite valuable.

A few minor drawbacks must be noted. There are several typographical errors that should have been caught in proofreading, such as 'studyied' for 'studied' (p. 137) and 'born' for 'borne' (p. 159). More seriously, there are several mistakes which affect the sense of his claims. For example, on p. 148 an intrinsic lower limit is defined as the "the minimum capacity upon which the active capacity *cannot* act," when it should clearly be the minimum capacity upon which the active capacity *can* act; on p. 150 the intrinsic upper limit of an active capacity is said to be illustrated in Figure 12 (B2), which instead illustrates an extrinsic upper limit—the correct reference is to Figure 12 (B1). In most cases the intended sense can be discovered.

This book is a solid contribution in a neglected field. But I must close bu registering a serious complaint against Reidel. Longeway's book is *not typeset*. It seems to be a serif typewriter font (probably Pica), photo-reduced on the page. The test is not justified; the footnote numbers, and there are footnotes galore, run into the line typed above; it is practically impossible to read for any length of time without eyestrain. All of this would be tolerable were the price commensurate with the product. After all, advanced texts in mathematics often appear in this format. But Reidel's hardcover edition with its current price tag is outrageous.