Solving Constrained OSNR Nash Game in WDM
Optical Networks with a Fictitious Player

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Abstract—Non-cooperative game theory is a powerful modeling
tool for resource allocation problems in modern communication
networks. However, practical concerns of capacity constraints
and allocation efficiency have been a challenge for network
engineers. In this paper, we base our results in the context
of link-level power control of optical networks and propose a
special form of games with an additional player to overcome
these difficulties. We introduce a novel framework with a fictitious
player (GFP) to extend the current OSNR Nash game framework
with capacity constraints. We characterize a more analytically
tractable solution in comparison to other approaches and propose
a first-order iterative algorithm to find the equilibrium.

I. INTRODUCTION

Recent technological advances have enabled a new gen-
eration of Optical Wavelength-Division Multiplexed (WDM)
communication networks. Devices such as Optical Add/Drop
MUXes (OADM), optical cross connects (OXC) and dynamic
gain equalizer (DGE) have provided essential building blocks
for smart optical networks [1]. With advent of these new
technologies, current networks are evolving towards dynamic
networks, able to respond to changes in traffic and require-
ments. A static network management mechanism can no
longer service such networks. Therefore, intelligent network
management and control systems need to be part of future
network design. Complex in their own structure, networks
need control on different levels. The first level is an optical
device level control, where smart feedback algorithms are
used to reduce noise and stabilize the device. Examples have
been seen in [2] and [3] where control principles are applied
to study EDFA and SOA, respectively. The next level of
management is on the link level, where we need to optimize
the quality of transmission and reduce the interference and
noise in transmission. Optimization-based models have been
seen in the case of wireless networks in [4], [5]. However, the
unique physical structure of optical networks imposes different
challenges on modeling and solution concepts.

The third level is the network level, where problems of
interests are optimal routing and congestion control. These
problems are on a higher level and they have been well studied
in a general network setting such as in [6]. The last but
not least is the system level control. This level of research
captures optical network as a dynamical system as seen
in [7]. Interesting problems are usually on the robustness and
stability of large scale networks.

Our focus here is on the link level. Channel optical signal-
to-noise ratio (OSNR) is an important performance factor at
this level as it directly relates to the bit error rate (BER) in
the transmission [8]. In recent years, research work on OSNR-
based optimization is making an effort to derive iterative de-
centralized OSNR optimization algorithms in optical networks.
Two dominant methods are commonly seen in literature.
One is the centralized optimization as in [9], [10] and the
other is non-cooperative game theory as in [11], [12]. The
centralized approach embeds OSNR targets in constraints and
indirectly minimizes the total power consumption in optical
networks. It is relatively easy to find a closed form solution
with this approach, however, its indirect minimization of total
power consumption doesn’t fully make use of the network
resource for communication purposes. On the other hand, the
non-cooperative game approach naturally deals with OSNR
optimization in a decentralized and direct manner. However,
it is a well-known fact that the resulting Nash equilibrium
may not be Pareto efficient [13], [14]. In addition, under
the OSNR game framework, it has been a challenge to find
an analytical solution for a game with capacity constraints.
Research efforts have been made to solve this problem by
integrating constraints into utility functions [15], [16]. And,
in particular, work has been done in [17], [18] to deal with
such constraints based on classical Lagrangian duality theory.
However, complexity of the solution grows in an undesirable
way and it is exceedingly difficult to give an analytical solution
for OSNR Nash game.

In this paper, we propose a different approach to deal with
constraints in OSNR Nash games. We formulate a Nash game
with a fictitious player to give a closed form solution to the
constrained OSNR Nash game. We may also use the role
of fictitious player to achieve an efficient Nash equilibrium
under certain conditions. The fictitious player, in reality, can
be implemented via a service channel or a transmission channel
which only needs a target OSNR.

This paper is organized as the following. In section 2, we
review a network OSNR model and give a brief introduction
to unconstrained non-cooperative game approach. In section 3,
OSNR is defined as the ratio of the channel output power (at Rx), and the vector of all channels’ input powers. Let $u_i$ be the $i$th channel input optical power (at Tx), and $n_i$ the optical noise power in the $i$th channel bandwidth at Rx. The $i$th channel optical OSNR is defined as $OSNR_i = \frac{u_i}{n_i}$.

Under A1 and A2, the ASE noise accumulation will be the dominant impairment in the model. The OSNR for the $i$th channel is given as

$$OSNR_i = \frac{u_i}{n_{o,i} + \sum_{j \in N_i} \Gamma_{i,j} u_j},$$

where $\Gamma$ is the full $n \times n$ system matrix which characterizes the coupling between channels. $n_{o,i}$ denotes the $i$th channel noise power at the transmitter. System matrix $\Gamma$ encapsulates the basic physics present in optical fiber transmission and implements an abstraction from a network to an input-output system. This approach has been used in [12] for the wireless case to model CDMA uplink communication. Different from the system matrix used in wireless case, the matrix $\Gamma$ given in (2) is commonly asymmetric and is more complicatedly dependent on parameters such as spontaneous emission noise, wavelength-dependent gain, and the path channels take.

$$\Gamma_{i,j} = \sum_{i \in R,s} \sum_{k=1}^{K_s} \frac{G_{l,i}^k}{G_{l,s}^k} \left( \prod_{q=1}^{l-1} \frac{T_{q,i}}{T_{q,j}} \right) \frac{ASE_{l,k,i}}{P_{0,l}}, \forall j \in N_i.$$  

where $G_{l,k,i}$ is the wavelength dependent gain at $k$th span in $l$th link for channel $i$; $T_{l,i} = \prod_{q=1}^{l-1} T_{q,i} G_{l,i}$ with $L_{l,k}$ being the wavelength independent loss at $k$th span in $l$th link; $ASE_{l,k,i}$ is the wavelength dependent spontaneous emission noise accumulated across cascaded amplifiers; $P_{0,l}$ is the power output at each span.

### B. Non-cooperative Game Approach

Let’s review the basic game-theoretical model for power control in optical networks without constraints. Consider a game defined by a triplet $(N, (A_i), (J_i))$. $N$ is the index set of players or channels; $A_i$ is the strategy set $\{u_i | u_i \in [u_{i,min}, u_{i,max}]\}$; and, $J_i$ is the cost function, chosen such that minimizing the cost is related to maximizing OSNR level. In [11], $J_i$ is defined as

$$J_i(u_i, u_{-i}) = \alpha_i u_i - \beta_i \ln \left( 1 + a_i \frac{u_i}{X_{-i}} \right), \forall i \in N$$  

where $\alpha_i, \beta_i$ are channel specific parameters, that quantify the willingness to pay the price and the desire to maximize its OSNR, respectively. $a_i$ is a channel specific parameter, $X_{-i}$ is defined as $X_{-i} = \sum_{j \neq i} \Gamma_{i,j} u_j + n_{o,i}$. This specific choice
of utility function is non-separable, nonlinear and coupled. However, $J_i$ is strictly convex in $u_i$ and takes a specially designed form such that its first-order derivative takes a linear form with respect to $u_i$, i.e., is in the class of linear games defined in section 2.

The solution from the game approach is usually characterized by Nash equilibrium (NE). Provided that $\sum_{j \neq i} \Gamma_{i,j} \leq a_i$, the resulting NE solution is given in a closed form by

$$\Gamma u^* = \tilde{b}, \tag{4}$$

where $\Gamma_{i,j} = a_i$, for $j = i$; $\Gamma_{i,j} = \Gamma_{j,i}$, for $j \neq i$ and $\tilde{b}_i = \frac{a_i \tilde{b}_i}{a_i} - n_{0,i}$.

Similar to the wireless case [12], we are able to construct iterative algorithms to achieve the Nash equilibrium. A simple deterministic first order parallel update algorithm can be found by $u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{X_i(a_i)}{a_i} (\Gamma_{i,i}^{-1}(n) - \Gamma_{i,i}) u_i(n)$. \(\tag{5}\)

As proved in [11], the algorithm (5) converges to Nash equilibrium $u^*$ provided that $\frac{1}{\alpha_i} \sum_{j \neq i} \Gamma_{i,j} \leq 1, \forall i$.

III. Game with a Fictitious Player (GFP)

In optical networks, a saturation power level exists in each link of channel paths [15]. A launched power has to be below or equal to this threshold so that the nonlinear effects in the span following each amplifier are kept minimum [19]. We can easily interpret this effect as a capacity constraint on an optical link in the network. In this section, we tackle the game described in section 2 with such constraint by considering a non-cooperative game with an additional fictitious player, labeled $F$. The fictitious player can be regarded as an additional player implemented via a channel that doesn't participate in the game for its need for quality of transmission. An example is the service channel in optical networks. It only requires certain amount of power to transmit network information and doesn't aim for OSNR optimization. It rather behaves as a player to regulate the performance of the network. We will use this interpretation to solve an $(N+1)$-person non-cooperative game with constraint of

$$\sum_{i \in N \cup \{F\}} u_i \leq C. \tag{6}$$

Let the payoff function of user $i \in N$ given by Equation (3) and we choose the payoff function of user $F$ to be

$$J_F(u_F, u_{-F}) = \alpha_F u_F - \beta_F \left( C - \sum_{j \neq F} u_j \right) \ln a_F u_F. \tag{7}$$

Function $J_F$ is convex when $\sum_{j \neq F} u_j \leq C$. Since the fictitious player may not ask for an optimal quality of transmission, we do not design function (7) directly related to OSNR, but in terms of power and capacity constraint instead. It is composed of two parts with the first term describing the cost on power usage $u_F$ and the second term the capacity-dependent utility. With the assumption of convexity, the best response function for $J_F$ is given by an implicit expression in (8).

$$\omega_F u_F + \sum_{j \neq F} u_j = C. \tag{8}$$

where $\omega_F = \frac{\alpha_F}{\beta_F}$. We let $u_i \in [u_{i,\text{min}}, u_{i,\text{max}}]$, where $u_{i,\text{min}} \in \mathbb{R}^+$ and $u_{i,\text{max}} \in \mathbb{R}^+$ can be chosen to be sufficiently small and large so that they will not be the solution to the minimization of the cost function $J_i, i \in N \cup \{F\}$.

**Proposition 3.1:** If $\omega_F \geq 1$, then any solution $u$ that satisfies (8) is within the feasible set described by the constraint (6).

**Proof:** Observe from (8), we can conclude that for any $u \in \{u \mid \sum_{i \in N} u_i + \omega_F u_F \}$, the following holds.

$$C = \sum_{i \in N} u_i + \omega_F u_F \geq \sum_{i \in N} u_i + u_F, \forall \omega_F \geq 1. \tag{9}$$

Therefore, $u \in \{u \mid \sum_{i \in N \cup \{F\}} u_i \leq C.\}$

Following the proof, we also can observe that when $\omega_F = 1$, the best response function of user $F$ will impose an equality capacity constraint of $\sum_{i \in N \cup \{F\}} u_i = C$ and the solution will be efficiently achieved on the boundary of the feasible set. However, increasing $\omega_F$ to be strictly greater than 1 will result in less efficient solution.

The construction of the best response function (8) can be seen as a slack constraint from (10)

$$\omega_F u_F + u_F + \sum_{j \neq F} u_j = C. \tag{10}$$

where $\omega_F = \omega_F - 1$, and $\omega_F u_F > 0$ as a slack variable. Similar to the interpretation of Lagrangian multiplier in classical Lagrangian theory, $\omega_F u_F$ can be seen as an analog of Lagrange multiplier and tells how efficient the system is with respect to the constraint.

**A. Characterization of Nash Equilibrium**

We use the same approach in [11] to characterize the equilibrium of the game. By the definition of Nash equilibrium in [20], a Nash equilibrium $u^F$ with a fictitious player is a point which jointly satisfies the best response functions as follows.

$$a_i u_i^F + X_F^i = \frac{a_i \beta_i}{\alpha_i}, \text{ for } i \in N. \tag{11}$$

$$\omega_F u_F^F + \sum_{j \neq F} u_j = C, \text{ for } i = F. \tag{12}$$

Expressed in matrix form, they become

$$\Gamma u^F = \tilde{B}. \tag{13}$$

where $u^F = [u_1^F, \ldots, u_N^F, u_F^F]^T$,

$$\Gamma = \begin{bmatrix} a_1 & \Gamma_{12} & \cdots & \Gamma_{1N} & \Gamma_{1F} \\ \Gamma_{21} & a_2 & \cdots & \Gamma_{2N} & \Gamma_{2F} \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ \Gamma_{N1} & \Gamma_{N2} & \cdots & a_N & \Gamma_{NF} \\ 1 & 1 & \cdots & 1 & \alpha_F / \beta_F \end{bmatrix}, \tilde{B} = \begin{bmatrix} \frac{a_1 \beta_1}{\alpha_1} - n_{0,1} \\ \vdots \\ \frac{a_N \beta_N}{\alpha_N} - n_{0,N} \\ C \end{bmatrix}.$$
A necessary and sufficient condition for Nash equilibrium to exist is to require
\[ \mathbb{B} \in \mathbb{R}(\Gamma). \]
with $\mathbb{R}(\cdot)$ denotes the range space. However, to ensure the existence and uniqueness of Nash equilibrium, we may need to assume some special features of the game, for example, diagonal dominance of the matrix $\mathbb{B}$ and the convexity of the utility functions $J_i$. Theorem 3.2 summarizes these conditions and gives a sufficient condition on the uniqueness and existence of the Nash equilibrium to GFP.

**Theorem 3.2:** Let $\rho(\cdot)$ denote the spectral radius of a matrix. If $\frac{\max_i b_i \sqrt{N+1}}{\rho(\Gamma^T \Gamma)} \leq C$ and $a_i > \sum_{j \neq i} \Gamma_{ij}$, $\omega_F > N$, then the game with a fictitious player (GFP) will have a unique Nash equilibrium.

**Proof:** First of all, we need to show that the utility functions are convex and there exists a minimizing $u^F$. It has been proved in [11] that functions (3) is convex in $u_i$. Due to the fact that $u_i$ that guarantees the pricing and utility functions are convex. The functions are convex and there exists a minimizing $u^F$. Theorem 3.2 summarizes these conditions and gives a sufficient condition on the uniqueness and existence of the Nash equilibrium to GFP.

Due to the convexity of $J_i$, the convexity of $\Gamma_i$ will hold.

Lastly, we prove that there exists a unique solution under the assumption of diagonal dominance of matrix $\mathbb{B}$. With $a_i > \sum_{j \neq i} \Gamma_{ij}$, $\omega_F > N$, matrix $\mathbb{B}$ becomes diagonal dominant. From Gershgorin’s Theorem [22], it follows that $\mathbb{B}$ is nonsingular and there exists a unique solution to linear system (13).

**Remark 3.1:** If we further assume that $C > \frac{a_i d_i}{\alpha_i} - n_0, \forall i$, then it will reduce the condition to $\rho(\Gamma^T \Gamma) \geq N + 1$. This result allows to the maximum number of channels to be admitted in the network for a fixed capacity.

Though we notice that some portion of the power is allocated to the service channel or the fictitious player, we need to accept that this amount of power is a necessary allocation for the network to operate. Furthermore, this power consumption can be adjusted through parameter $\omega_F$. On the other hand, we should also note that the strong assumption of diagonal dominance, in particular, $\omega_F > N > 1$ may not lead to an efficient solution as has been indicated by Inequality (9). However, letting $\omega_F = 1$ may still give rise to a unique and efficient solution, since Theorem 3.2 only describes a sufficient condition.

**B. Iterative Algorithm**

Following (5), the algorithm for the game with a fictitious player is given by a synchronous algorithm given in (15). A step of update includes two sub-steps: an initial update on $u_i(n+1), i \in N$ and a update sub-step on $u_F(n+1)$.

\[
\begin{align*}
& \left\{ u_i(n+1) = \frac{\partial_i}{\alpha_i} - \frac{1}{\omega_F} \left( \sum_{j \neq i} \Gamma_{ij} u_j(n) \right), \quad \forall i \in N; \\
& u_F(n+1) = \frac{1}{\omega_F} \left( C - \sum_{j \neq F} u_j(n) \right), \quad \text{for } F.
\end{align*}
\]

(15)

**Proposition 3.3:** The algorithm described by (15) converges to $u^F$ provided that $a_i > \sum_{j \neq i} \Gamma_{ij}$ and $\omega_F > N$.

**Proof:** Define $e_i(n) = u_i(n) - u_i^F$ and $e_i(n+1) = \frac{1}{\alpha_i} \sum_{j \neq i} \Gamma_{ij} e_j(n)$ will follow. Letting $\epsilon(n) = [e_1(n), \ldots, e_N(n), e_F(n)]^T$ and taking the infinity norm on $\epsilon(n+1)$, we can arrive at

\[ \| \epsilon(n+1) \|_\infty = \max_{i \in N} |e_i(n+1)| \]

Under the assumption of strictly diagonal dominance, i.e., $\frac{1}{\alpha_i} \sum_{j \neq i} \Gamma_{ij} < 1, \forall i$, the contraption mapping theorem will show $\epsilon(n) \rightarrow 0$ and hence, $u_i(n) \rightarrow u_i^F$.

For user $F$’s algorithm, in a similar way, we define $e_F(n) = u_F(n) - u_F^F$ and $e_F(n+1) = \frac{1}{\omega_F} \sum_{i \neq F} (u_i(n) - u_i^F)$.

\[ |e_F(n+1)| = \frac{1}{\omega_F} \left| \sum_{i \neq F} (u_i(n) - u_i^F(n)) \right| \]

\[ \leq \frac{N}{\omega_F} \max_{i \neq F} |u_i(n) - u_i^F(n)| \]

\[ = \frac{N}{\omega_F} \| \epsilon(n) \|_\infty. \]

Using inequality (16),

\[ |e_F(n+1)| \leq \frac{N}{\omega_F} \max_{i \neq F} \left( \frac{1}{\alpha_i} \sum_{j \neq i} \Gamma_{ij} \right) \| \epsilon(n) \|_\infty. \]

Since $\| \epsilon(n) \|_\infty \rightarrow 0$, then $|e_F(n+1)| \rightarrow 0$; and thus, $u_F(n)$ will converge to $u_F^F$.

Parameters $\alpha_i, i \in N$, and $\omega_F$, as shown in the proof, determines the rate of convergence. On average, increasing
where $a_i, i \in N$ results in a faster convergence for $u_i, i \in N \cup \{F\}$. And increasing $\omega_F$ will lead to a boost in convergence speed of user $F$’s algorithm.

We also can observe a similarity with the algorithm derived based on duality theory in [17], where $u_F$ is more closely related to the dual variable $\mu$. The difference between the two is that we used a fictitious player in the game in the position of the dual variable and the player has it own rule of interactions with other players.

We need to point out that this similarity is not surprising to us because we can see the way a constraint is associated with an additional player in a constrained Nash game in analogy to the way constraints are associated with lagrangian multipliers in classical optimization theory. The user $F$’s algorithm turns can be seen analogously as the algorithm for the Lagrangian multiplier.

**C. GFP with OSNR constraint**

In the above game with fictitious player, we have considered a utility function for user $F$ without OSNR requirement. However, being an internode communications channel for management and user data, the optical service channel may require a certain OSNR level to support intelligent optical network communication [1], [23]. In this regard, we may require a certain OSNR level to support intelligent optical network communication [1], [23]. In this regard, we may impose a target OSNR as a constraint for the user $F$ to guarantee its minimum requirement of quality of transmission. Let $\gamma_F$ be the target OSNR for $F$ and require $OSNR_F \geq \gamma_F$, that is, by (1),

$$OSNR_F \geq \gamma_F.$$  \hspace{1cm} (19)

that is,

$$\sum_{j \in N} \frac{u_F}{F} \geq \gamma_F,$$

$$u_F - \gamma_F \sum_{j \in N} \frac{F}{j} u_j \geq \gamma_F n_0/F \geq 0,$$ \hspace{1cm} (21)

where $q_F = [-\gamma_F F_{F1}, -\gamma_F F_{F2}, \ldots, 1]^T$, and $u_F = [u_1, u_2, \ldots, u_N, u_F]$. We show the OSNR constraint of user $F$, together with the capacity constraint, will give a nonempty convex feasible set when $C \geq \gamma_F n_0/F$ or $\gamma_F \leq C/n_0$. It is illustrated in Figure 19 and stated in Proposition 3.4.

**Proposition 3.4:** The feasible set $X_F = F_1 \cap F_2$ is nonempty if and only if $C \geq \gamma_F n_0/F$, i.e., $\gamma_F \leq C/n_0/F$, where $F_1 = \{u | \sum_{j \in N \cup \{F\}} u_i \leq C\}$ and $F_2 = \{u_F | q_F u_F \geq \gamma_F n_0/F\}$.

**Proof:** Let’s prove the necessity first, i.e., if the feasible set $X_F$ is nonempty, then $C \geq \gamma_F n_0/F$. The proof starts with $q_F u_F \geq \gamma_F n_0/F$ and substitute the inequality $u_F \leq C - u_1 - u_2 - \cdots - u_N$ into $u_F$. We obtain the following inequality if both constraints are satisfied.

$$-(\gamma_F F_{F1} + 1) u_1 - \cdots - (\gamma_F F_{FN} + 1) u_N + C \geq \gamma_F n_0/F.$$  \hspace{1cm} (21)

Since $\gamma_F, F_{F1}, u_i$ are nonnegative,

$$C \geq -(\gamma_F F_{F1} + 1) u_1 - \cdots - (\gamma_F F_{FN} + 1) u_N + C \geq \gamma_F n_0/F.$$  \hspace{1cm} (21)

Therefore, we have $C \geq \gamma_F n_0/F$.

In the following, we show the sufficiency, i.e., if $C \geq \gamma_F n_0/F$, then there always exist a point in $X_F$ that satisfies both constraints. Suppose there exists a point $u_F = [0, \ldots, u_F]$ that satisfies the constraint $q_F u_F \geq \gamma_F n_0/F$. This yields $u_F \geq \gamma_F n_0/F$. With the condition $C \geq \gamma_F n_0/F$, we can find at least one $u_F$ such that $C \geq u_F$ that satisfies the constraint $q_F u_F \geq \gamma_F n_0/F$. This shows that we can find points $u_F$ that satisfy the inequality $\sum_i u_i \leq C$. Therefore, there always exists feasible points in the feasible set and the feasible set is not empty.

**Proposition 3.5:** Suppose the non-cooperative game with a fictitious player has a unique solution $u_F^*$, then if target OSNR $\gamma_F$ is met, then $\gamma_F$ should satisfy the inequality:

$$\ln \gamma_F \leq \min \{\ln(C/n_0/F), \ln(1/\gamma) - \gamma (\mathbf{T}^{-1} \mathbf{B})\}.$$  \hspace{1cm} (21)

**Proof:** From Proposition 3.4, we require $\gamma_F < C/n_0/F$. Use the OSNR expression in (1) and we obtain

$$OSNR_F(u_F^*) = \frac{\sum_{j \in N} \frac{F}{j} u_j + n_0/F}{u_F}.$$  \hspace{1cm} (21)

where $\gamma = \gamma F_{F1}$. Inequality (23) comes from the arithmetic-geometric mean inequality [24]. With $\gamma F_{Fi} = 1$, $u_i, \gamma F_{Fi} > 0$, we have

$$\gamma F_{F1} u_1, \gamma F_{F2} u_2, \ldots, \gamma F_{FN} u_N \leq \gamma F_{F1} u_1 + \gamma F_{F2} u_2 + \cdots + \gamma F_{FN} u_N.$$  \hspace{1cm} (23)

Equality holds only when $u_1 = u_2 = \cdots = u_N$. Requiring $\gamma_F \leq OSNR_F(u_F^*)$, we can further determine an upper
bound on the target $\gamma_F$.

$$
\ln \gamma_F \leq \ln(1/\gamma) + \ln(u_1^{-1/\gamma}) + \cdots + \ln(u_N^{-1/\gamma}) + \ln u_F
$$

$$
= \ln(1/\gamma) - \Gamma'_F \ln(u_1) - \cdots - \Gamma'_F \ln(u_N) + \ln u_F
$$

$$
= \ln(1/\gamma) - \hat{\gamma}^T \ln(u)
$$

$$
(25)
$$

$$
\ln(1/\gamma) - \hat{\gamma}^T \ln(\Gamma^{-1} b)
$$

$$
(26)
$$

where $\hat{\gamma}^T = [\Gamma'_F, \cdots, \Gamma'_{FN}, -1]$ and $\Gamma'_F = \Gamma_{FJ} / \gamma$.

From the above result, by assuming the power allocated to user $F$ is negligible, we can further simplify the inequality into an estimate of $\gamma_F$ by

$$
\gamma_F \leq \min\{C_0/n_{0,F}, 1/\gamma\},
$$

because $\gamma^T \ln(u) > 0$ if $u_F$ is sufficiently small. Therefore, we have a rough estimate on the upper bound on $\gamma_F$, i.e., $\gamma_F < \frac{1}{\gamma}$. It also means that we can’t not make the transmission for user $u_F$ better than $1/\gamma$ or $C_0/n_{0,F}$ in terms of OSNR.

IV. DIRECTION OF FUTURE WORK

This paper outlines a way to deal with coupled constraints by including a fictitious player. In the context of optical networks, we have only considered the coupled capacity constraints. It can be further extended to include multiple linearly coupled constraints by including more fictitious players. As a result, analogous to classical Lagrangian method in which each constraint is associated with a Lagrangian multiplier, in non-cooperative game, we deal with each constraint by associating it with a fictitious player. It will be possible to crystallize this analogy into a theory for dealing with a general class of Nash games with coupled constraints.

V. CONCLUSION

In this paper, we study the constrained OSNR game in the context of optical networks. We addressed the issue of constrained optimization and its efficiency in non-cooperative games. We characterize the Nash equilibrium with a closed form solution by including a fictitious player with her target OSNR due to the linearity of the coupled constraint and the best response function. This unique approach allows us to derive an iterative algorithm similar to the duality approach in a much simpler way.

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