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Model Order Reduction Technique for Steady-state Interconnect Simulation

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Outline

1. Circuit and Circuit Simulation
2. Transmission line Problem
3. Technique of Model Order Reduction
4. Results and Discussion
5. Future Work

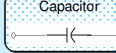
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Basic Circuits

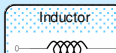
Passive Circuit Element



Resistor




Capacitor

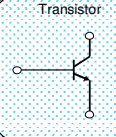


Inductor

Active Circuit Element



Diode



Transistor

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V-I Characteristics

- Resistor
 – Ohm's Law: $I(t) = C \frac{dV(t)}{dt} \rightarrow I(s) = sCV(s)$
- Capacitor:
 – V-I Relation: $V(t) = L \frac{dI(t)}{dt} \rightarrow V(s) = sLI(s)$
- Inductor:
 – V-I Relation: $V(t) = I(t)R \rightarrow V(s) = I(s)R$

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KVL and KCL

KVL

Algebraic sum of the currents in all the branches which converge in a common node is equal to zero

$$\sum I_{in} = \sum I_{out}$$

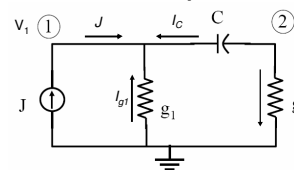
KCL

Algebraic sum of the voltages between successive nodes in a closed path in the network is equal to zero

$$\sum V = \sum IR$$

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Example



$$\begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + s \begin{bmatrix} C & -C \\ -C & C \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} J \\ 0 \end{bmatrix}$$

$(G+sC)X(s)=b$
 How to get those matrices by inspection?

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Resistor Stamp

Node i \xrightarrow{g} Node j

row i $\begin{bmatrix} g & -g \end{bmatrix}$

row j $\begin{bmatrix} -g & g \end{bmatrix}$

col i col j

Node i \xrightarrow{g} \parallel

row i $\begin{bmatrix} g \end{bmatrix}$

col i

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Capacitor Stamp

Node i \xrightarrow{C} Node j

row i $\begin{bmatrix} sC & -sC \end{bmatrix}$

row j $\begin{bmatrix} -sC & sC \end{bmatrix}$

col i col j

Node i \xrightarrow{C} \parallel

row i $\begin{bmatrix} sC \end{bmatrix}$

col i

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Inductor Stamp

Node i \xrightarrow{L} Node j

row i $\begin{bmatrix} +1 \end{bmatrix}$

row j $\begin{bmatrix} -1 \end{bmatrix}$

New row $\begin{bmatrix} +1 & -1 & -sL \end{bmatrix}$

col i col j New col

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Current Source Stamp

Node i \xrightarrow{J} Node j

row i $\begin{bmatrix} J \end{bmatrix}$

row j $\begin{bmatrix} -J \end{bmatrix}$

Node i \xrightarrow{J} \parallel \rightarrow Ground

row i $\begin{bmatrix} J \end{bmatrix}$

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Circuit Simulator

User interface
Schematic Capture
Extraction
Netlist input

Numerical Solvers

Output User interface

Network Formula
Generating systems of equations

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Motivation for Transmission Line

- The assumption of circuit theory is that signal from node i appears **simultaneously** at node j. This assumption can be justified if the wavelength of the signal is long enough, i.e. **low frequency**.
- For **high frequency** application, e.g. computer bus, and microwave circuits, we need to fix this problem by introducing:

Transmission-line

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Physical Transmission Line

- In physical world, transmission line is a microstrip on chip, chip interconnects, coaxial cable, etc.

Interconnect

Microstrip

Coaxial Cable

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Transmission Line: Distributed Circuit Model

Electrical

Distributed TR Line

Physical

Resistor: Ohmic losses

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Transmission Line Model: Telegraphy Equation

From $\Delta x \rightarrow \partial x$

$$\begin{cases} \frac{\partial V(x,s)}{\partial x} = -I(x,s)(R + sL) \\ \frac{\partial I(x,s)}{\partial x} = -V(x,s)(G + sC) \end{cases}$$

Two Sets of Partial Differential Equations!

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Solution to Telegraphy Equation

$$\begin{cases} \frac{\partial V(x,s)}{\partial x} = -I(x,s)(R + sL) \\ \frac{\partial I(x,s)}{\partial x} = -V(x,s)(G + sC) \end{cases}$$

$$\frac{\partial}{\partial x} \begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix} = (D + sE) \begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix}$$

where $D = \begin{bmatrix} 0 & -D \\ -G & 0 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & -L \\ -C & 0 \end{bmatrix}$

Solution: $\begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix} = e^{(D+sE)x} \begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix}$

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MNA of Transmission Line

- From distributed model to analytical TL governing equation.
- How to design a stamp for Transmission line Simulation?
- Partial Differential Equations?
- Going back to distributed model and we can make use of existing stamps for known circuit elements

'Finite' instead of 'infinite'

$(G+sC) X(s) = b$

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Problem with Discretization

- Large matrix system due to number of blocks need to be big.
- Number of the building block is determined by the wavelength of the signal.
- How to save memory and CPU time?

Model Order Reduction

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Model Order Reduction

$GX(s) + sCX(s) = b$
 Q^T ($q \times N$) G ($N \times N$) Q ($N \times q$) \rightarrow \hat{G} ($q \times q$)
 $\hat{G}\hat{X}(s) + s\hat{C}\hat{X}(s) = \hat{b}$

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Model Order Reduction

$(G + sC)X(s) = b$

$\hat{C} = Q^T C Q$
 $\hat{G} = Q^T G Q$
 $\hat{b} = Q^T b$
 $\hat{X} = Q^T X$

$(\hat{G} + s\hat{C})\hat{X}(s) = \hat{b}$

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Model Order Reduction

Large
 $GX(s) + sCX(s) = b$
High order
 $\dim(G, C) \gg \dim(\hat{G}, \hat{C})$
Small
 $\hat{G}\hat{X}(s) + s\hat{C}\hat{X}(s) = \hat{b}$
Reduced order
 $X(s) \equiv Q\hat{X}$

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Numerical Example

From Anu

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Multiple Transmission Lines

$V(z, s)$
 $I(z, s)$
 $I(0, s)$
 $V(0, s)$
 $V(d, s)$
 $I(d, s)$
 \dots
Reference Conductor

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Y-Parameter

$\begin{cases} (G + sC)x = Ru \\ I = R^T x \end{cases}$

$C \frac{\partial x}{\partial t} + Gx = Ru \quad I = Lx$

Congruent Transformation

$\bar{G} = Q^T G Q \quad x \rightarrow Q\bar{x}$
 $\bar{C} = Q^T C Q \quad \bar{B} = Q^T B$

$\bar{C} \frac{\partial \bar{x}}{\partial t} + \bar{G}\bar{x} = \bar{R}u \quad I = \bar{L}\bar{x}$

$Y(s) = R^T(G + sC)^{-1}R$

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Carbon Nanotube Model

L_K : Kinetic energy per unit length
 → Sum of the kinetic energies of left and right movies

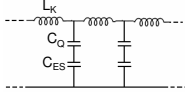
$$L_K = \frac{h}{2e^2 v_F}$$
 Typically, 16 nH/um

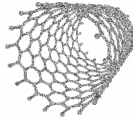
C_{ES} : Electrostatic capacitance
 → Capacitance between wire and ground plane

$$C_E = \frac{2\pi\epsilon}{\cosh^2(2h/d)} \approx \frac{2\pi\epsilon}{\ln(h/d)}$$
 Typically, 50 aF/um

C_Q : Quantum Capacitance
 → Associated with energy to add an extra electron

$$C_Q = \frac{2e}{hv_F}$$
 Typically, 100 aF/um





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Reference

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- [5] Burke, P.J., "An RF circuit model for carbon nanotubes", *IEEE Transactions on Nanotechnology*, Vol. 2, Iss. 1, March 2003, Pages:55 – 58

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