Model Order Reduction Technique for Steady-state Interconnect Simulation

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Outline
1. Circuit and Circuit Simulation
2. Transmission line Problem
3. Technique of Model Order Reduction
4. Results and Discussion
5. Future Work

Basic Circuits

Passive Circuit Element
- Resistor
- Capacitor
- Inductor

Active Circuit Element
- Diode
- Transistor

V-I Characteristics

- Resistor
  - Ohm's Law: \( I(t) = C \frac{dV(t)}{dt} \rightarrow I(s) = sCV(s) \)
- Capacitor
  - V-I Relation: \( V(t) = L \frac{dI(t)}{dt} \rightarrow V(s) = sLI(s) \)
- Inductor
  - V-I Relation: \( V(t) = I(t)R \rightarrow V(s) = I(s)R \)

KVL and KCL

- KVL: Algebraic sum of the currents in all the branches which converge in a common node is equal to zero
  \[ \sum I_{in} = \sum I_{out} \]
- KCL: Algebraic sum of the voltages between successive nodes in a closed path in the network is equal to zero
  \[ \sum V = \sum I \]

Example

\[
\begin{bmatrix}
 g_1 & 0 & v_1 \\
 0 & g_2 & v_2
\end{bmatrix}
+ s
\begin{bmatrix}
 C & -C & v_1 \\
 -C & C & v_2
\end{bmatrix}
= \begin{bmatrix}
 J \\
 0
\end{bmatrix}
\]

\[(G+sC) \times (s) = b\]

How to get those matrices by inspection?
Motivation for Transmission Line

- The assumption of circuit theory is that signal from node $i$ appears simultaneously at node $j$. This assumption can be justified if the wavelength of the signal is long enough, i.e. low frequency.

- For high frequency application, e.g. computer bus, and microwave circuits, we need to fix this problem by introducing:

  Transmission-line
Physical Transmission Line

- In physical world, transmission line is a microstrip on chip, chip interconnects, coaxial cable, etc.

Transmission Line: Distributed Circuit Model

Electrical

Distributed TL Line

Physical

Resistor: Ohmic losses

Solution to Telegraphy Equation

\[
\begin{align*}
\frac{\partial V(x,t)}{\partial t} - \frac{1}{R} \frac{\partial V(x,t)}{\partial x} &= -I(x,t)(R + sL) \\
\frac{\partial I(x,t)}{\partial x} - \frac{1}{L} \frac{\partial I(x,t)}{\partial t} &= V(x,t)(G + sC)
\end{align*}
\]

where

\[
D = \begin{bmatrix}
0 & -D \\
-G & 0
\end{bmatrix} \quad \text{and} \quad E = \begin{bmatrix}
0 & -L \\
-C & 0
\end{bmatrix}
\]

Solution:

\[
\begin{bmatrix}
V(x,t) \\
I(x,t)
\end{bmatrix} = e^{(D + sE)t} \begin{bmatrix}
V(0,t) \\
I(0,t)
\end{bmatrix}
\]

Transmission Line Model: Telegraphy Equation

From \( \Delta x \to \Delta t \)

\[
\begin{align*}
\frac{\partial V(x,t)}{\partial t} - \frac{1}{R} \frac{\partial V(x,t)}{\partial x} &= -I(x,t)(R + sL) \\
\frac{\partial I(x,t)}{\partial x} - \frac{1}{L} \frac{\partial I(x,t)}{\partial t} &= V(x,t)(G + sC)
\end{align*}
\]

Two Sets of Partial Differential Equations!

MNA of Transmission Line

- From distributed model to analytical TL governing equation.
- How to design a stamp for Transmission line Simulation?
- Partial Differential Equations?
- Going back to distributed model and we can use make use of existing stamps for known circuit elements

\[
(G + sC) X(s) = b
\]

Problem with Discretization

- Large matrix system due to number of blocks need to be big.
- Number of the building block is determined by the wavelength of the signal.
- How to save memory and CPU time?

Model Order Reduction
Model Order Reduction

\[ GX(s) + JCX(s) = b \]

\[ \hat{G}X(s) + J\hat{C}X(s) = \hat{b} \]

\[ X(s) = Q\hat{X} \]

Model Order Reduction

\[ (G + sC)X(s) = b \]

\[ \hat{C} = Q^T CQ \]
\[ \hat{G} = Q^T GQ \]
\[ \hat{b} = Q^T b \]
\[ \hat{X} = Q^T X \]

\[ (\hat{G} + s\hat{C})\hat{X}(s) = \hat{b} \]

Numerical Example

From Anu

\[ Y(s) = \frac{R}{1 + (G + sC)X} \]

Multiple Transmission Lines

\[ V_1(s) \]
\[ I(s) \]

Reference Conductor

\[ Y(s) = R'(G + sC)^{-1}R \]
Carbon Nanotube Model

- $L_w$: Kinetic energy per unit length
- $L_w$: Sum of the kinetic energies of left and right movies
- $L_w$: Typically, 100 aF\mu m
- $C_{es}$: Electrostatic capacitance
- $C_{es}$: Capacitance between wire and ground plane
- $C_{es}$: Typically, 56 aF\mu m
- $C_{sq}$: Quantum Capacitance
- $C_{sq}$: Associated with energy to add an extra electron
- $C_{sq}$: Typically, 100 aF\mu m

Reference

Major Papers:

2. R. Khazaka, “Projection Based Techniques for the Simulation of RF Circuits and High Speed Interconnects”, PhD Dissertation, Carleton University, Canada, 2002
4. R. Khazaka, “ECSE 596 Circuit Simulators”, Lecture Notes, McGill University, 2005