

1. Any non-empty open will contain closed rectangles which we know are not of measure zero.
2. Nope, take A to be irrationals.
3. Nope, take projection of any straight line in the plane. For the pre-image, we can take any constant function as a counter example. For domain and codomain being of the same dimension we can take the Hilbert Curve as a counter example (I think).
4. Let U be some open set in \mathbb{R}^n . If U does not intersect A^c then $U \subset A$ but this implies that A is not of measure zero. For the counterexample for the converse we take $A = \mathbb{R} - \mathbb{Q}$.
5. Define $h = f - g$. Then the set of points where f and g differ is $h^{-1}(\mathbb{R}^m - \{0\})$ which is open as h is continuous. By assumption it is non-empty hence is not of measure zero.
6. The fat Cantor set has an empty interior as it does not contain any intervals therefore must be equal to its boundary. Doing the computation, we find that the total length of intervals removed is $\frac{1}{2}$. Hence the set cannot be of measure zero.