Pricing in Telecommunication Networks: From Wireless to Optical Networks

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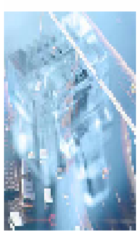
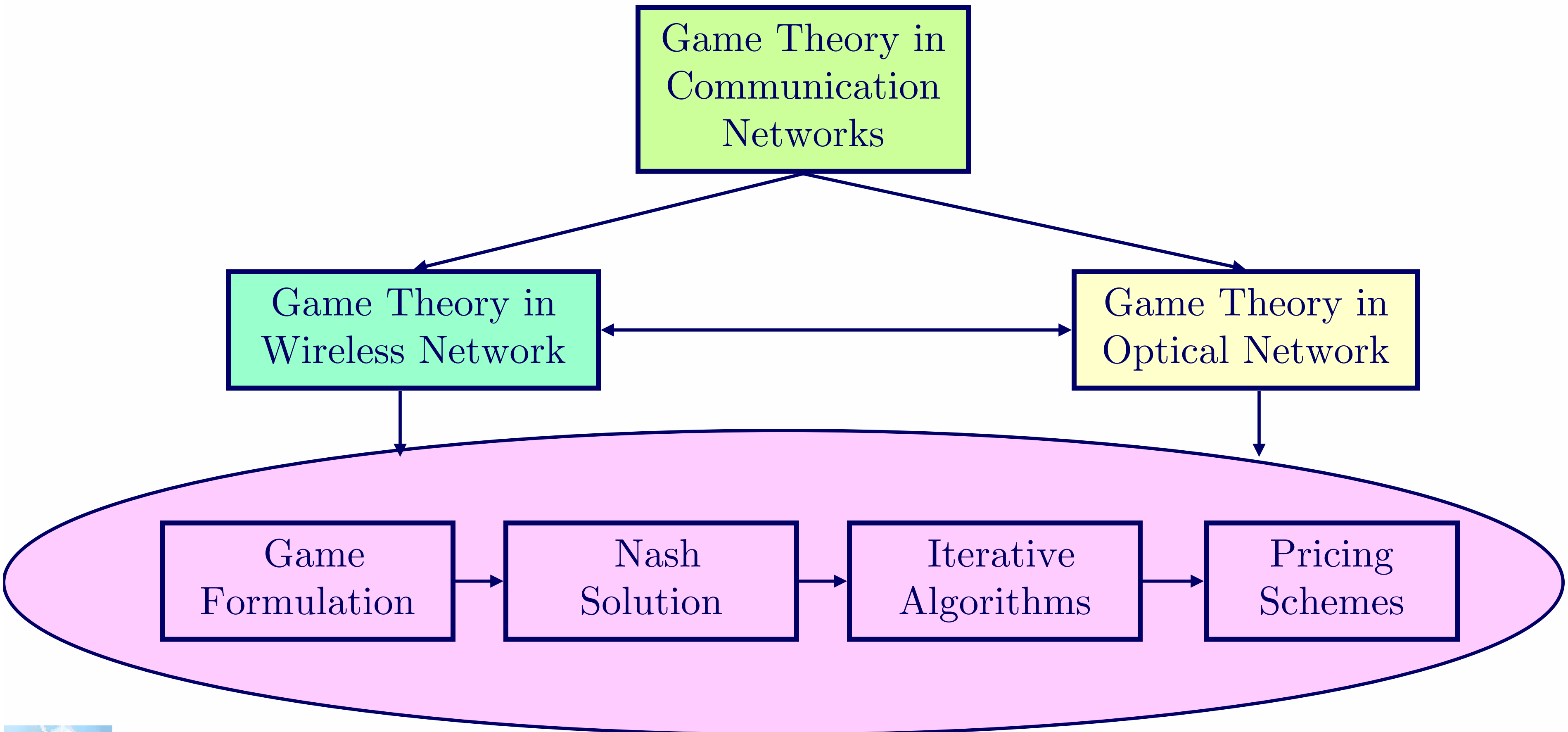
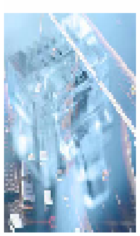


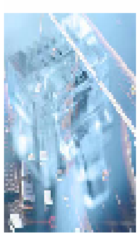
Outline


Game Theory in Communication Networks
Game Theory in Wireless Network
Game Theory in Optical Network
Game Formulation
Nash Solution
Iterative Algorithms
Pricing Schemes



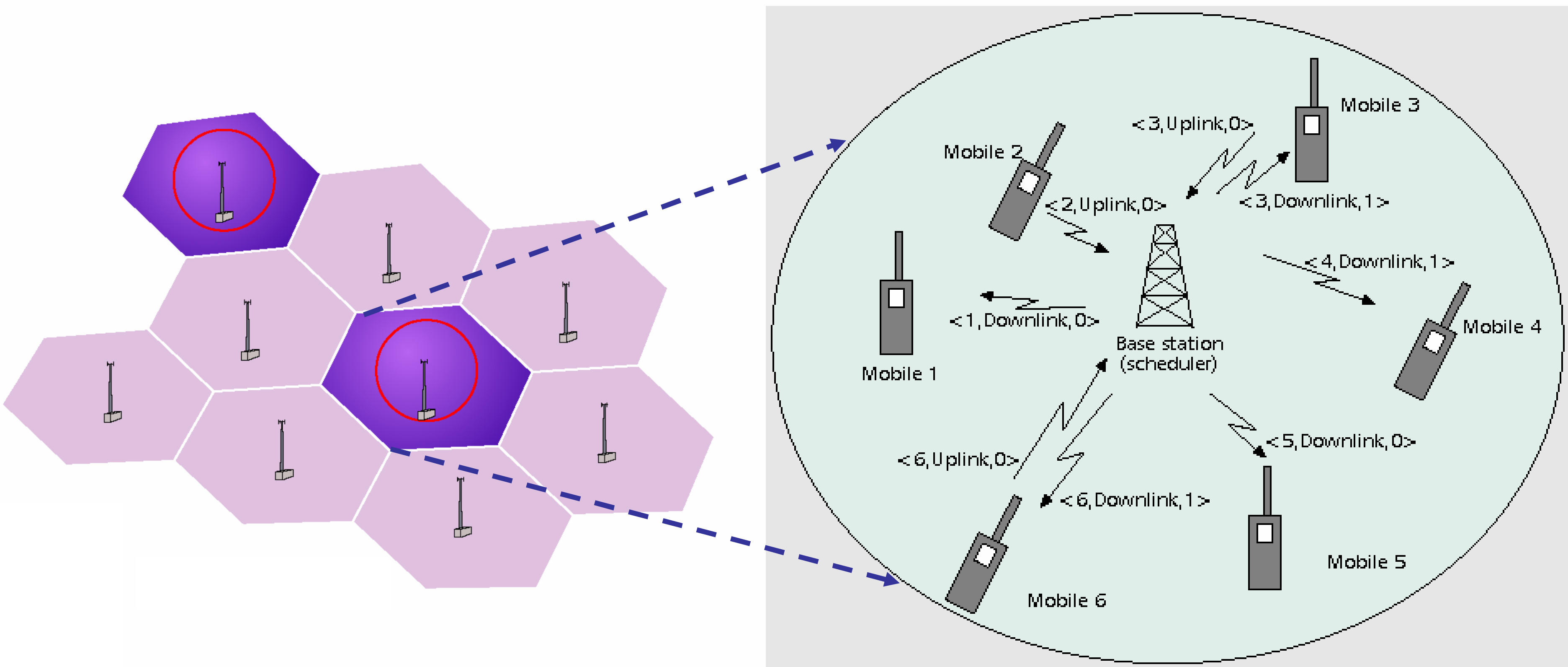
Motivation

- From centralized to distributed protocols
  - Network size growth
  - Resource allocation (power, bandwidth,etc.)
- Tools: Optimization vs. Game theory
  - Agents: different channels
  - Strategy: feasible power consumptions
  - Preference: utility function
  - Information: local and myopic
  - Non-cooperative
- Implementation
  - Pricing schemes (proportional pricing, decentralized pricing, auctions, etc.)
- Example: Wireless and Optical Networks



Wireless Network

- CDMA uplink power control:
  - Conserve battery energy
  - Minimize the effect of interference → SNR



Game Formulation

- A game defined by $\langle N, (A_i), (J_i) \rangle$
  - N=(1,2,…M), number of players
  - $(A_i)_{i \in N}$: set of actions
  - $(J_i)_{i \in N}$: preference relations represented by payoff functions:

$$J_i(p_i, p_{-i}) = \underbrace{\lambda_i p_i}_{\text{cost}} - \underbrace{\ln(1+\gamma_i)}_{\text{utility}} \qquad \forall i$$

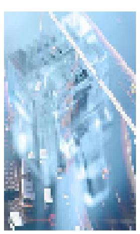
$$\text{(SNR)} \qquad \gamma_i = L \frac{h_i p_i}{\sum_{i \neq j} h_i p_j + \sigma^2}$$

- Each user i has her own best response as function:

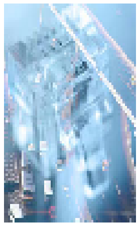
$$BR_i(p_i, p_{-i}) = \arg\max_{p_i} J_i(p_i, p_{-i})$$

- Nash equilibrium is unique when only one p* satisfies

$$BR_1(p^*) = BR_2(p^*) = \ldots = BR_N(p^*)$$



Nash Equilibrium



"Best result comes when one is doing what is best for himself and the group."

Nash Solution and Algorithms

Unique solution of $Ap^* = b \;\Leftarrow\;$ A is strictly diagonally dominant

$$BR_1(p^*) = BR_2(p^*) = \ldots = BR_N(p^*)$$

Parallel Update Algorithm

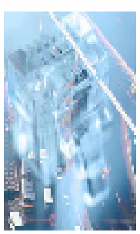
$$\begin{cases} p_i^{(n+1)}(p_{-1}^{(n)}, \lambda_i) = \dfrac{1}{\lambda_i} - \dfrac{1}{Lh_i}\left(\displaystyle\sum_{j \neq i} h_j p_j^{(n)} + \sigma^2\right) & \text{if } \displaystyle\sum_{j \neq i} h_j p_j^{(n)} \leq \dfrac{Lh_i}{\lambda_i - \sigma^2} \\ p_i^{(n+1)}(p_{-1}^{(n)}, \lambda_i) = 0 & \text{otherwise} \end{cases}$$

- Myopic iteration at each step

Random Update Algorithm

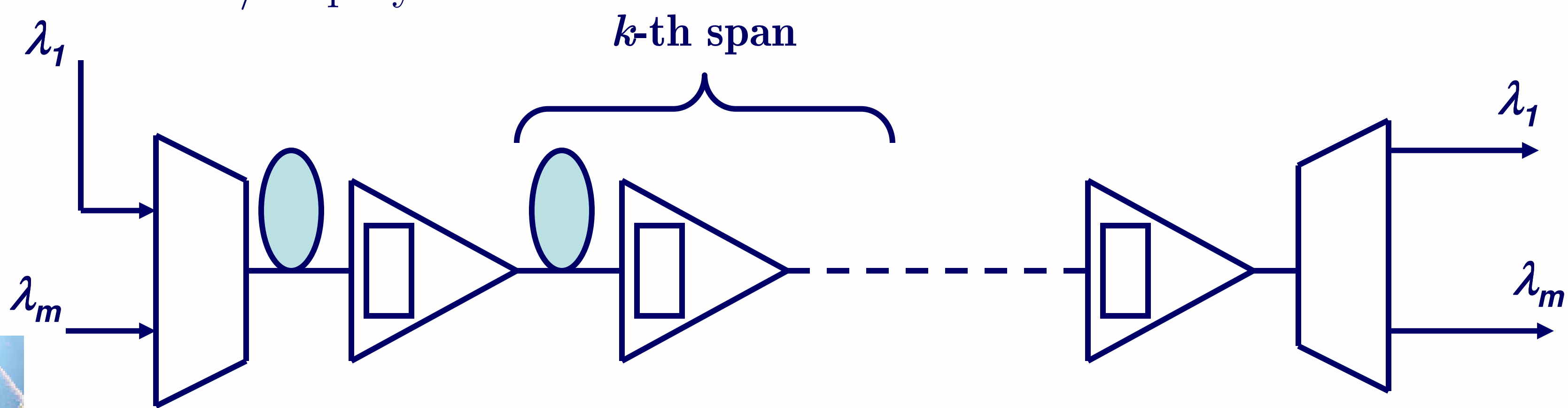
$$\begin{cases} p_i^{(n+1)} = p_{i,PUA}^{(n+1)}(p_{-i}^{(n)}, \lambda_i) & \text{with probability } \pi_i \\ p_i^{(n+1)} = p_i^{(n)} & \text{with probability } 1 - \pi_i \end{cases}$$

- Myopic iteration and randomized delays in update

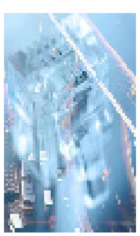


Optical Networks

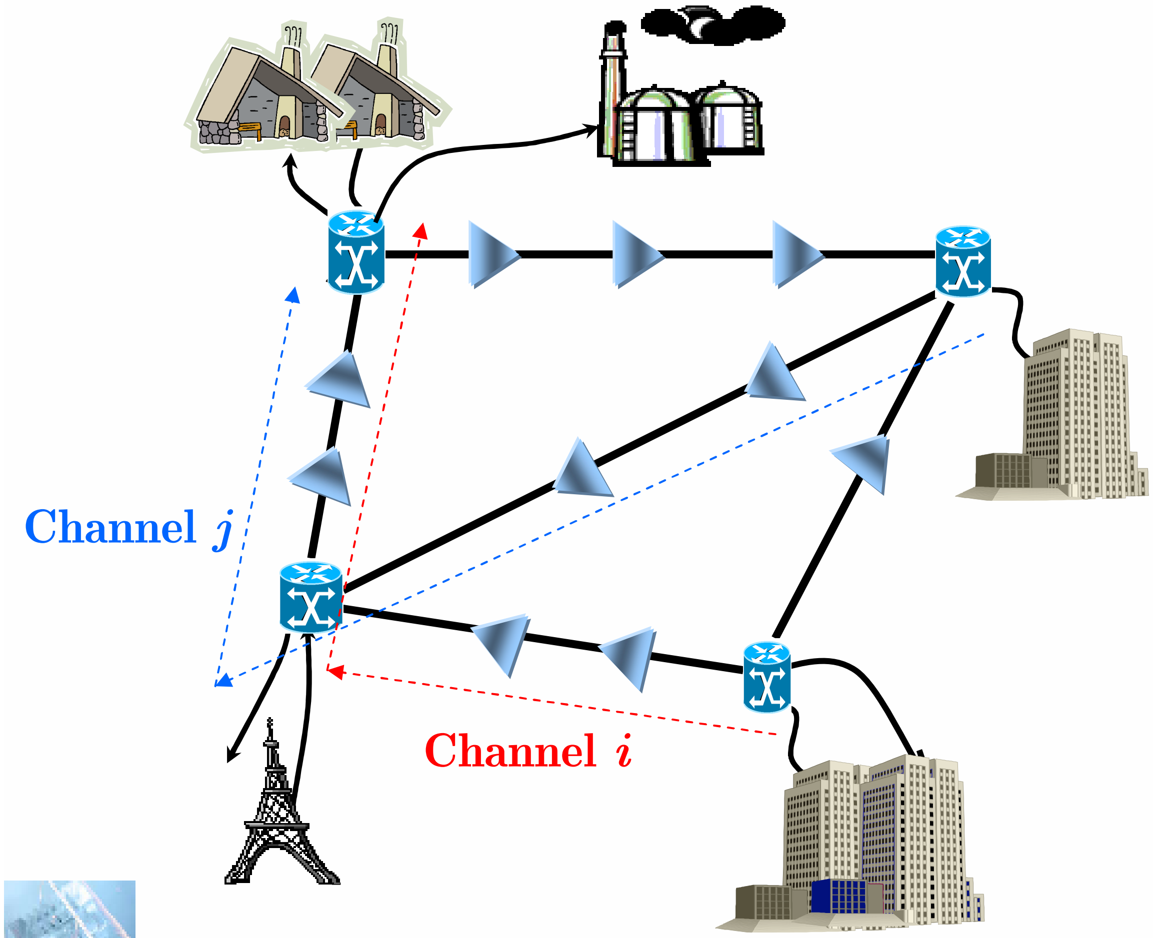
- A preferred means of transmission for signals
  - Low loss
  - Low levels of undesirable transmission impairment
  - Strong immunity to electromagnetic interference
  - Long life-span
- Network Management:
  - Centrally managed optical layer → one controlled in distributed fashion
  - Add/drop dynamics



l-th link, $l \in L$

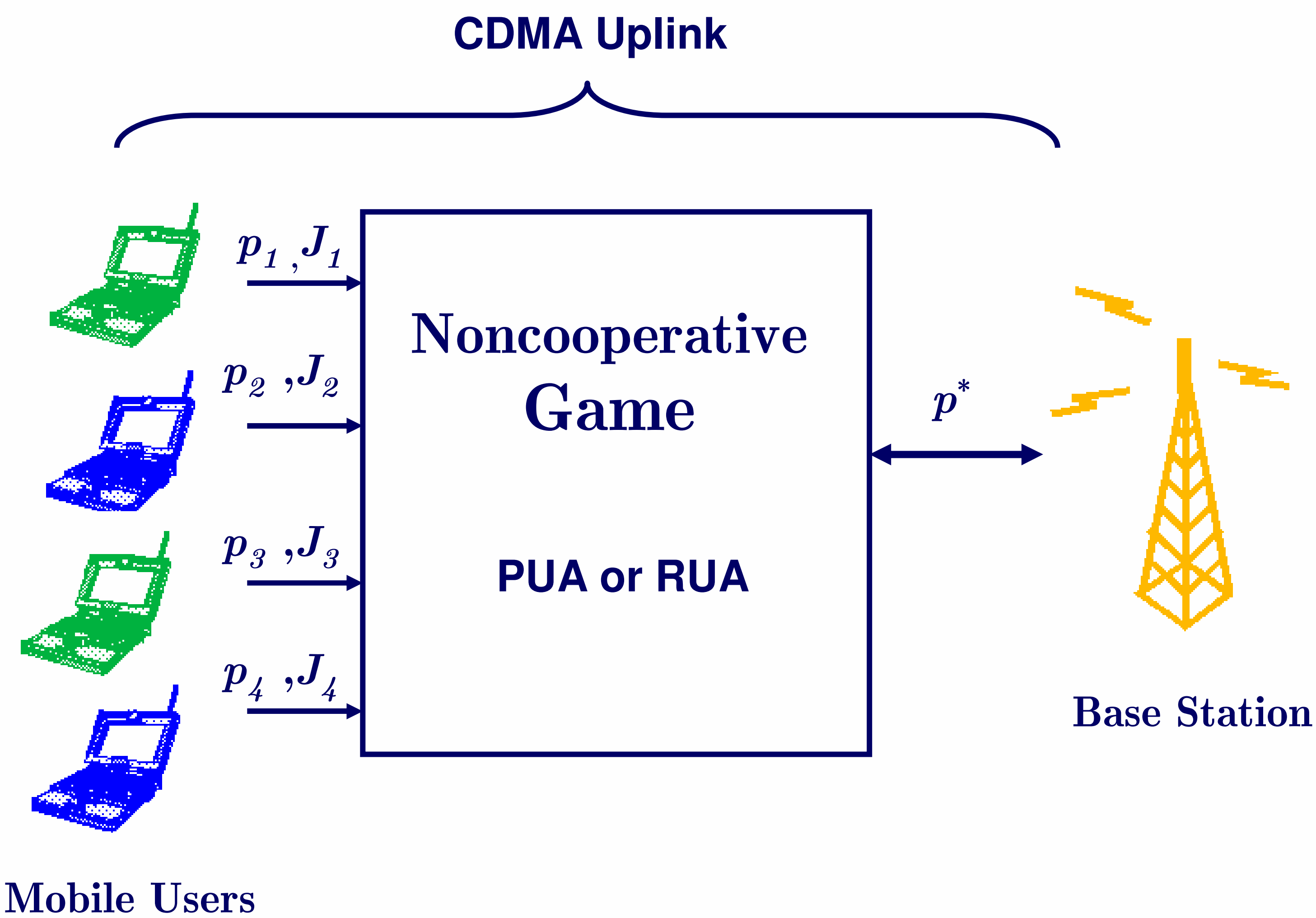


An Example of Optical Network Topology

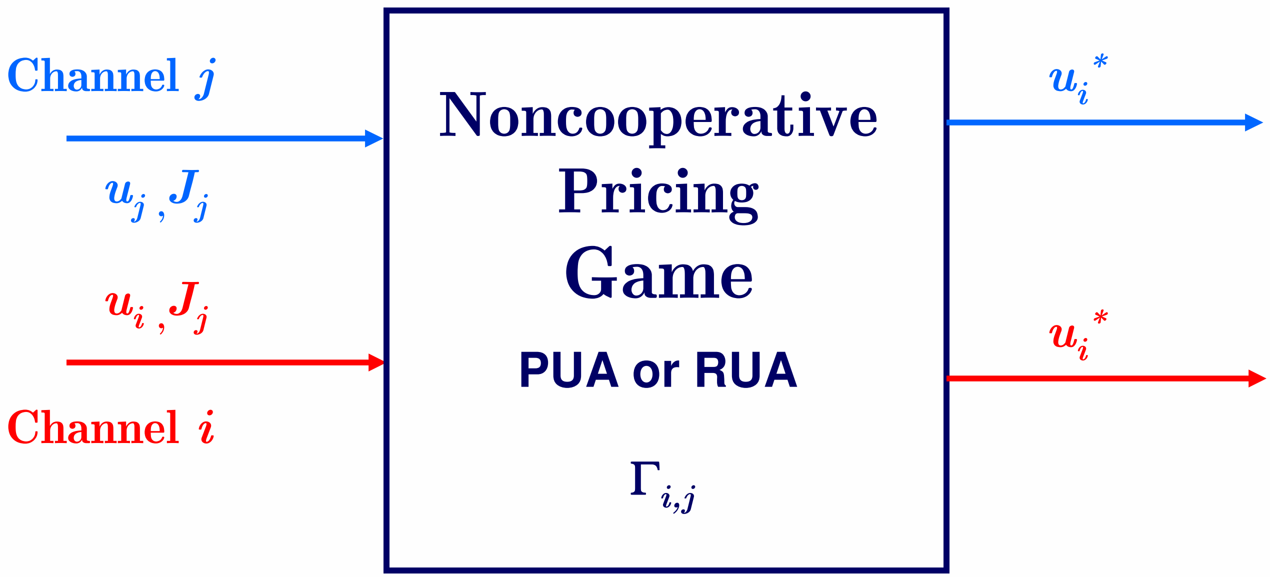


1. Channel-channel interference
2. Spontaneous Emission Noise (ASE)
3. Multi-stage amplifications
4. More complicated network topology

From Wireless to Optical Networks

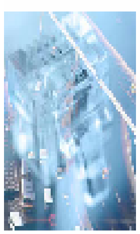


From Wireless to Optical Network



$$OSNR_i = \frac{u_i}{n_{0,i} + \sum_{j \in \mathcal{M}} \Gamma_{i,j} u_j} \quad \forall i \in \mathcal{M}$$

$$\Gamma_{i,j} = \sum_{i \in \mathcal{R}_i} \sum_{k=1}^{N_l} \frac{G_{l,k}^k}{G_{l,i}^k} \left(\prod_{q=1}^{l-1} \frac{\mathbf{T}_{q,j}}{\mathbf{T}_{q,i}} \right) \frac{ASE_{l,k,i}}{P_{o,l}}, \quad j \in \mathcal{M}_l$$



Game and Nash Solution

- A game defined by $\langle\, N, (A_i), (J_i) \,\rangle$
  - N=(1,2,…M), number of players
  - $(A_i)_{i \in \mathrm{N}}$: set of strategies $[0, u_{\max}]^{\mathrm{N}}$
  - $(J_i)_{i \in \mathrm{N}}$: preference relations represented by payoff functions:

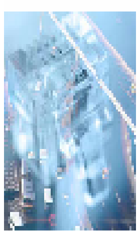
$$J_i(u_i, u_{-i}) = \alpha_i u_i - \beta_i \ln(1 + a_i \frac{u_i}{X_{-i}})$$

- Implicit Best response function $BR_i(u_i, u_{-i})$

$$a_i u_i^* + X_{-i}^* = \frac{a_i \beta_i}{\alpha_i}, \text{ where } X_{-i} = \sum_{j \neq i} \Gamma_{i,j} u_j + n_{0,i}$$

Aggregate into a matrix:

$$\tilde{\Gamma} u^* = \tilde{b}$$



Game and Nash Solution (Cont'd)

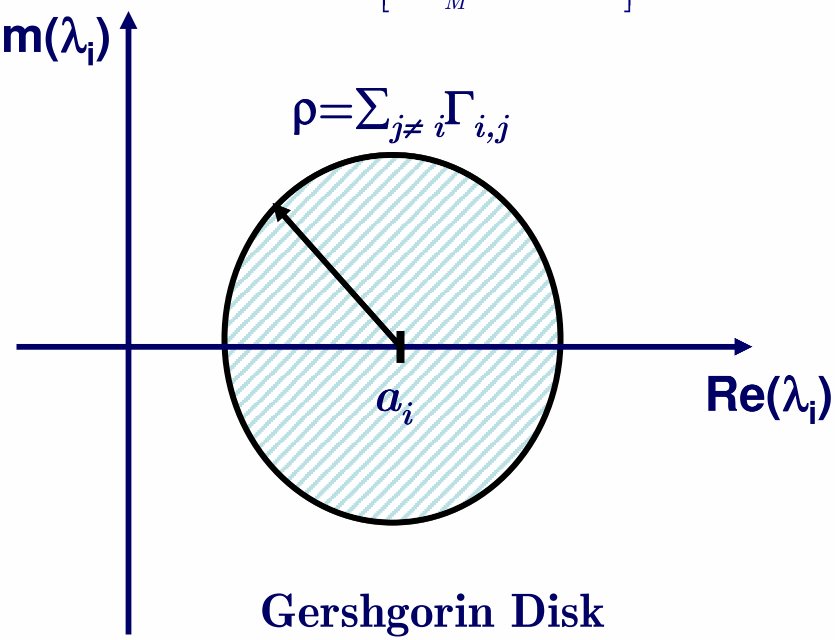
$$\tilde{\Gamma} u^* = \tilde{b}$$

$$\begin{bmatrix} a_1 & \Gamma_{1,2} & \cdots & \Gamma_{1,M} \\ \Gamma_{2,1} & a_2 & \cdots & \Gamma_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{M,1} & \Gamma_{M,2} & \cdots & a_M \end{bmatrix} \begin{bmatrix} u_1^* \\ u_2^* \\ \vdots \\ u_M^* \end{bmatrix} = \begin{bmatrix} \frac{a_1 \beta_1}{\alpha_1} - n_{0,1} \\ \frac{a_2 \beta_2}{\alpha_2} - n_{0,2} \\ \vdots \\ \frac{a_M \beta_M}{\alpha_M} - n_{0,M} \end{bmatrix}$$

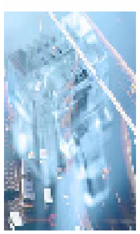
Existence and Uniqueness of Solution
under a sufficient condition:

Strictly Diagonal Dominance:

$$\sum_{j \neq i} \Gamma_{i,j} < a_i$$



Gershgorin Disk



$a_i > 0 \rightarrow \tilde{\Gamma}$ **strictly positive definite**

Nash Equilibrium and Iterative Algorithm

$$a_i u_i^* + X_{-i}^* = \frac{a_i \beta_i}{\alpha_i}$$

$$u^* = \tilde{\Gamma}^{-1} \tilde{b}$$

Nash Equilibrium Solution
(Static)

$$u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{X_{-i}(n)}{a_i}$$

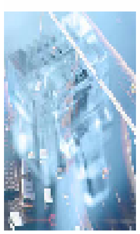
$$u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{1}{a_i}\left(\frac{1}{OSNR_i(n)} - \Gamma_{i,i}\right) u_i(n)$$

Iterative Algorithm
(Dynamic)

- The game is in strategic form and the decisions are just made once
- The iterative algorithm is distributed.

Some questions:

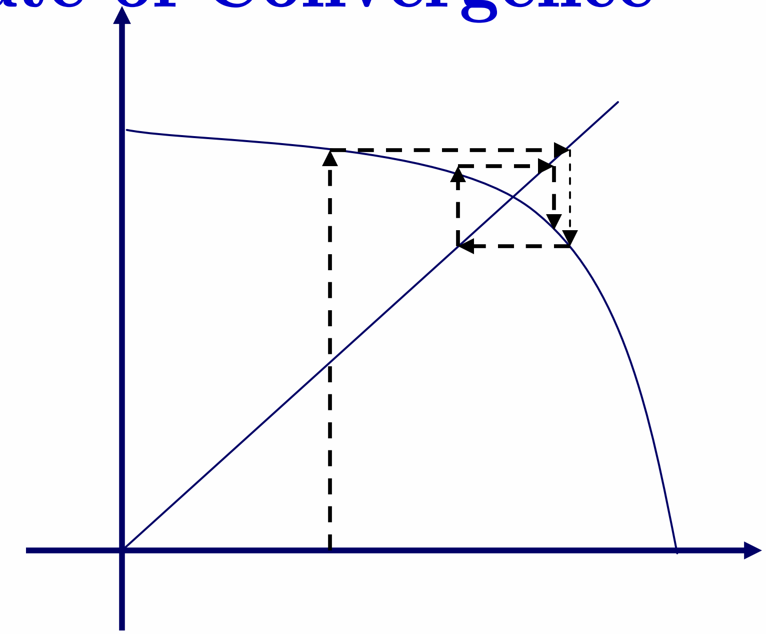
- What is the rate of convergence in the iterative algorithm?
- How robust is the algorithm?



Iterative Algorithm: Rate of Convergence

$$u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{1}{a_i}\left(\frac{1}{OSNR_i(n)} - \Gamma_{i,i}\right)u_i(n)$$

$$u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{X_{-i}(n)}{a_i}$$

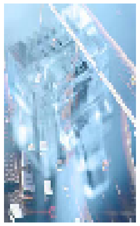


Contraction Mapping!

Define $e_i(n) = u_i(n) - u^*$

$$e_i(n+1) = -\frac{1}{a_i}\sum_{j\neq i}\Gamma_{i,j}e_j(n)$$

$$\|e_i(n+1)\|_\infty = \max_i |e_i(n+1)| \leq \underbrace{\max_i\left(\frac{1}{a_i}\sum_{j\neq i}\Gamma_{i,j}\right)}_{\text{Rate of convergence}} \|e_i(n)\|_\infty$$

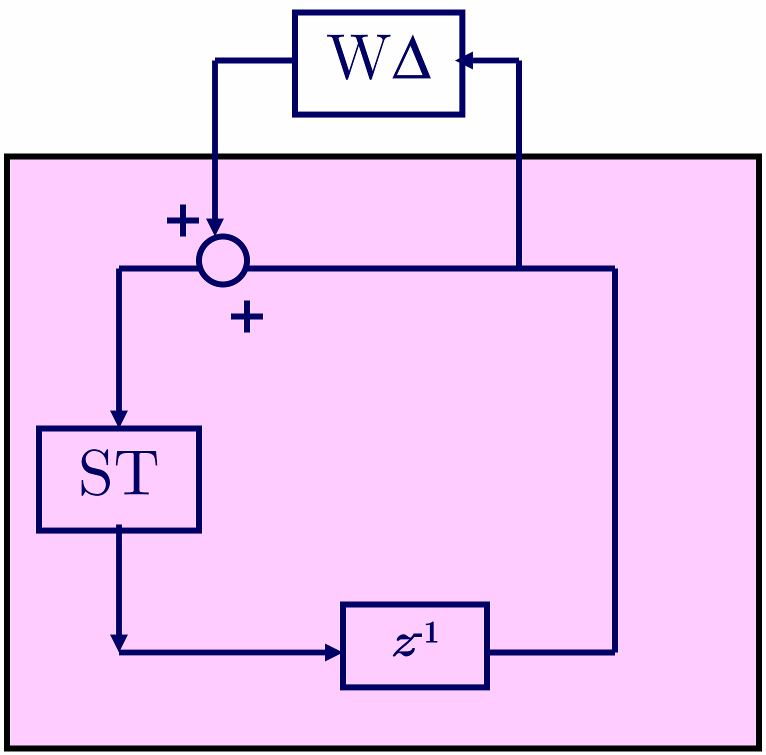


Parameter a_i determines the rate of convergence!

Iterative Algorithm: Robustness

$$u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{1}{a_i}\left(\frac{1}{OSNR_i(n)} - \Gamma_{i,i}\right)u_i(n)$$

$$u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{X_{-i}(n)}{a_i}$$

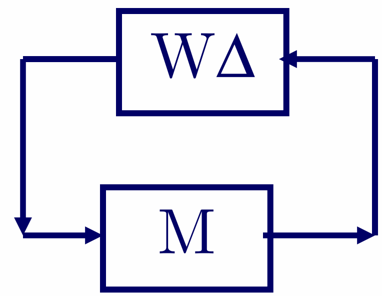


Example (3 channels):

$$e_1(z) = -z^{-1}\left[\frac{1}{a_1}\Gamma_{1,2}e_2(z) + \frac{1}{a_1}\Gamma_{1,3}e_3(z)\right]$$

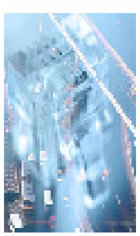
$$e_2(z) = -z^{-1}\left[\frac{1}{a_1}\Gamma_{2,1}e_1(z) + \frac{1}{a_1}\Gamma_{2,3}e_3(z)\right]$$

$$e_3(z) = -z^{-1}\left[\frac{1}{a_1}\Gamma_{3,1}e_1(z) + \frac{1}{a_1}\Gamma_{3,2}e_2(z)\right]$$



Robustness Criteria:

$||W\Delta|| < 1/\beta$ **if** $||M|| \leq \beta$



Random Update Algorithm

$$u_i(n+1) = \frac{\beta_i}{\alpha_i} - \frac{1}{a_i}\left(\frac{1}{OSNR_i(n)} - \Gamma_{i,i}\right)u_i(n) \quad \text{with probability } \pi_i$$
$$u_i(n+1) = u_i(n) \quad \text{with probability } 1-\pi_i$$

- Can deal with the randomized delay in updates

Convergence in Expectation:

$$\mathbf{E}[e_i(n+1)] = \mathbf{E}[u_i(n+1) - u_i^*] = \pi\mathbf{E}[(-\frac{1}{a_i}\sum_{j\neq i}\Gamma_{i,j}e_j(n)] + (1-\pi_i)\mathbf{E}[e_j(n)]$$

$$\leq \underbrace{[-\frac{1}{a_i}\sum_{j\neq i}\Gamma_{i,j}\pi + (1-\pi)]}_{<\mathbf{1}}\mathbf{E}(e_j(n)).$$

Convergence in Probability (Almost surely):

$$\sum_{n=1}^{\infty} P(|e_i(n)| > \epsilon) \leq \sum_{n=1}^{\infty}\frac{\mathbf{E}(|e_i(n)|)}{\epsilon} \leq \frac{1}{\epsilon}\sum^{\infty}\|e(n)\|_\infty$$

$$\|e(n)\|_\infty \leq \kappa\|e(n-1)\|_\infty \leq \ldots \leq \kappa^n\|e(0)\|_\infty, \Rightarrow \sum_{n=1}^{\infty} P(|e_i(n)| > \epsilon) \leq \frac{K}{\epsilon(1-\kappa)} \Rightarrow$$

Converge In Probability

Borel-Cantelli

Pricing Schemes

$$J_i(u_i, u_{-i}) = \alpha_i u_i - \beta_i \ln(1 + a_i \frac{u_i}{X_{-i}})$$

- α_i: directly determines the pricing of unit power
- a_i : influences the rate of convergence
- β_i: closely related to the bounds on desired OSNR γ^*

Example: proportional pricing, i.e., $\alpha_i = \Gamma_{i,i} k_i$

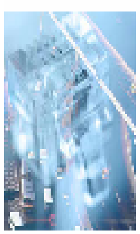
$$\frac{1}{\gamma_i^*} u_i^* = (\Gamma_{i,i} - a_i) u_i^* + \frac{a_i}{\Gamma_{i,i}} \frac{1}{k_i} \quad \forall i \qquad \textbf{or} \qquad \mathbf{\Sigma u^* = v}.$$

a_i doesn't affect the upper bound of γ_i if $\rho(\tilde{\Gamma}\Sigma^{-1}) < 1$

$$\gamma^* < \frac{1}{\sum_j \Gamma_{i,j}}$$

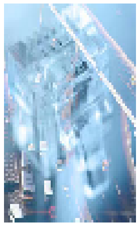
$$\beta_i > \frac{1 + (a_i - \Gamma_{i,i}\gamma_i^*)}{1 - \Gamma_{i,i}\gamma_i^*} \frac{\alpha_i}{a_i} X_{-i} \quad \forall i$$

$$\beta_i < \frac{\alpha_i}{a_i} [u_{max}(a_i + \sum_{j \neq i} \Gamma_{i,j}) + n_0] \quad \forall i$$



Pricing Schemes

- More in-depth pricing schemes
- Criteria on pricing schemes
  - Stability and convergence
  - Computationally efficient
  - Fairness and allocation efficiency
- More economical basis needs to be investigated.



Extension of Current Framework

- Unanswered question under the current framework
  - Algorithms that deal with capacity constraints
  - Deal with nonlinearity effects as optimization constraints
  - Robustness of the algorithm with the presence of the uncertainties

- From strategic form to extensive form
  - Each players are allowed to make multi-stage decisions
  - Algorithm based on multi-stage decision making, that truly describes the dynamics of players.

- Two kinds of extensive forms
  - Time-based extensive game: Nash bargaining game, auctions
  - Space-based extensive game: unique to the optical networks