Nonlinear Control of MEMS:
Modeling, Analysis and Design

Quanyan Zhu, *IEEE Member*

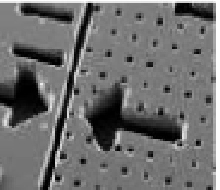
Software for Analysis and Design

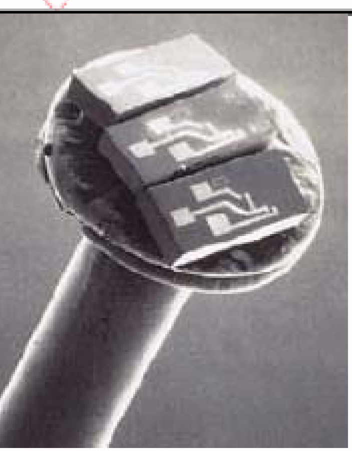
Department of Electrical and Computer Engineering
Faculty of Engineering

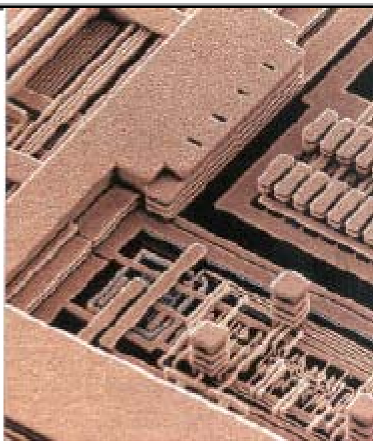
McGill University
Montreal, Canada

Presentation details on http://www.ece.mcgill.ca/~qzhu4

Contact: qzhu4@ieee.org or qzhu4@ece.mcgill.ca

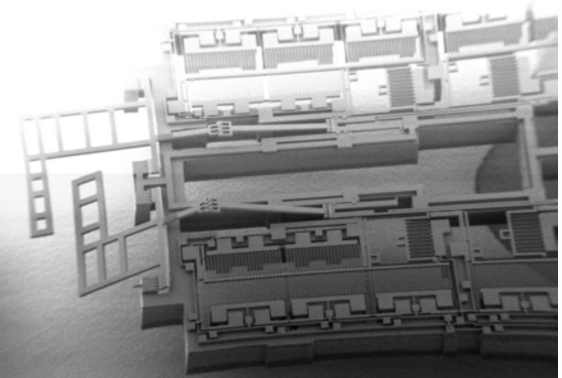


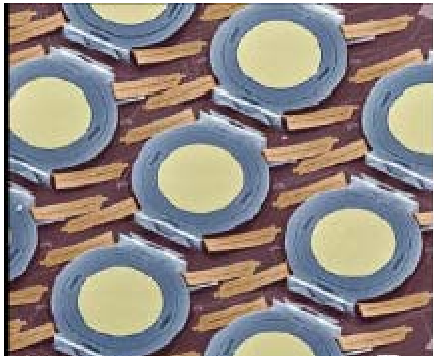




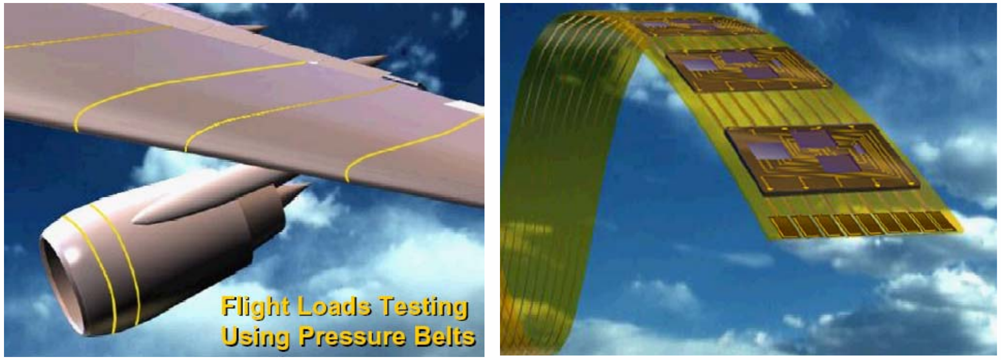
Outline

1. Introduction
2. Modeling and Analysis
3. Finite Element Analysis
4. Issues of Control
5. Future Work
6. Conclusion





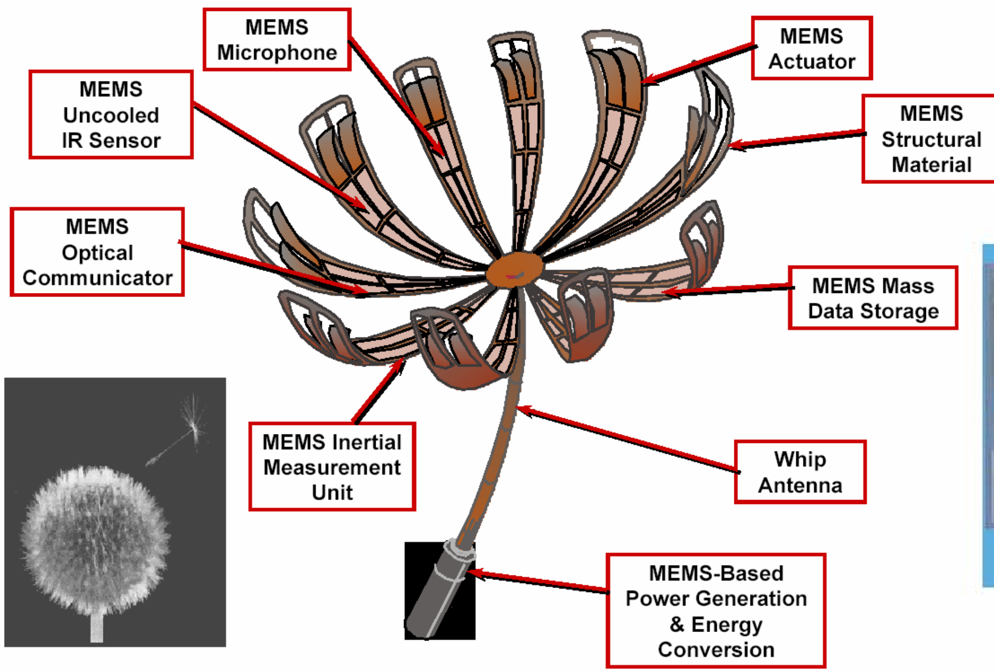
MEMS: Micro-Electro-Mechanical Systems

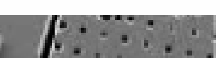


Boeing Pressure Belt using MEMS

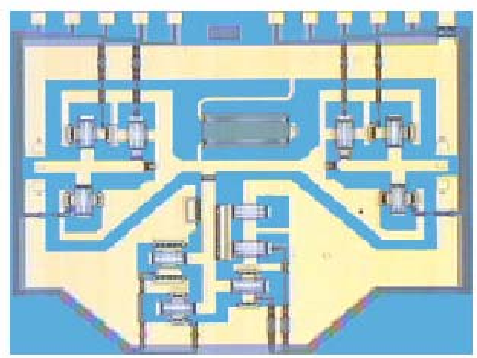
Wide Applications:

- Signal Processing
- Biomimetic Sensors
- Mass Data Storage
- Integrated micro-optomechanical components
- Embedded sensors
- Integrated RF Circuit, etc.

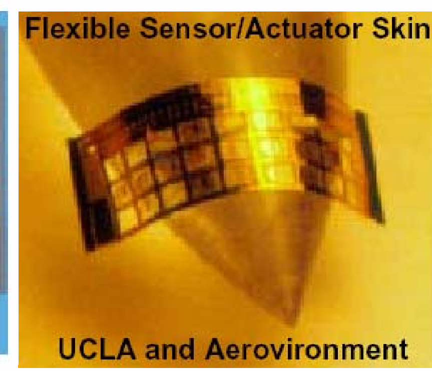




airborne surveillance dust



MEMS RF Tunable Filter



Sensor/Actuator

Introduction

Fabrication

1. Analytical Approach
2. Semi-Analytical Approach
3. Numerical Analysis: Finite Element Method

Modeling

Analysis

1. Classical Mechanics
2. Thermal Physics
3. Electromagnetism
4. Fluid Dynamics
5. Quantum Physics

etc.

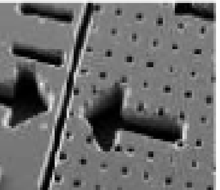
Control

Design

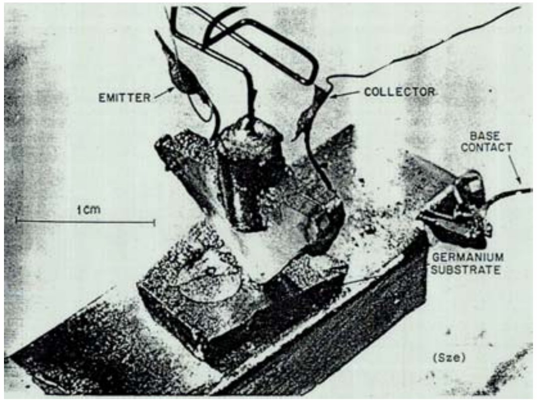
1. Robust Control
2. Distributed Control
3. Hybrid Control

Etc.

- Design Software
- Prototype Design

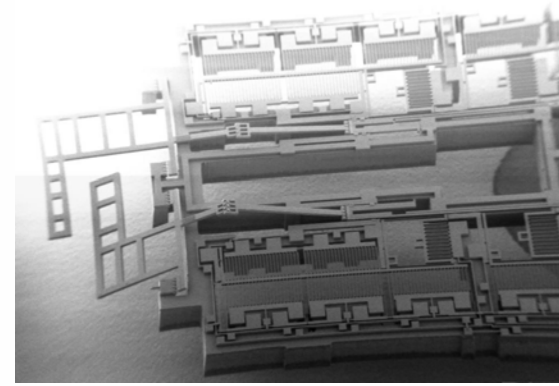


MEMS Fabrication

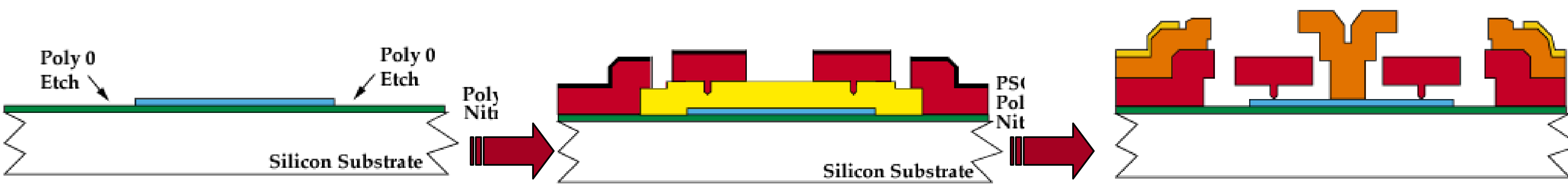


J. Bardeen, W.H. Brattain, "*The first transistor, a semiconductor triode*", Phys. Rev., 74, 230 (1948).

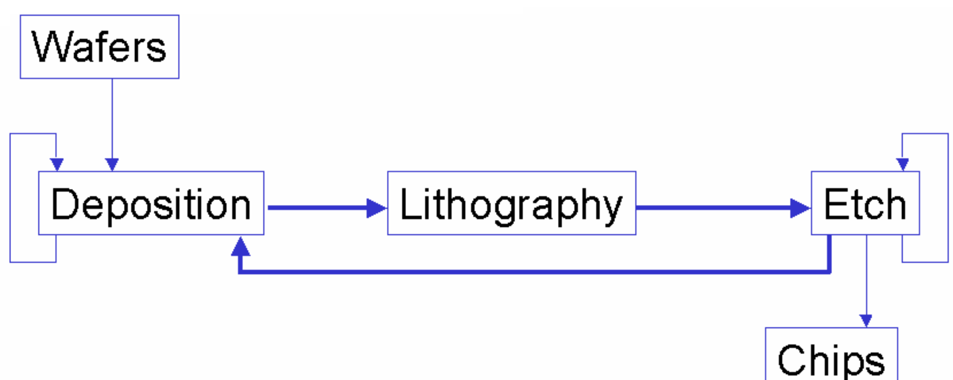
- Bulk micromachining
- Surface micromachining
- Go beyond microelectronics fabrication

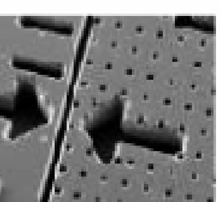


Microfabrication lab, Berkeley

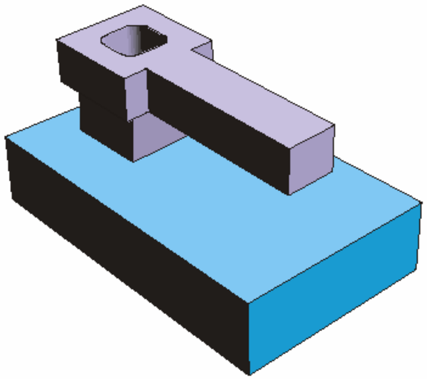


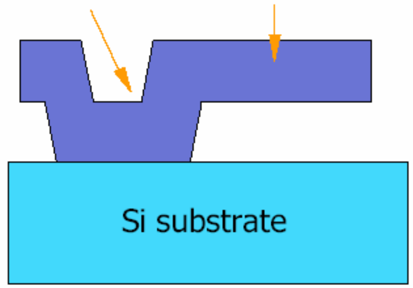
Fundamental Fabrication Procedures

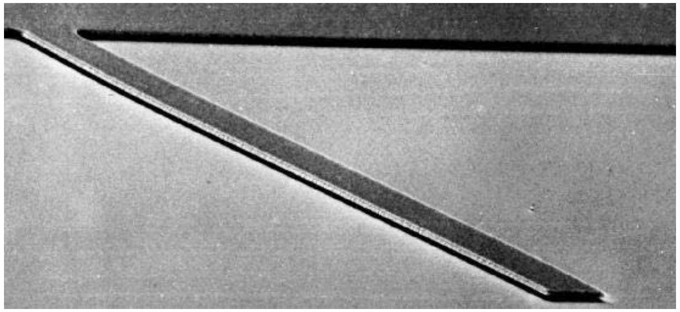




Fundamental Structures: Beam

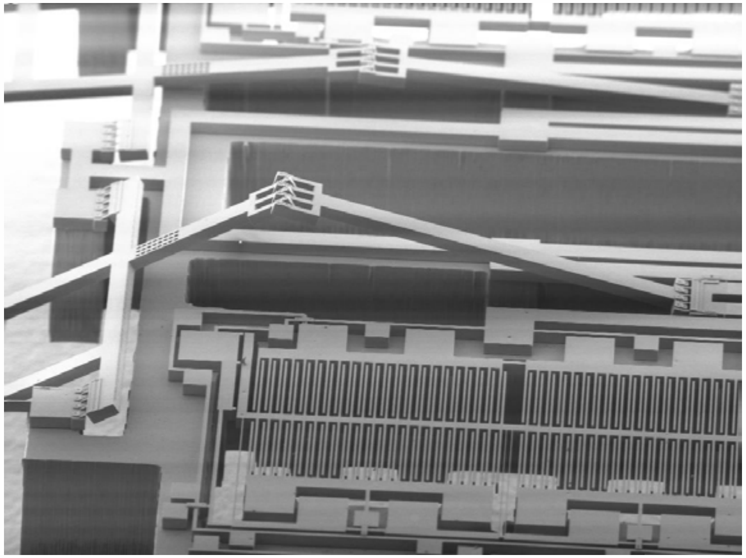






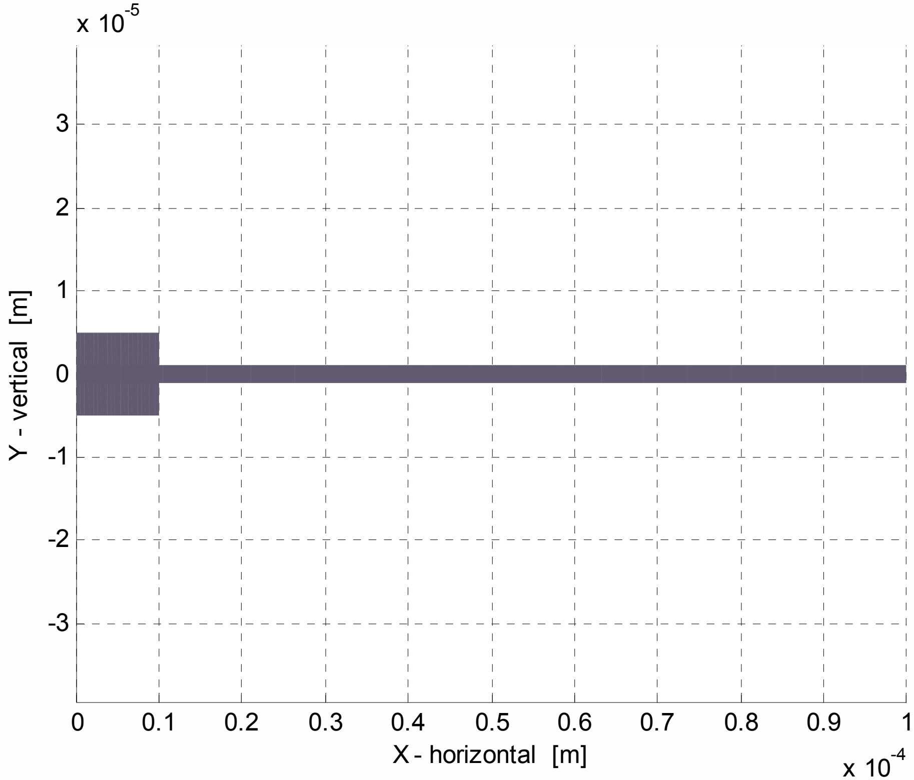
Cantilever-based devices:

- Versatile for a variety of applications
- Fabrication simplicity
- Better Sensitivity



Governing equation:

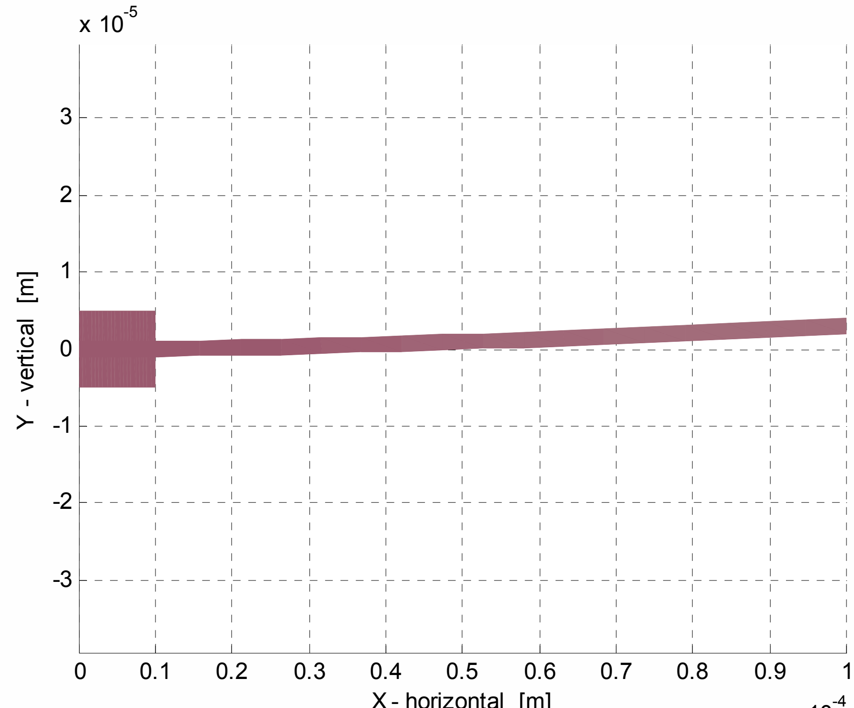
$$\rho A\frac{\partial^2 u}{\partial t^2} - \mu\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 u}{\partial x^2}\right) = f(x,t)$$



2μN on the tip produces a displacement of 3.0 μm

E=165 GPa

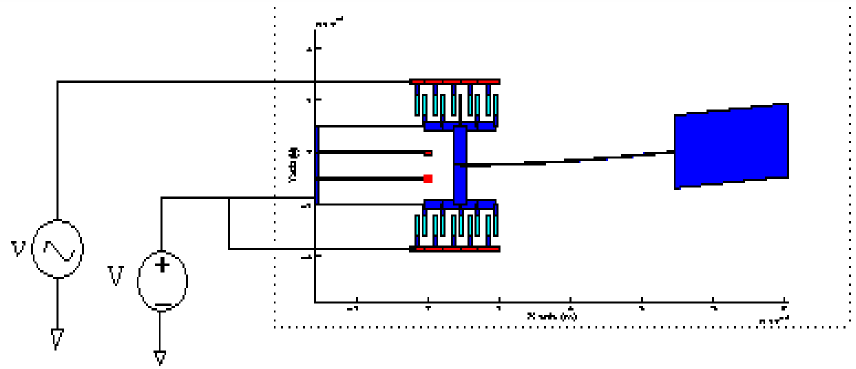
Material: Polysilicon



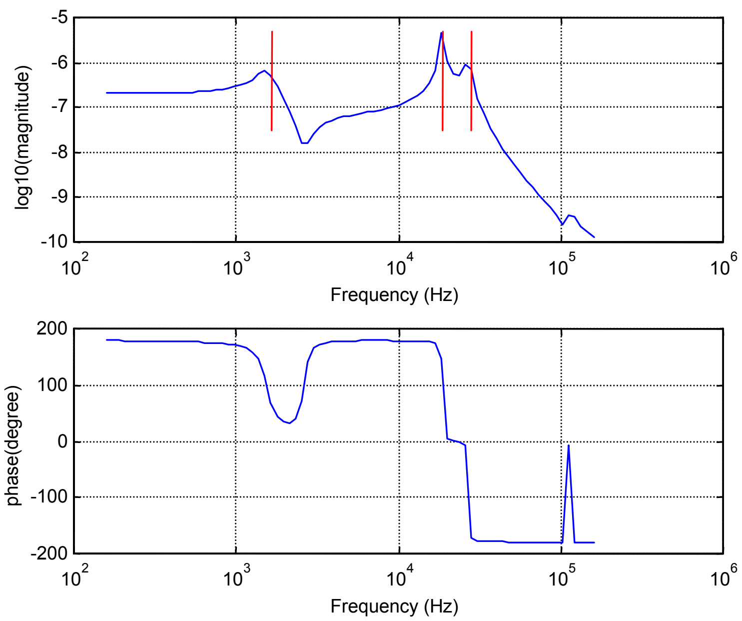
How about Coupling?

- Electro-elastic problem
- Thermo-elastic problem
- Thermo-magnetic problem, etc

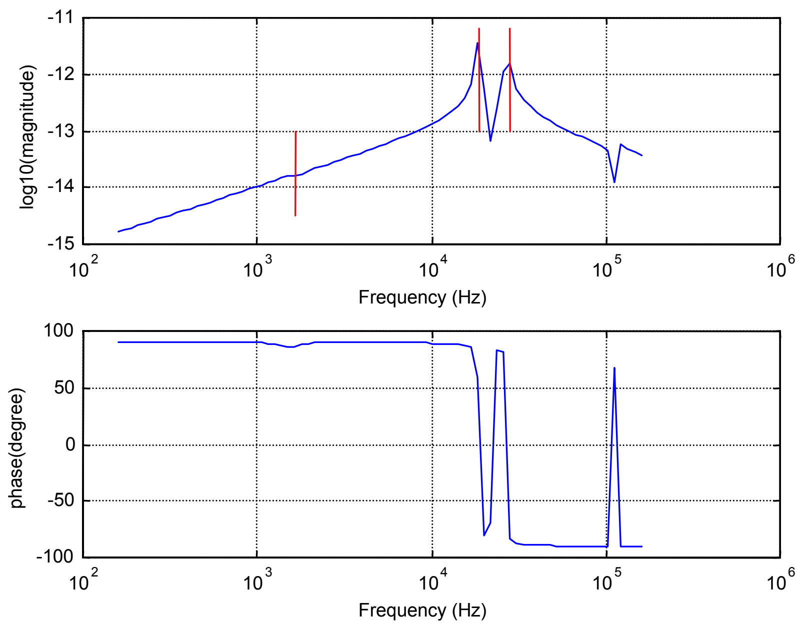
Simulation of a Linear Multiple Mode Resonator

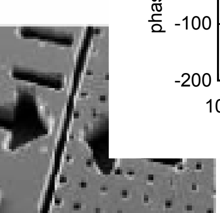


The response of vertical displacement of mass

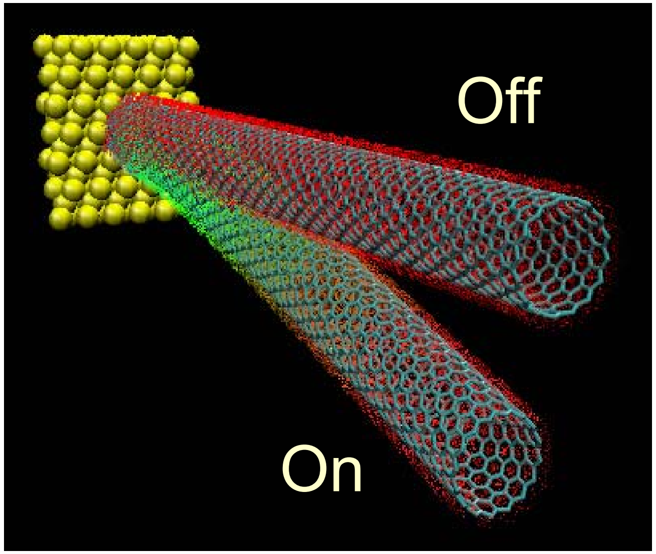


The response of induced current in lower comb

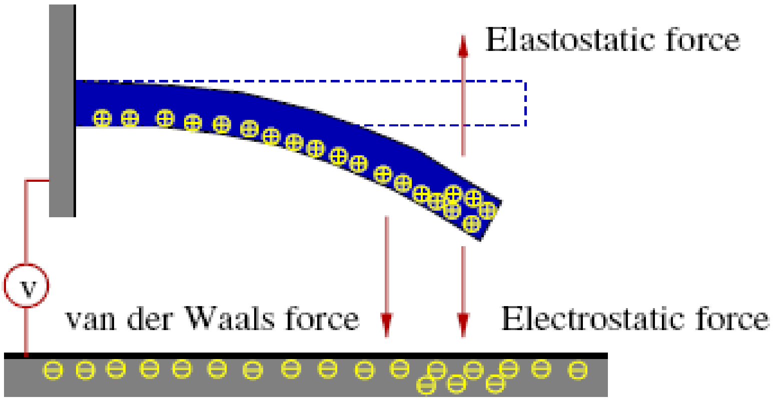


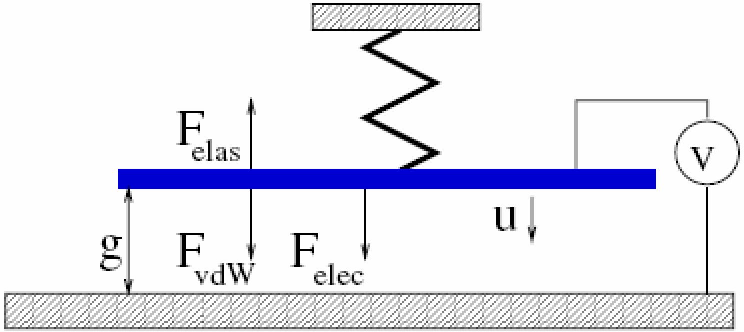


From MEMS to NEMS



SWNT Cantilever





Van der Waals Force per unit length:

$$q_{vdW} = \frac{\mathrm{d}(E_{vdW} / L)}{\mathrm{d}r}$$

where $$E_{vdW}(r) = \iint_{V_1,V_2} \frac{n_1 n_2 C_6}{r^6(V_1,V_2)} \mathrm{d}V_1 \mathrm{d}V_2$$

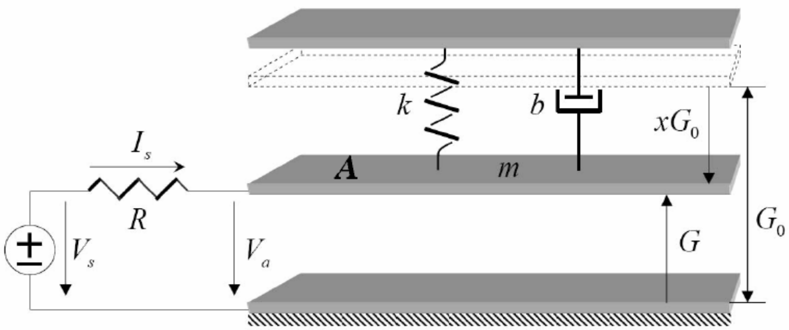
Simple Model of Electro-mechanical Coupling problem

1-D Governing equation

Coupled term

Mechanical: $$m\ddot{G}(t)+b\dot{G}(t)+k(G(t)-G_0)=\boxed{-\frac{Q^2(t)}{2\varepsilon A}}$$

Electrical: $$\dot{Q}(t)=\frac{1}{R}\left(V_s(t)-\frac{Q(t)G(t)}{\varepsilon A}\right)$$



Coupled, Nonlinear, Second-order set of Equations:

Potentially Unstable beyond a critical point: pull-in Voltage

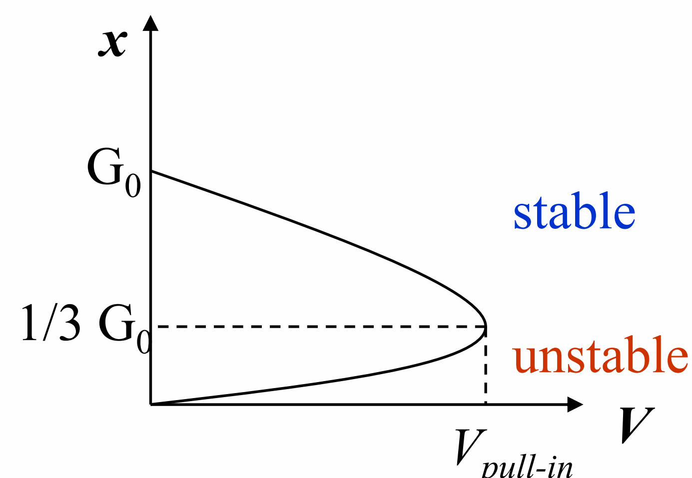
$$\begin{bmatrix}\dot{x}\\ \dot{v}\\ \dot{q}\end{bmatrix}=\begin{bmatrix}v\\ -2\zeta v-x+\frac{1}{3}q^2\\ -\frac{1}{r}q(1-x)\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \frac{2}{3r}\end{bmatrix}u$$

Normalized State-Space form of the dynamics

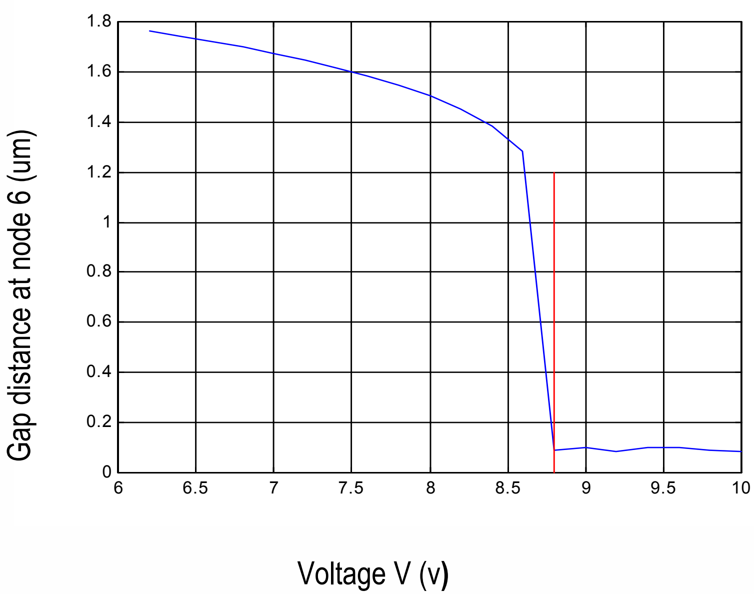
Some Nonlinear Control Scheme Might be Necessary!

1-D Simplified Analytical Solution

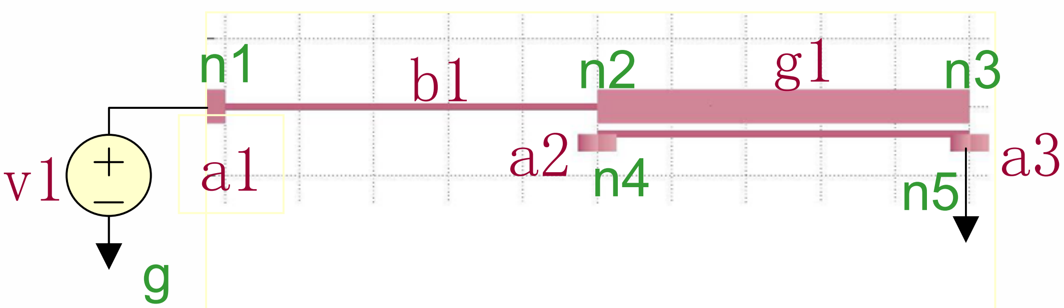
$$F=\frac{\varepsilon_0 V^2}{2(s_0-x)^2}=kx \Rightarrow V_{pull-in}=\sqrt{\frac{8kG_0^3}{27A\varepsilon_0}}$$

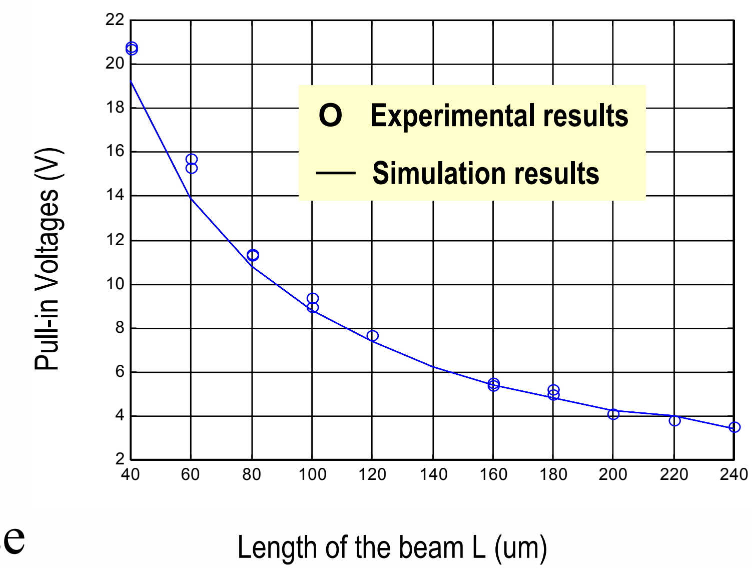


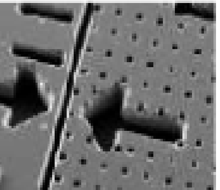
Simulation of Simplified Model



- Beam: Nominal Lb=100um, w=2um, h=2um. Measured : L=100um, w=1.74um, h=2.003um (Error of 0.13%)
- Gap plate: Lg=100um, w=10um, h=2.003um.
- Young's Modulus: assume 165GPa.
- Fringing Effects Considered
- Pull-in Voltage Predicted at 8.56V

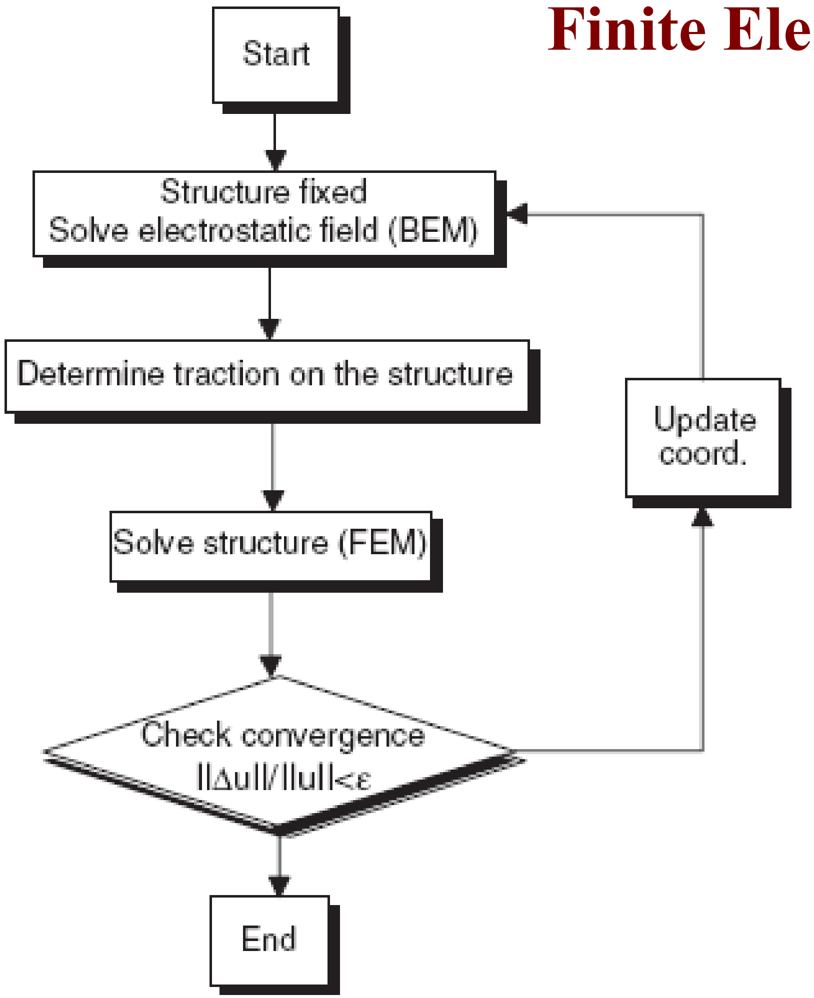


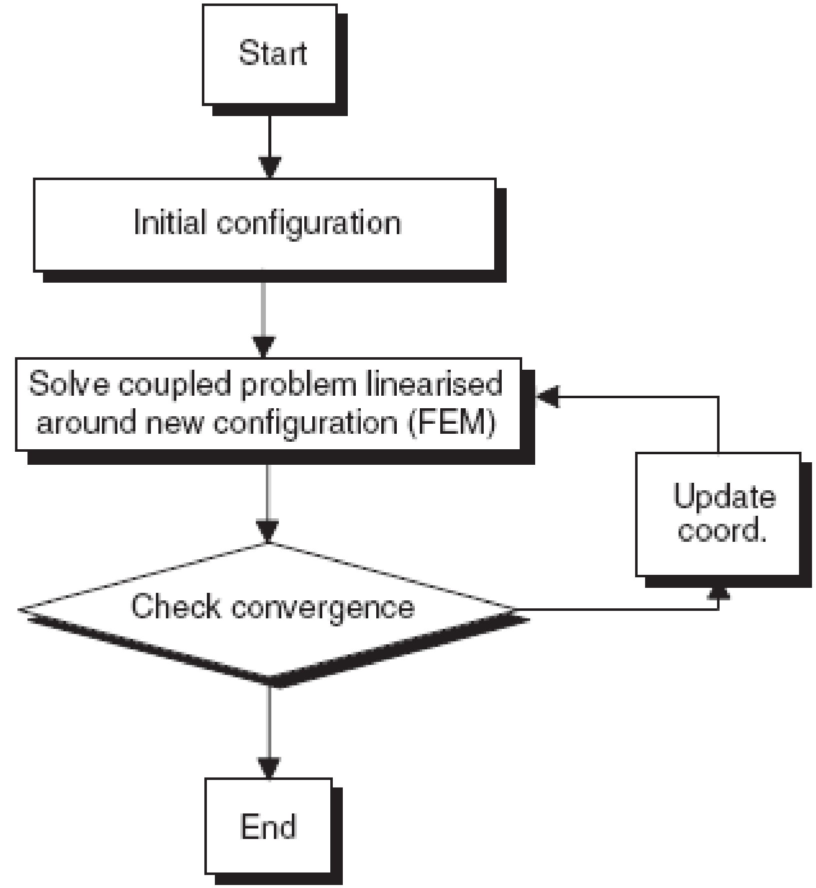




How to make this prediction more precise to include fringing effects, beam nonlinearity, and manufacturing error?

Finite Element Analysis





Conventional Week coupling procedure:

Iterative Finite Element Analysis in separate solves for different physics domain

Strong Coupling procedure:

Simultaneous analysis in one solver

Finite Element Analysis (Cont'd)

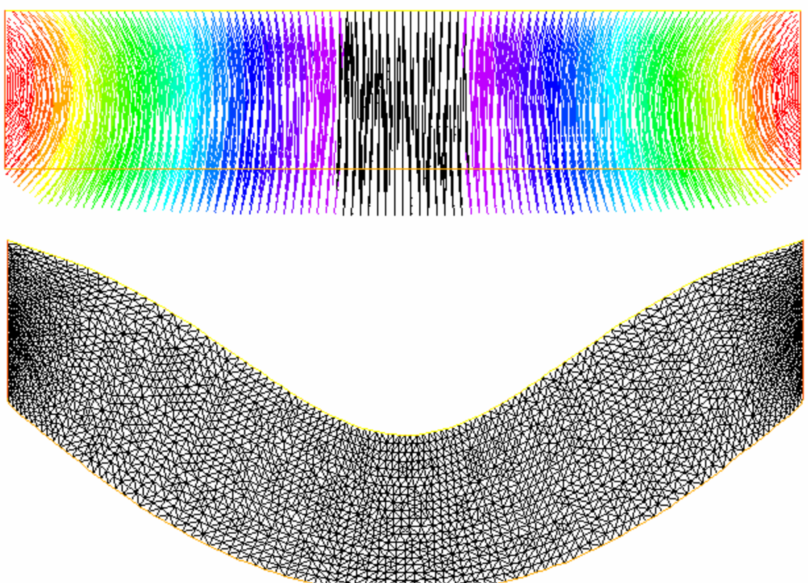
Discretization of Element:

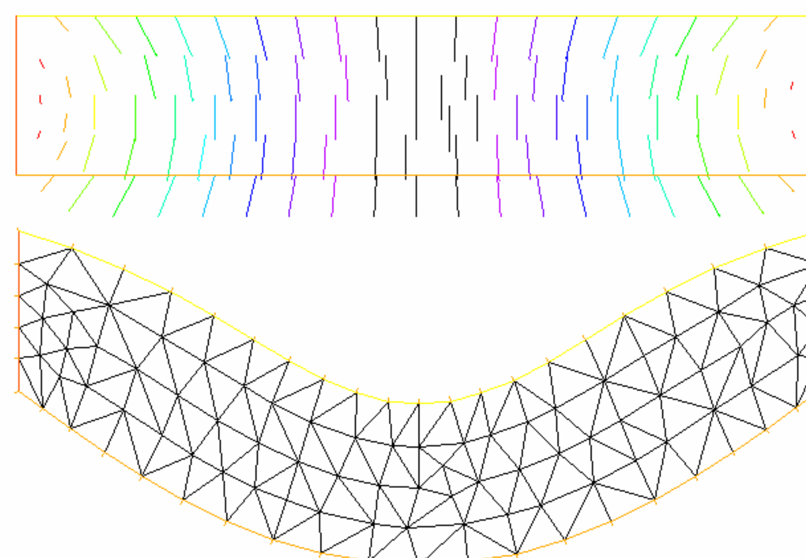
$$\hat{\mathbf{u}}(x, y) = \sum_i \beta_{U_i}(x, y) U_i = \boldsymbol{\beta}_U \mathbf{U}$$

$$\hat{\phi}(x, y) = \sum_i \beta_{\Phi i}(x, y) \Phi_i = \boldsymbol{\beta}_\Phi \boldsymbol{\Phi}$$

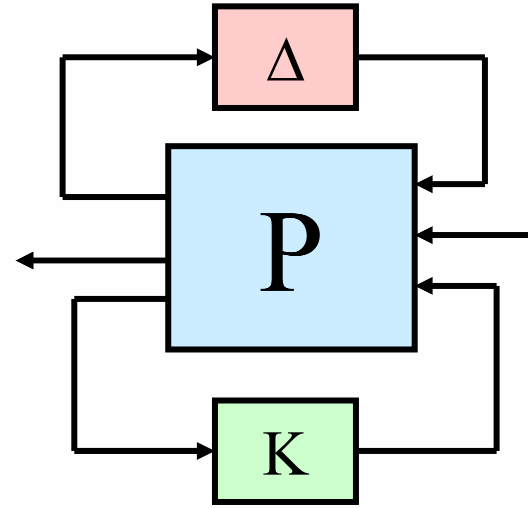
Monolithic Formulation:

$$\begin{pmatrix} \mathbf{K}_{UU} + \mathbf{K}^*_{UU} & \mathbf{K}_{\Phi U} \\ \mathbf{K}_{U\Phi} & \mathbf{K}_{\Phi\Phi} \end{pmatrix} \begin{pmatrix} \delta \mathbf{U} \\ \delta \boldsymbol{\Phi} \end{pmatrix} = \begin{pmatrix} \Delta \mathbf{F}_{\text{ext}} \\ \Delta \mathbf{Q}_{\text{ext}} \end{pmatrix}$$





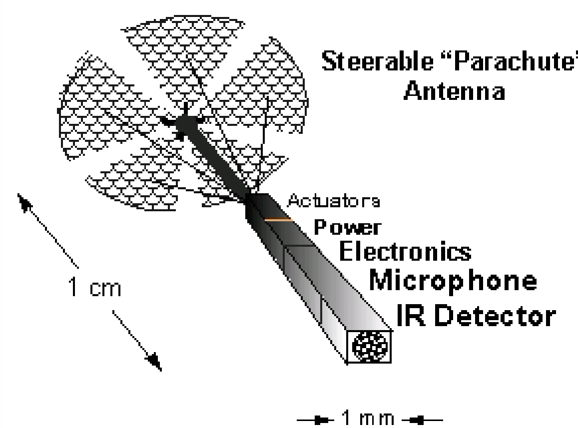
MEMS Control Challenges



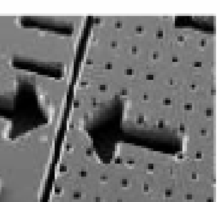
- Single Device Stability
  - Nonlinearity from the device
  - Ensure the device operate in stability
  - Manage the manufacture uncertainty and chaotic behavior

Multi-variable Robust Control

- Increasing Complexity of Device
  - Conventional model-based optimization process sensitive to modeling errors.
  - Learning-based approaches, such as neural network: hard to know the model.
  - Both model-based and learning-based

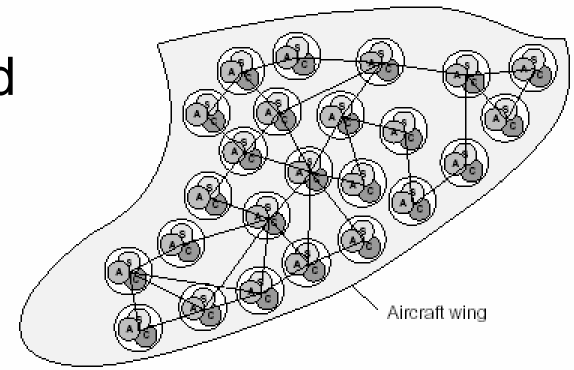


Model-based Neural Networks Control



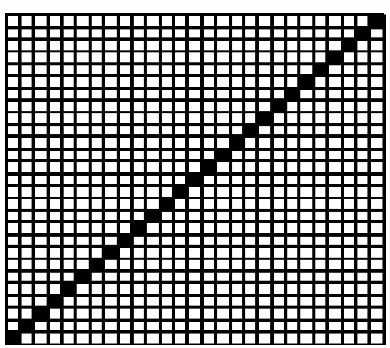
MEMS Control Challenges (Cont'd)

- Multi-device System Stability
  - Distributed nature of MEMS: many embedded sensors and actuators interaction
  - Need robust coordination: may elements can exhibit failures, delays and limited modeling ability in stochastic environment

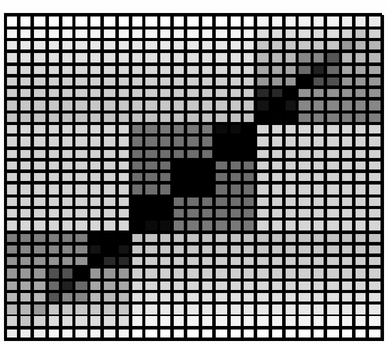


Distributed Market-based Control
(Game Theoretically Based Control)

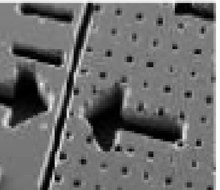
http://ho.seas.ucla.edu.



(local)

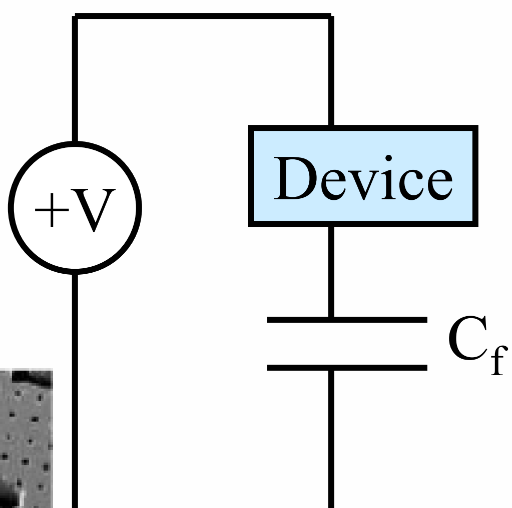


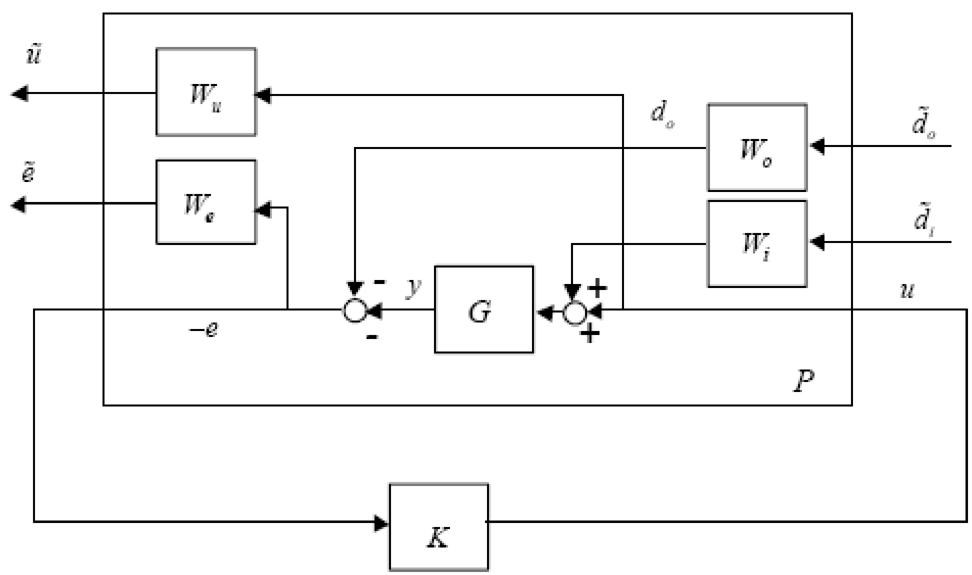
(hierarchy)

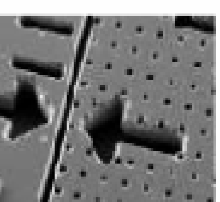


Multivariable Robust Control

- Present Control Design of the parallel-plate micro-actuator
  - Simple feedback with a capacitor may stabilize the system
- Proposed Robust Control Design of the benchmark problem
  - Provide more feedback control perspective for more sophisticated control
  - Directly deal with state-space of a control system
  - Deal with parametric uncertainty arising from manufacturing process
- Strong coupled finite element analysis for parameter identification







Conclusion

- Study of MEMS involves the multidisciplinary study of multi-physics problems. Finite element analysis is a flexible tool for solving physical quantities.

- Different levels of control problem exist, both on device level and system-network level. Intelligent schemes such as market-based control and neural networks are worthy of investigation

- We will establish a link between finite element analysis and control design, which will be used for parameter identification

- **Some Prospective Future Work**
  - Other control scheme:
    - Hybrid system optimal control
    - Neural Network based control
    - Game-theoretically based control
  - Adaptive finite element scheme
  - Nano-beam structure