MAT257 RSG 11

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1. This may have been brought to your attention when answering Q2 in the previous assignment or even when looking at the Partitions of Unity Theorem in general, but it's an interesting enough fact that I felt we should mention it at least: show that if A is any subset of $\mathbb{R}^n$ then any open cover of A has a finite subcover. (*Hint*: One way to show this is to use the Partitions of Unity Theorem).

2. Let $A = [1, 4]$. Let $\mathcal{U} = \{(0, 2), (1, 3), (2, 5)\}$. Find smooth functions ϕ_i for $i = 1, 2, 3$ so that they form a partition of unity for A subordinate to $\mathcal{U}$.

(Partitions of Unity Theorem)

3. Show that there exist smooth step functions, i.e. a smooth function $\theta : \mathbb{R} \to \mathbb{R}$ such that $\theta(x) = 0$ for $x < 0$ and $\theta(x) = 1$ for $x > 1$.

4. Let $U \subset \mathbb{R}^n$ be an open set and C a compact set contained in it. Show there exists a smooth $\psi : \mathbb{R}^n \to [0, 1]$ that is 1 on C and whose support is contained in U.

5. Let A be a compact set and $\mathcal{U} = \{U_i\}$ some (finite) open cover of it. Show that for every $i = 1, \ldots, p$ there exists a compact set $D_i \subset U_i$ such that $A \subset \bigcup_{i=1}^{p} \text{int} D_i$.

6. Use the previous questions to show there exists a partition of unity for compact sets.