1. We claim that if $C \subset \mathbb{R}^n$ is closed then $\text{int}(\text{Bd}(C)) = \phi$.

   Recall that $\text{Bd}(C) = \overline{C} - \text{int}(C)$. Since C is closed $\overline{C} = C$. Then $\text{Bd}(C) = C - \text{int}(C)$. Then if $x \in \text{int}(\text{Bd}(C))$, then there would have to be an open U that contains x and is contained in $\text{Bd}(C)$. But then this U would be contained in C so $x \in \text{int}(C)$ leading to a contradiction.

   Then $\text{Bd}(\text{Bd}(\text{Bd}(A))) = \text{Bd}(\text{Bd}(A)) - \text{int}(\text{Bd}(\text{Bd}(A)))$. Since the boundary of a set is always closed, we use the above fact.

2. This problem immediately reduces down to finding what conditions for A guarantee that $\text{int}(\text{Bd}(A)) = \phi$. I really want to say that this is true exactly when $\text{Bd}(A) \neq \mathbb{R}^n$ (i.e. when A is not dense) but have not been able to prove this yet.

3. We know that if $|D_i f_j(a)| \leq M$, then $|f(x) - f(y)| \leq nmM|x - y|$. We just take $M = 0$.

4. We define $h : \mathbb{R} \to \mathbb{R}^2$, given by $h(x) = (f(x), g(x))$ and $p : \mathbb{R}^2 \to \mathbb{R}, p(x, y) = xy$. Then

$$\begin{aligned}(p \circ h)'(x) &= p'(h(x))h'(x) \\ &= \begin{pmatrix} g(x) & f(x) \end{pmatrix} \cdot \begin{pmatrix} f'(x) \\ g'(x) \end{pmatrix} \\ &= f'(x)g(x) + f(x)g'(x)\end{aligned}$$

5. By assumption if f is tiny then given any $\epsilon > 0$, we can find some $\delta' > 0$ such that $|h| < \delta$ implies that $|f(h)| < \epsilon|h|$. We can then take our δ from the $\delta - \epsilon$ definition of continuity to be $\delta = \min\{\delta, 1\}$. This implies that $|f(h)| < \epsilon|h| \leq \epsilon$.

   The identity map on $\mathbb{R}^n$ is such a map. This question is just meant to illustrate that being 'tiny' is a stronger condition than being continuous.

6. We simply note that

$$\min(a, b) = \frac{a + b - |a - b|}{2}$$

   The minimum function on higher dimensions is simply the composition min with itself, hence is continuous. I am fairly sure that $\min : \mathbb{R}^2 \to \mathbb{R}$ is differentiable everywhere except the diagonal.