Model Order Reduction Technique for Steady-state Interconnect Simulation

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Outline

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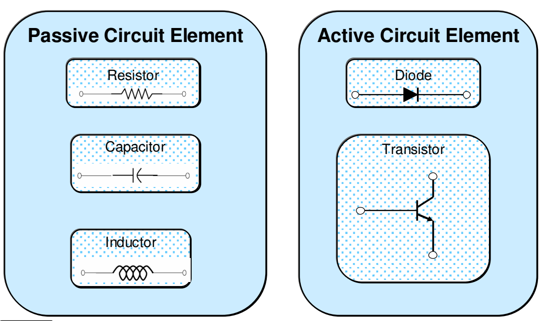
Basic Circuits



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V-I Characteristics

- Resistor
  - Ohm's Law: $I(t) = C\frac{dV(t)}{dt}$ ➡ $I(s) = sCV(s)$
- Capacitor:
  - V-I Relation: $V(t) = L\frac{dI(t)}{dt}$ ➡ $V(s) = sLI(s)$
- Inductor:
  - V-I Relation: $V(t) = I(t)R$ ➡ $V(s) = I(s)R$

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KVL and KCL

KVL

Algebraic sum of the currents in all the branches which converge in a common node is equal to zero

$\sum I_{in} = \sum I_{out}$

KCL

Algebraic sum of the voltages between successive nodes in a closed path in the network is equal to zero

$\sum V = \sum IR$

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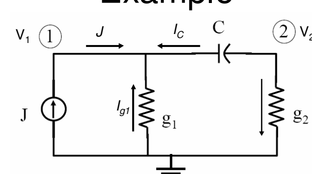
Example

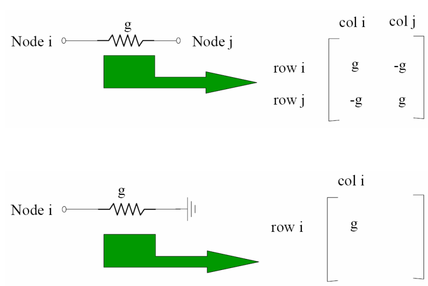


$$\begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + s\begin{bmatrix} C & -C \\ -C & C \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} J \\ 0 \end{bmatrix}$$

(G+sC) X(s) =b

How to get those matrices by inspection?

Resistor Stamp



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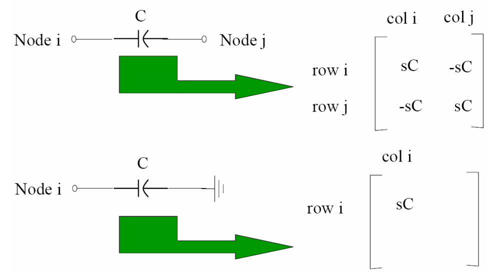
Capacitor Stamp



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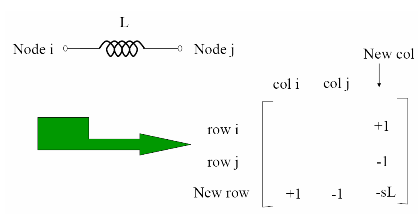
Inductor Stamp



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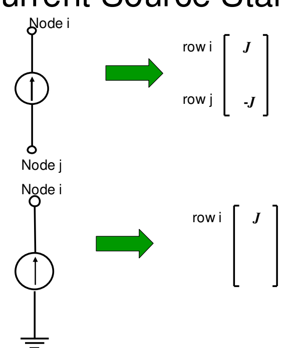
Current Source Stamp



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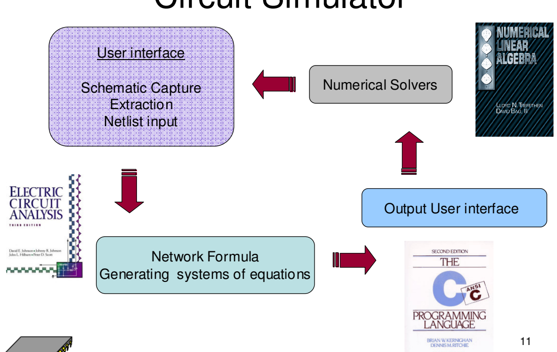
Circuit Simulator



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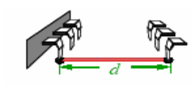
Motivation for Transmission Line

- The assumption of circuit theory is that signal from node i appears **simultaneously** at node j. This assumption can be justified if the wavelength of the signal is long enough, i.e. **low frequency**.
- For **high frequency** application, e.g. computer bus, and microwave circuits, we need to fix this problem by introducing:

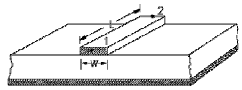
Transmission-line

Physical Transmission Line

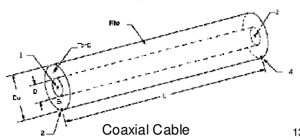
- In physical world, transmission line is a microstrip on chip, chip interconnects, coaxial cable, etc.



Interconnect



Microstrip



Coaxial Cable

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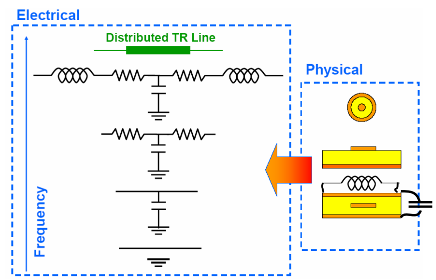
Transmission Line: Distributed Circuit Model



Resistor: Ohmic losses

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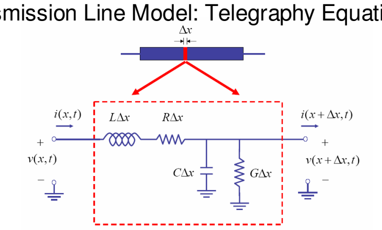
Transmission Line Model: Telegraphy Equation



From $\Delta x \rightarrow \partial x$

$$\begin{cases} \dfrac{\partial V(x,s)}{\partial x} = -I(x,s)(R+sL) \\ \dfrac{\partial I(x,s)}{\partial x} = -V(x,s)(G+sC) \end{cases}$$

Two Sets of Partial Differential Equations!

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Solution to Telegraphy Equation

$$\begin{cases} \dfrac{\partial V(x,s)}{\partial x} = -I(x,s)(R+sL) \\ \dfrac{\partial I(x,s)}{\partial x} = -V(x,s)(G+sC) \end{cases}$$

$$\frac{\partial}{\partial x}\begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix} = (D+sE)\begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix}$$

where $D = \begin{bmatrix} 0 & -D \\ -G & 0 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & -L \\ -C & 0 \end{bmatrix}$

Solution: $\begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix} = e^{(D+sE)x}\begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix}$

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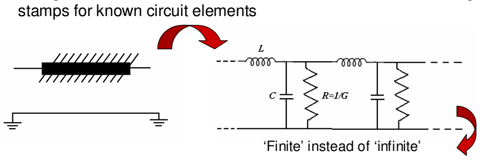
MNA of Transmission Line

- From distributed model to analytical TL governing equation.
- How to design a stamp for Transmission line Simulation?
- Partial Differential Equations?
- Going back to distributed model and we can make use of existing stamps for known circuit elements



'Finite' instead of 'infinite'

(G+sC) X(s) = b

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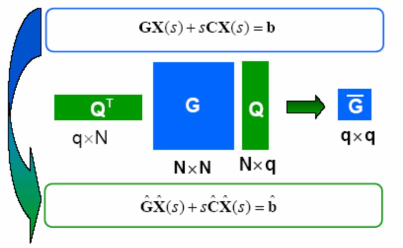
Problem with Discretization

- Large matrix system due to number of blocks need to be big.
- Number of the building block is determined by the wavelength of the signal.
- How to save memory and CPU time?

Model Order Reduction

Model Order Reduction



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Model Order Reduction

(G + sC) X(s) = b

$$\hat{C}=Q^TCQ$$
$$\hat{G}=Q^TGQ$$
$$\hat{b}=Q^Tb$$
$$X=Q^TX$$

$$(\hat{G}+s\hat{C})\hat{X}(s)=\hat{b}$$

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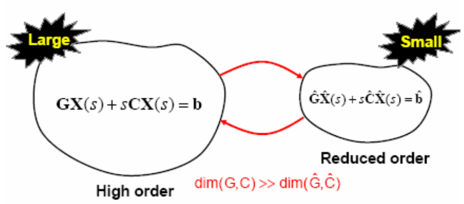
Model Order Reduction



$$\mathbf{X}(s)\cong \mathbf{Q}\hat{\mathbf{X}}$$

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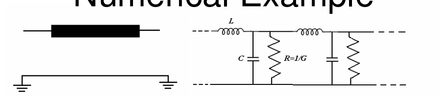
Numerical Example



From Anu

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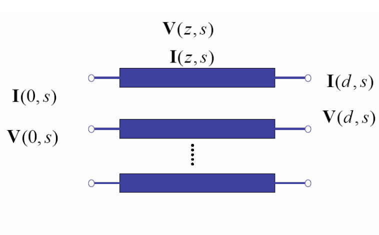
Multiple Transmission Lines



Reference Conductor

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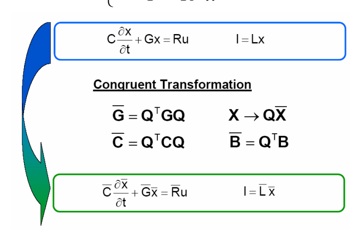
Y-Parameter

$$\begin{cases}(G+sC)x=Ru\\ I=R^Tx\end{cases}$$



$Y(s)=R^T(G + sC)^{-1}R$

Carbon Nanotube Model

L_K: Kinetic engery per unit length

Sum of the kinetic energies of left and right movies

$L_K = \frac{h}{2e^2 v_F}$ Typically, 16nH/um

C_{ES}: Electrostatic capacitance

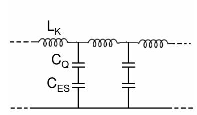
Capacitance between wire and ground plane

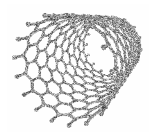
$C_E = \frac{2\pi\varepsilon}{\cosh^{-1}(2h/d)} \approx \frac{2\pi\varepsilon}{\ln(h/d)}$ Typically, 50 aF/um

C_Q: Quantum Capacitance

Associated with energy to add an extra electron

$C_Q = \frac{2e}{h v_F}$ Typically, 100 aF/um





Reference

Major Papers:

[1] E.Gad, R.Khazaka, M.Nakhla, R.Griffith, “Circuit Reduction Technique for Finding the Steady State Solution of Nonlinear Circuits”, *IEEE MTT-S Digest*, 2000

[2] R.Khazaka, *Projection Based Techniques for the Simulation of RF Circuits and High Speed Interconnects*, PhD Dissertation, Carleton University, Canada, 2002

[3] J.Ogrodzki, *Circuit Simulation and Method and Algorithm*, CRC Press, 1994

[4] R.Khazaka, “ECSE 596 Circuit Simulators”, Lecture Notes, McGill University, 2005

[5] Burke, P.J., “An RF circuit model for carbon nanotubes”, *IEEE Transactions on Nanotechnology*, Vol. 2, Iss. 1, March 2003, Pages:55 – 58